COMPENSATION OF BEND-PLANE EMITTANCE GROWTH IN A 180 DEGREE BEND *

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Abstract

Emittance preservation in beam bending systems is vitally important in the production of bright, high-current electron microbunches. Generally, the emittance increase occurs in the bend plane and results from changes in the microbunch energy distribution as the beam transits the bend. This redistribution of electron energies increases the beam's divergence, and hence the emittance, by spoiling the achromatic transport of the bending system. In this paper we investigate the correlated emittance growth in a 180 degree isochronous bend due to coherent synchrotron radiation (CSR). Introducing sextupole fields in the high dispersion region of the bend partially cancels the CSR-induced correlation thereby reducing the bend plane emittance growth. The generalization of this emittance compensation scheme is discussed.

1 INTRODUCTION

Emittance growth in beam transport sytems typically occurs in the plane of the bend and is due to changes in the beam microbunch energy distribution. While there are many effects capable of producing this redistribution, the most common is the space charge force which tends to increase the microbunch head and tail energies relative to that of the center, thereby correlating an angle change with longitudinal position in the microbunch. The other mechanisms which can redistribute the electron energies are wake fields and CSR. In a sense, these three phenomena are similar in their effects upon the beam in a bend. They all change the energy distribution of the microbunch, which leads to a combination of both correlated and uncorrelated emittance growth. This work concentrates upon CSR-induced emittance, but the basic principles can be applied to any combination of these three effects.

2 COHERENT SYNCHROTRON RADIATION INDUCED EMITTANCE GROWTH

CSR occurs when the bending of a relativistic electron beam allows the synchrotron radiation emitted by the tail of the microbunch to "catch up" with the head electrons. If the arc length of the bend is long enough, this radiation sweeps along the entire length of the microbunch and transfers energy from the tail to the head. Therefore CSR tends to increase the energy of the head while lowering that of the tail[1].

The transport of the microbunch through a multi-magnet bending system is computed using ray tracing with the first order matrix formalism of TRANSPORT [2] in combination with an energy redistribution using CSR formulae. In this procedure, one defines an initial four dimensional phase space distribution consisting of the bend-plane transverse coordinates, x and θ, and the longitudinal coordinates, z and $\Delta E/E$. The non-bend plane degrees of freedom are ignored. Each macroparticle is incremented through the bending system with each step from location 0 to location 1 consisting of first the linear matrix transformation,

$$\begin{pmatrix} x_i(1) \\ \theta_i(1) \\ z_i(1) \\ \frac{\Delta E}{E}_i(1) \end{pmatrix} = R(\text{stepsize}) \begin{pmatrix} x_i(0) \\ \theta_i(0) \\ z_i(0) \\ \frac{\Delta E}{E}_i(0) \end{pmatrix}, \qquad [1]$$

followed by the CSR-produced change in the energy of the i-th macroparticle as given by,

$$\left.\frac{\Delta E}{E}\right|_i(1) = \left.\frac{dE(z_i(0))}{cdt}\right|_{CSR} * \text{stepsize} * \frac{1}{E_{beam}} + \left.\frac{\Delta E}{E}\right|_i(0)\,. \qquad [2]$$

Here R(stepsize) is the 4x4 TRANSPORT transformation matrix which propagates the electrons the distance equal to stepsize through the bend. The CSR energy change of each macroparticle, $\left.\frac{dE(z_i(0))}{cdt}\right|_{CSR}$, is evaluated at the z-location of the i-th macroparticle and the fractional energy change added to its current energy fraction. The details for computing CSR energy loss gradient are given in Reference [3].

3 THE REDUCTION OF CSR-INDUCED, CORRELATED EMITTANCE

The energy redistribution impressed upon the microbunch by phenomena such as CSR, leads to a transverse angle vs longitudinal position (θ-z) correlation at the end of the bending system[3]. In order to understand the emittance compensation process, consider an effect similar to CSR, but which increases quadratically both the head and tail energies relative to the microbunch center. Such an effect leads to an final angle change along the microbunch given by,

$$\theta(z) = B(E(z) - E(z=0))^2 \qquad [3]$$

Here B includes all the details of the bending system as well as the strength of the hypothetical quadratic force.

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Introducing a sextupole field in the bend where the energy dispersion is a maximum adds a second term to the equation

$$\theta(z) = B(E(z) - E(z=0))^2 + T_{211}(x(z) - x(z=0))^2 \quad [4]$$

Given that the transverse position of the each electron is well-correlated with the energy deviation impressed upon it by the quadratic force, then it is possible to cancel any correlated emittance growth with a sextupole field. In principle, it should be possible to compensate for more complicated correlations by introducing octupole and higher-order fields.

4 EMITTANCE GROWTH AND COMPENSATION IN AN ISOCHRONOUS 180 DEGREE BEND

4.1 Description of the Dundee Bend

As an example for emittance compensation we choose the 180 degree bend capable of adjustable non-isochronism proposed by Gillespie [4], hereafter referred to as the Dundee bend. An isochronous version of the bend is shown in Figure 1, and consists of three pairs of quadrupoles and sextupoles. The quadrupole strengths and magnet spacings are adjusted to obtain a nearly diagonal first-order unity matrix with the exception of R_{12} which is 96.3 cm/mrad for the entire bending system. The transverse dispersion term, R_{16}, peaks at the Q2 quadrupoles and the S2 sextupoles. In Gillespie's orginal design, the sextupoles are used to minimize second-order chromatic terms such as T_{516} and T_{566} which distort the longitudinal shape of the microbunch. Here the S2 sextupole pair are used for emittance compensation.

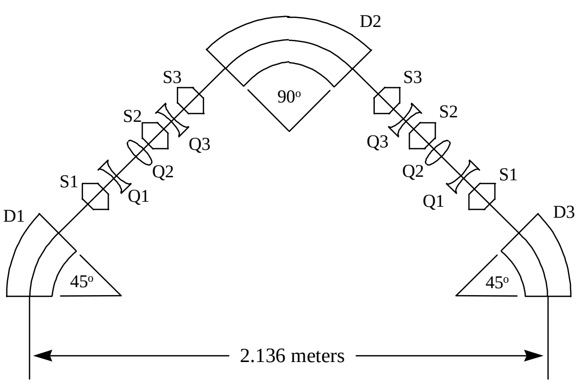

Figure 1. The isochronous 180 degree Dundee bend used to compute CSR emittance growth and emittance compensation.

4.2 Emittance Growth in the Dundee Bend Due to CSR.

The calculation assumes the beam is initially at 100 MeV with a rms emittance of 1 π mm-mrad, a rms microbunch length of 1.4 mm, negligible energy spread and a microbunch charge of 5 nC. This corresponds to a peak current of approximately 500 amperes. The final, CSR-induced emittance is 8 π mm-mrad. Figure 2 gives the transverse phase space and Figure 3 shows the θ-z correlation at the exit of D3 for these conditions.

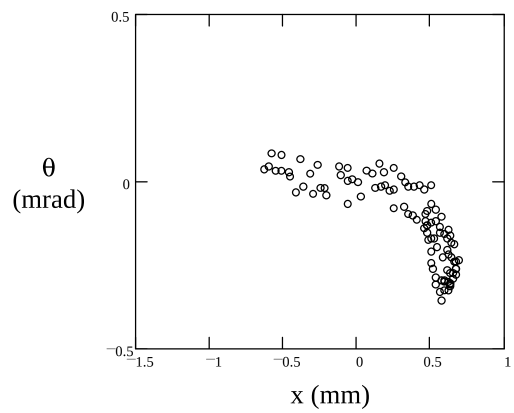

Figure 2. The transverse phase space at the exit of the Dundee bend for sextupoles $S2_{tip}$ =0 KG.

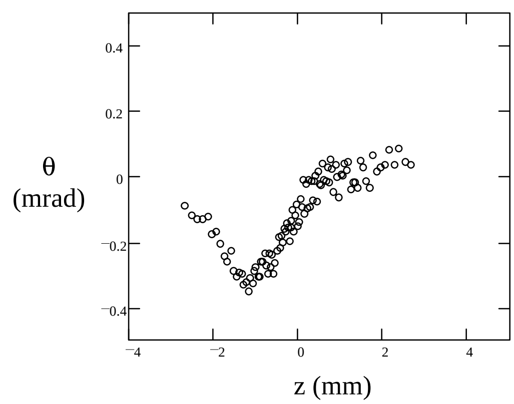

Figure 3. The correlation between beam divergence and longitudinal position in the microbunch for sextupoles $S2_{tip}$ =0 KG.

A comparison of these two figures shows that a significant portion of the emittance growth is correlated. That is, the tail electrons which are at lower energies exit the bend at smaller angles compared to those at the head. The electrons near z = -1 mm are approximately 0.5% lower in energy than the head electrons.

4.3 Compensated Emittance in the Dundee Bend

The final emittance as a function of S2 sextupole tip field from 0 to 8 KG upon is shown in Figure 4. The minimum emittance of 4.5 π mm-mrad occurs at 4 KG (aperture diameter = 25 mm and effective length = 5 cm).

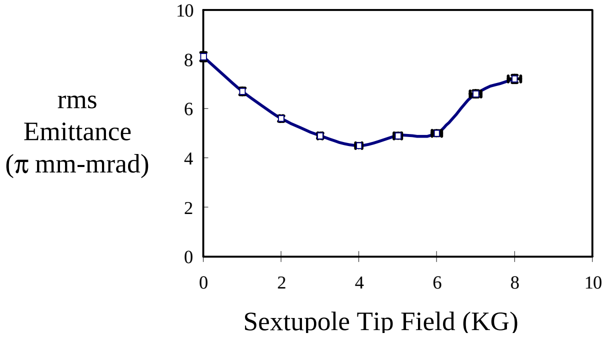

Figure 4. Final emittance as a function of S2 sextupole field.

The microbunch transverse phase space and θ-z correlation are plotted in Figures 5 and 6 for $S2_{tip}$ = 4 KG. The sextupole removes the half of the CSR-induced correlated emittance growth associated with the quadratic correlation. The remaining portion appears to include higher-order correlations which may require the addition of octupole fields. However, the reduction of the emittance growth by a factor of two illustrates the benefit of this correction scheme.

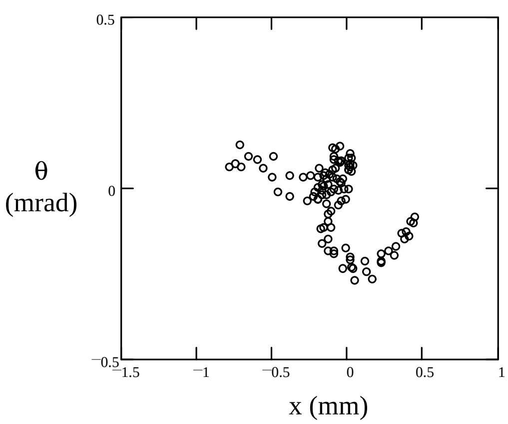

Figure 5. Transverse phase space at the exit of the bend for a S2 sextupole tip field of 4 KG.

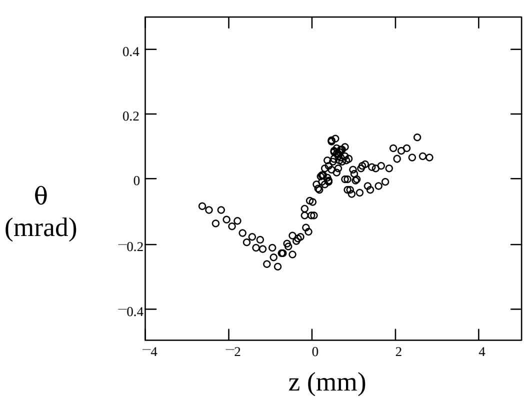

Figure 6. The correlation between beam divergence and longitudinal position in the microbunch for S2 at 4 KG.

5 DISCUSSION

The energy distributions produced by space charge forces, wakefields and CSR in a bending system typically lead to correlations between electron angle and longitudinal position in the microbunch. The result is a component of the emittance growth in bends which is correlated to various orders of the z-position in the microbunch. This work shows it is possible to cancel the quadratic piece of the correlation by introducing a sextupole field in a high dispersion section of the bend. In the case considered here, the CSR-induced emittance growth is reduced by a factor of two.

The next step involves generalizing Equation [4] to include higher-order correlations, i.e.,

$$\theta(z) = \sum_{j=2}^{n}[B_j(E(z) - E(z=0))^j - S_j(x(z) - x(z=0))^j]$$

where B_j result from the distorting forces and the bend transport, and S_j are the strengths of the multipole fields introduced in the high-dispersion section of the bend. As long as there is a strong correlation between the energy and beam size, it should be possible to cancel any correlated emittance growth term by term.

A final comment: while in these calculations the pair of S2 sextupoles were coupled and varied together, only the one in the last half of the bend affected the emittance. This is because there is significantly less CSR-induced energy spread at the first S2 location compared to the second S2. Therefore additional flexibility in this compensation scheme is possible by deliberately programming an energy slew along the length of the microbunch. This energy-z correlation can then be used by multipoles at the high dispersion position in the first half of the bend to correct for θ-z correlations induced later in the bend by phenomena such as CSR. This should be most useful in microbunch compressors where there is already an energy slew present to bunch the beam.

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