TRANSVERSE INSTABILITY IN HIGH INTENSITY PROTON RINGS*

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Abstract

Most high intensity proton rings are at low energy below transition. Several aspects of the beam dynamics of this kind of rings are different from the electron or high energy rings. The transverse microwave instabilities will be discussed in this article.

1 INTRODUCTION

In recent years, many applications are being considered for low energy high intensity proton synchrotrons, see for example[1]. This kind of rings are different from the electron or high energy proton rings in several aspects of beam dynamics. The transverse microwave instability is the subject to be discussed in this report.

The transverse beam dynamic equation will be presented, where it is indicated that among several factors responsible for the instabilities, the most concerned issue is the impedance, especially the transverse space charge impedance.

It will be shown that the conventional transverse space charge impedance is related to the difference of the space charge coherent and incoherent tune shifts. The space charge incoherent tune spread is an important stabilizing force for the transverse microwave instabilities. Thus, the transverse space charge impedance is relevant to both coherent motion and the tune spread for the Landau damping.

This scenario dominates the transverse beam dynamics for the low energy proton rings. Many important issues in the beam dynamics for the electron and high energy machines become secondary or even negligible effects. On the other hand, for high intensity rings, the space charge incoherent tune spread has to be limited, which is likely to affect the stability margin.

2 BEAM DYNAMIC EQUATION

Consider the transverse bunched beam dynamic equation with the azimuthal mode $m = 0$[2],

$$\omega - \omega_\beta = \frac{j\beta e I_0}{2Rm_0\gamma\nu_0\omega_0} \sum_{n=-\infty}^{\infty} Z_T(n)\Lambda_0^2(n') \qquad (1)$$

where ω_β and ω_0 are the betatron and revolution frequencies, respectively, I_0 is the average beam current, R is the machine average radius, and m_0 is the rest mass of proton. Z_T is the transverse impedance, and Λ_0 is the spectrum of the beam line density for $m = 0$ mode. The notation n' denotes the beam spectrum frequency shift due to the chromatic effect.

In (1), the beam line density spectrum, the effective spectrum lines, the impedance, and the chromatic effect for bunched beams are relevant to the beam instabilities.

For high intensity proton rings, the bunch has to be long to reduce the space charge effect. Therefore, the beam line density spectrum will be narrow, approaching the situation of coasting beams. The effective spectrum lines will be few. In other words, the coasting beam criterion will be more relevant to the transverse instabilities. Now it appears that the most concerned issue in the beam instabilities is the impedance, especially the transverse space charge impedance.

3 TRANSVERSE SPACE CHARGE IMPEDANCE

The transverse space charge impedance is conventionally defined as[3],

$$Z_{TSC} = j\frac{RZ_0}{\beta^2\gamma^2}\left(\frac{1}{b^2} - \frac{1}{a^2}\right) \qquad (2)$$

where Z_0 is the impedance of free space, a and b are the average radius of the beam and the average half chamber height, respectively.

For coasting symmetric beam with non-penetrating fields, the space charge incoherent and coherent tune shifts are defined as[4],

$$\Delta\nu_{inc} = \frac{-NRr_0}{\pi\nu_0\beta^2\gamma}\left(\frac{\epsilon_1}{b^2} + \beta^2\frac{\epsilon_2}{g^2} + \frac{1}{2a^2\gamma^2}\right) \qquad (3)$$

and

$$\Delta\nu_{coh} = \frac{-NRr_0}{\pi\nu_0\beta^2\gamma}\left(\beta^2\frac{\epsilon_1}{b^2} + \beta^2\frac{\epsilon_2}{g^2} + \frac{\xi_1}{b^2\gamma^2}\right) \qquad (4)$$

where N is the total number of particles, r_0 is the classical radius of proton, ν_0 is the betatron tune with zero beam current, g is the half pole gap, and ϵ_1 and ϵ_2 are the Laslett incoherent electric and magnetic coefficients, respectively. The coefficient ξ_1 is the Laslett coherent electric coefficient.

For the simplified model, we consider circular chamber, which gives rise to,

$$\epsilon_1 = \epsilon_2 = 0, \ \xi_1 = 0.5 \qquad (5)$$

then the incoherent and coherent tune shifts become,

$$\Delta\nu_{inc} = \frac{-NRr_0}{2\pi\nu_0\beta^2\gamma^3 a^2} \qquad (6)$$

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and

$$\Delta\nu_{coh} = \frac{-NRr_0}{2\pi\nu_0\beta^2\gamma^3b^2} \tag{7}$$

For low energy synchrotrons, since γ is small, the simplification (5) is approximately right even the chamber is not circular, see (3) or (4). For bunched beams, we take a simplified approach, by adding the bunching factor B_f to the denominators of Eqs. (6) and (7).

In the following, we show that,

- The tune shifts shown in (6) and (7) can be obtained by substituting a proper part of the space charge impedance into the dynamic equation (1).
- The transverse space charge impedance represents the difference between the coherent and incoherent tune shifts.
- The incoherent tune shift will be cancelled in the dynamic equation, and therefore, it pays no role in the coherent motion.

3.1 Impedance and dynamic equation

First, we take the chamber part of the impedance (2),

$$Z_T = j\frac{RZ_0}{\beta^2\gamma^2}\frac{1}{b^2} \tag{8}$$

For coasting beams, the beam power spectrum is a delta function, with the amplitude $1/2\pi$ [2],

$$\Lambda_0^2(n) = \frac{1}{2\pi}\delta(n) \tag{9}$$

Thus, the summation in (1) is removed.

Now we use $I_0 = Ne\omega_0/2\pi$, and $Z_0 = 1/\epsilon_0 c$, where ϵ_0 is the permittivity in free space. Also using

$$r_0 = \frac{1}{4\pi\epsilon_0}\frac{e^2}{m_0c^2} \tag{10}$$

and $\omega_0 = \beta c/R$, then substituting (8) into (1), we get exactly the space charge coherent tune shift shown in (7). In other words, the impedance of (8) represents the space charge coherent tune shift. Similarly, the beam part of the impedance (2) is relevant to the space charge incoherent tune shift.

3.2 Transverse space charge impedance

Now substituting the transverse space charge impedance (2) into the dynamic Eq. (1), we have,

$$\omega - \omega_\beta = \Delta\nu_{coh}\omega_0 - \Delta\nu_{inc}\omega_0 \tag{11}$$

i.e., the transverse space charge impedance represents the difference between the coherent and incoherent tune shifts.

The impedance (2) is defined based on the deflecting electromagnetic fields distributed between the beam and the perfectly conducting wall[5]. Exactly what it means to the beam motion has to come from the Eq. (11).

Specifically, the transverse space charge impedance represents neither coherent nor incoherent tune shift. In the case that $a \ll b$, the implied tune shift will be approximately equal to the incoherent tune shift. However, this tune shift is increased, while the space charge incoherent tune shift should be decreased.

3.3 Incoherent tune shift and coherent motion

Writing on the left side of the Eq. (1) by the following convention ,

$$\omega_\beta = \omega_{\beta 0} + \Delta\nu_{inc}\omega_0 \tag{12}$$

and also using (11), the Eq. (1) becomes,

$$\omega - \omega_{\beta 0} - \Delta\nu_{inc}\omega_0 = \Delta\nu_{coh}\omega_0 - \Delta\nu_{inc}\omega_0 \tag{13}$$

where the incoherent tune shift is cancelled. This shows that the incoherent tune shift plays no role in the transverse coherent motion. Therefore, the beam part of the transverse space charge impedance, i.e., the contribution of $1/a^2$, is a redundancy in the dynamic equation.

The writing of (12) is following the longitudinal case, where one has to write the synchrotron oscillation frequency in the way of $\omega_S = \omega_{S0} + \Delta\nu_{S,inc}\omega_0$, because the incoherent frequency shift affects the longitudinal focusing, which is often called the potential well effect. In the transverse case, the similar effect is negligible. This is one of the fundamental differences between the transverse and longitudinal beam dynamics.

4 TRANSVERSE LANDAU DAMPING

For long bunches, the power spectrum of the perturbation can be a delta function $\delta(n - n_1)/2\pi$, where the spectrum line n_1 represents the frequency $(n_1 + \nu_0)\omega_0$, because only the perturbation at these frequencies has a chance to grow.

Also substituting the beam peak current I_p for the average current I_0, the Eq. (1) becomes,

$$\omega - \omega_\beta = \frac{j\beta eI_p}{4\pi Rm_0\gamma\nu_0\omega_0}Z_T(n_1) \tag{14}$$

To proceed further, we write the left side of the Eq. (14) as the frequency spread $\Delta\omega = \Delta\nu\omega_0$, which will be responsible for the Landau damping.

Note that the Landau damping has two implications.

- If the impedance is real and positive, the system is stable and the Landau damping is not needed. If it is negative, then the frequency spread must be larger than the growth rate to suppress the instability.
- If the impedance is imaginary, then the frequency spread on the left side must be larger than the coherent frequency shift on the right side of (14). Otherwise, an infinitesimal perturbation may cause instability.

The microwave instability criterion is, therefore, obtained as follows,

$$\Delta\nu > \frac{\beta eI_p}{4\pi Rm_0\gamma\omega_\beta\omega_0}\,|Z_T(n_1)| \tag{15}$$

It remains to clarify the sources responsible for the incoherent and coherent tune shifts.

4.1 Incoherent tune spread

For the incoherent tune spread, we consider the following sources.

- Space charge incoherent tune spread, which is the largest stabilizing force for the low energy proton synchrotrons. For the high energy machine, the tune spread is decreased, and its contribution diminishes. This is one of the reasons that the transverse instabilities is more critical for the high energy machines.

- Chromatic tune spread. For bunched beams, the chromatic tune spread is not effective for the weak instabilities with the growth rate comparable to the synchrotron frequency. It is, however, effective for the strong instabilities.

- Frequency slippage. This tune spread could be cancelled by the chromatic tune spread, then the trick is to let the cancellation happen at a stable frequency region.

- Octupolar tune spread. This tune spread is betatron oscillation amplitude dependent.

- Finally, the synchrotron oscillation may help. Conventionally, this contribution is estimated as $\Delta\omega \approx \omega_S = \Delta\nu_S\omega_0$.

The combined tune spread can, therefore, be written for the effective frequency $(n_1 + \nu_0)\omega_0$ as,

$$\Delta\omega = ((n_1+\nu_0)\eta - \xi\nu_0)\frac{\Delta p}{p} + \Delta\nu_{inc} + \Delta\nu_{oct} + \Delta\nu_S)\omega_0 \tag{16}$$

where $\Delta p/p$ is the beam momentum spread.

4.2 Coherent tune shift

The coherent tune shifts simply come from mainly two sources.

- Space charge coherent tune shift.

- Broad band impedance induced tune shift.

It can be observed that if the conventional transverse space charge impedance (2) is used, then it is relevant to both incoherent and coherent tune shifts, and it takes effect on both sides of the Eq. (15).

However, the transverse microwave instability can be estimated by taking the sum of the transverse space charge impedance with the broad band impedances, such as in[6]. If the sum is negative, then the incoherent tune spread is dominant, and the system is stable. Otherwise, the space charge incoherent tune spread is not large enough to stabilize the system by itself. It has been shown that this approach is valid for low energy rings. For high energy rings, the image effect is often stronger than the direct effect, however, the image incoherent effect is approximately cancelled with the coherent effect. Therefore, the approach is also valid.

5 HIGH INTENSITY PROTON RINGS

Most high intensity proton rings are at low energy below the transition. At low energy, the space charge incoherent tune spread is relatively large. At high intensity, the bunch has to be long to reduce the space charge tune spread. Also to eliminate the longitudinal microwave instability, the beam momentum spread will be relatively large. Thus, the chromaticity has to be corrected, and the machine is likely to work at a region with a slightly negative chromaticity. Therefore, the high intensity synchrotrons are different from the electron or high energy proton machines in terms of beam instabilities.

The issues relevant to the transverse instability of high intensity proton rings are summarized as follows.

- The space charge incoherent tune spread is an important stabilizing force for the transverse microwave instabilities. The transverse mode coupling will not happen for the low energy machines, because the space charge incoherent tune spread will be larger than the synchrotron tune.

- The coherent tune shift comes from the broad band impedance and the chamber part of the space charge impedance, both of which are small for low energy rings, because of the large chamber height and the small ring radius. On the other hand, for high intensity rings, the space charge tune spread has to be reduced as much as one can, therefore, the stability margin will be affected.

- The chromatic tune spread and the frequency slippage effect are relatively small. The cancellation of these effects, of interest for high energy machines, is not much concerned. Also the higher order mode, such as $m = 1$ mode, is not important.

- The synchrotron oscillation can eliminate the damping effect of the space charge incoherent tune spread for the transverse rigid bunch instabilities, such as the resistive wall instability. Careful study, or correction, will be needed.

6 REFERENCES

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