# EVALUATON OF "ROUND COLLIDING BEAMS" FOR TEVATRON

V.V. Danilov, Budker Institute of Nuclear Physics, 630090, Novosibirsk, Russia,
V.D. Shiltsev, FNAL* ,P.O. Box 500, Batavia, Illinois 60510

*Abstract*

This paper presents investigation of the proposed use of round beams for increasing the luminosity in colliders. The main idea of round beams is briefly discussed. Numerical simulations of round colliding beams for the Tevatron are much in favor of round beams, because they provide reduction of harmful impact of beam-beam forces on beam sizes, particles diffusion and better stability with respect to errors and imperfections.

## 1 INTRODUCTION

The essential conditions of the round beams [1] are equality of horizontal and vertical emittances $\varepsilon_x = \varepsilon_y = \varepsilon$, beta functions at interaction point (IP) $\beta_x = \beta_y = \beta$, and tunes $\nu_x = \nu_y = \nu$. Consequently, the transformation matrix in between of IP's can be generally presented in the form of

$$R(\phi) \cdot \begin{pmatrix} T & 0 \\ 0 & T \end{pmatrix}$$

(where T is a $2 \times 2$ matrix with $detT = 1$ and R is the matrix of rotation over an angle $\phi$ ), therefore, the rotational symmetry of the kick from the round opposite beam, complemented with the $X - Y$ symmetry of the betatron transfer matrix between the collisions, result in an additional integral of motion $\mathcal{M} = xy' - yx'$ that is longitudinal component of the angular momentum. Thus, the transverse motion becomes equivalent to a one dimensional (1D) motion. Resulting elimination of all betatron coupling resonances is of crucial importance, since they are believed to cause the beam-lifetime degradation and blow-up. The reduction to 1D motion makes impossible the diffusion through invariant circles. Moreover, the beam-beam parameter for the round beams $\xi_{x,y} = \frac{Nr_0}{4\pi\gamma\varepsilon}$, does not depend on $s$ because the emittance $\varepsilon = \sigma^2/\beta$ is independent of the longitudinal coordinate. This leads to suppression of synchrobetatron resonances (one can find more detailed discussion of these questions in [2]).

One can expect, that for hadron colliders, where the beams are almost round from the beginning, the most useful predicted properties of the Round Colliding Beams (RCBs) lead to their better stability, lower losses and longer beam lifetime.

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## 2 BEAM-BEAM SIMULATIONS WITH ROUND BEAMS IN TEVATRON

### *2.1 Beam-beam simulation code and parameters of the Tevatron upgrade*

We employ a recently developed beam-beam simulation code BBC Ver.3.3 [3] developed by K.Hirata for the beam-beam interaction in "weak-strong regime" which is close to conditions of the Tevatron collider upgrade named TEV33 [4] where proton bunch population is several times the antiproton one. The "weak" (antiproton) bunch was presented by number of test particles, while the "strong" (proton) bunch appeared as an external force of Gaussian bunch.

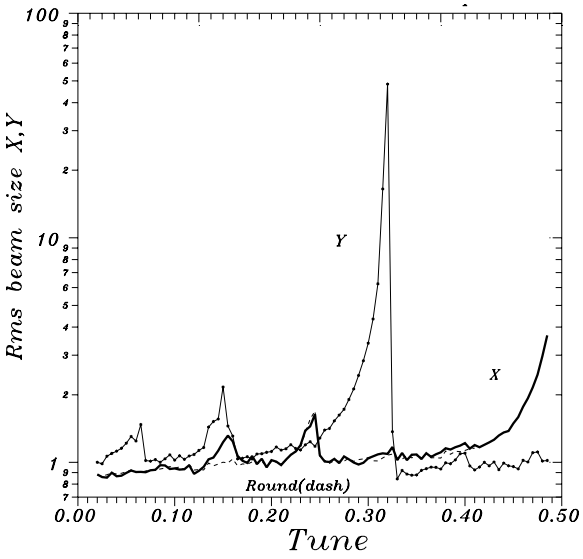


Figure 1: The rms beam size $\sigma/\sigma_0$ vs betatron tune $\nu_y = \nu_x = \nu$ for the round beams (dashed line), and the rms horizontal and vertical sizes $\sigma_{x,y}/\sigma_{0\,x,y}$ for non-round beams (solid and marked lines, respectively). ($\xi = 0.05$, $\Delta\vartheta = 0.002$, 50,000 turns).

Typically we tracked 100 (maximum 1000) test particles through five slices of strong bunch for (50-100)·$10^3$ turns. 50,000 turns in Tevatron correspond to about 1 s, some 200 synchrotron oscillation periods. No damping due to radiation or cooling is assumed to play role in the beam dynamics. Further increase of the number of particles or number of slices gave almost identical results.

The code outputs of greatest practical utility are luminosity, rms beam sizes and maximum betatron amplitudes which any of the test particles attained during tracking. These outputs are given with respect to unperturbed values, e.g. sizes and amplitudes are divided by their design rms values $\sigma_{x,y}/\sigma^0_{x,y}$ and $A^{max}_{x,y}/\sigma^0_{x,y}$, the luminosity is pre-

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sented by the reduction factor of $R = L/L_0$ where the bare design luminosity $L_0 = f_0 N_p N_{\bar{p}}/(4\pi\sigma_x^0\sigma_y^0)$ and $f_0$ is the rate of collisions. The relevant parameters of the simulations were chosen close to the TEV33 design ones.

We present here the results for the RCB scheme without rotation of betatron oscillations axis, although other schemes proposed originally for electron damping rings require such rotation, i.e. strong $x - y$ coupling. The comparison of the different schemes is made partly in [2].

### *2.2 Comparison of RBs and non-RBs. Random tune modulation.*

In order to make more realistic simulations we use noisy betatron phases jumps. The reason is that the weak resonances of high orders are usually not well seen after a small number of revolutions and in order to enhance them we used a method of the Ornstein-Uhlenbeck tune modulation (see, for example [5]) with correlation time of 100 turns.

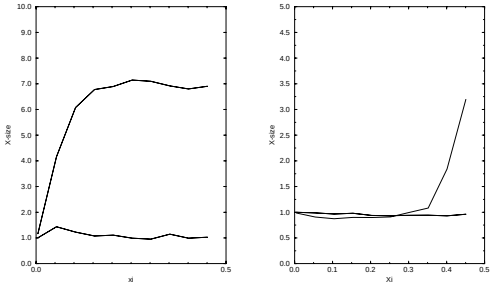

Figure 2: Beam size after 50,000 turns versus $\xi$. The upper curve corresponds to the short strong Gaussian bunch, the lower one — to the strong counter bunch with the "inverse beta-function" distribution. The beta-function at IP is 25 cm. **a** – left figure – tunes are equal to $\nu_x = \nu_y = -0.01$; **b** – right figure – tunes are equal to 0.05.

Now, with use of small noisy phase modulation (the parameter $\Delta\vartheta$ with the meaning of maximum changing of phase per turn in the Ornstein-Uhlenbeck process is equal to 0.002) , we compare the rms beam sizes after 50,000 turns for the round beams and the beams which are far from round. The colliding round beams satisfy to all the conditions:
$\varepsilon_x = \varepsilon_y = 3 \cdot 10^{-9}$ m· rad;
$\beta_x^* = \beta_y^* = 25$ cm; $\nu_x = \nu_y = \nu$,
while the "not-RBs" break them all:
$\varepsilon_x = 5/3\varepsilon_y = 5 \cdot 10^{-9}$ m· rad;
$\beta_x^* = 35/25\beta_y^* = 35$ cm; $\nu_x = \nu; \nu_y = \nu + 0.18 \neq \nu_x$.
As the result, the maximum $X, Y$ betatron amplitudes (see Fig.1) for the non-round beams are larger than the amplitude at the RBs case. Several strong resonances are seen in the non-RB curves while the RBs perform only the size increase at $\nu = 0.25$.

### *2.3 Simulations with "inverse beta function" charge distribution. Optimum bunch length.*

Everywhere above we deal with 2D motion, which can be reduced to 1D motion due to the angular momentum conservation. But 1D motion with the time-dependent Hamiltonian, generally speaking, is also stochastic, although it has more "regularity" in comparison with a general 2D motion. What we need to make the motion regular, is one more integral of motion for any value of the first one (angular momentum). It was proved in [6], that we obtain additional integral of motion if we take the betatron tunes near integer or half-integer resonance and the longitudinal charge distribution of the strong bunch (e.g. proton one in the Tevatron) proportional to the inverse $\beta$-function (one can find additional details of this system in [2]):

$$f(2s) = C/\beta(s) = C/(\beta^* + s^2/\beta^*), \qquad (1)$$

where $C$ is a constant, $\beta^*$ is the $\beta$-function value at the IP.

The beam-beam interaction of the bunches with the "inverse beta function" longitudinal charge distribution can provide integrable dynamics and better stability. We compare the behavior of such beams with the case of short round Gaussian colliding bunches at two working points. Note, that transverse sizes, bunch intensities, the weak bunch length of 15 cm and $\beta^* = 25$ cm are the same in both cases. Fig.2.2a presents the beam size growth vs. $\xi$ after 50,000 turns for $\nu = -0.01$.

From the upper curve one can see significant growth of the beam sizes of the short bunches with increase of $\xi$, while there is almost no effect for the integrable case (in fact, we allowed about $10\%$ deviation of the longitudinal charge distribution in the strong bunch from the exact $1/\beta(s)$ solution) – see the lower curve. There is only a small growth at $\xi \simeq 0.1$; if the charge distribution differs by about $1\%$ from $1/\beta(s)$ then there are no peaks at all and the beam size is not changing in time (this trivial result is not presented).

The second working point of $\nu = 0.05$ looks better for the both cases and Fig.2.2b shows a significant difference between the two cases only for large $\xi$.

If it's difficult to make such a distribution function, one can choose the best ratio of the length of the Gaussian bunch and beta-function at IP (the previous results for the Gaussian bunch were obtained with a very short strong beam). This optimum length depends on the working point. For better understanding of this fact, one can imagine a simple model of the "flat-top" (or rectangular) charge distribution over the full length of $l$ and with phase advance over half-turn equal to $d\psi = ds/\beta_0 = l/2\beta_0$, where $\beta_0$ is beta-function at IP. Let's assume, that the beta function is almost constant over the bunch length $\beta(s) \approx \beta_0$ and the longitudinal distribution is a constant within the coordinate interval of $\pm l/2$ and vanishes elsewhere (as well as the transverse kick) and in between of the tail of one bunch and the head of another we have the unity transformation $I$ of the betatron variables , then one can leave out the arcs and connect kicks from all our bunches together. As here is no dependence of the force on time so this dynamical system is integrable and has no resonances, so we have an optimum in beam lifetime for presented above relation of the phase advance and bunch length.

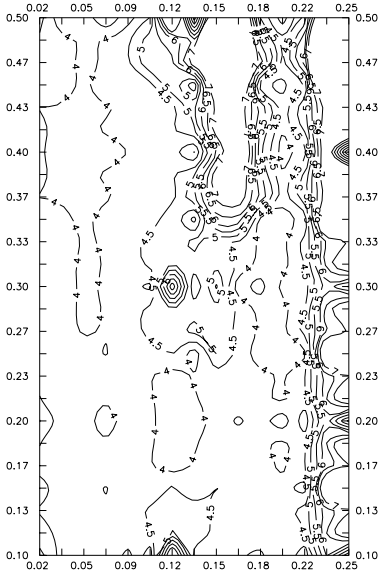

Figure 3: Contour plot of maximum betatron amplitude $A_{max}/\sigma_0$ vs rms bunch length $\sigma_s$ and tune $\nu_y = \nu_x = \nu$, $\xi = 0.05$, $\Delta\vartheta = 0.002$, 75,000 turns.

We performed a search for optimal $\sigma_s$ over tunes of $\nu_x = \nu_y = \nu = 0.02...0.25$ – see Fig.2.3 with the contour plot of the maximum betatron amplitude $A/\sigma_0$ vs. $\sigma_s$ and $\nu$ (75, 000 turns, $\xi$ =0.05, $\beta^* = 25$ cm, phase modulation of $\vartheta = 0.002$). The optimal bunch length (at which, say, $A/\sigma_0 \simeq 4$) depends on the tune and is about 30 cm for the tune around 0.2, about 20 cm for the tune around 0.12, and about 40 cm for the area of a good lifetime near the integer resonance. The last one corresponds to formula $\sigma \simeq \sqrt{2}\beta^*$. One of the probable explanations of that relation can be that the first terms in Taylor expansion of the Gaussian distribution $f(s) \propto \exp -s^2/2\sigma_s^2$ and the "inverse beta function" distribution $f(s) \propto 1/(1+(s/2\beta^*)^2)$ are equal if $\sigma_s = \sqrt{2}\beta^*$. It is interesting to note, that similar results on the optimum bunch length were observed in RCBs simulations for electron-positron colliders [7].

### *2.4 Asymmetry between two IPs*

The degradation of the collider performance due to beam-beam effects is often thought to be more significant if there are several asymmetric interaction points. Fig.4 present results of the maximum amplitude simulations for the scheme with two IPs. If one denotes the phase advance between the first IP and the second one as $\nu$ and between the second one and the first one as $\nu + \Delta\nu_{1,2}$ then the horizontal axis is for $\nu$ and the vertical axis is for $\Delta\nu_{12}$. The lighter areas correspond to smaller maximum betatron amplitude after 10,000 turns, the contour spacing goes as follows: $(A_{max}/\sigma_0)$=4, 5, 7, 10, 15, 20, 25, 30, 40, 50.

It is interesting to note, that over large tune space the optimum in $A_{max}$ lays out of the condition of symmetry, i.e. at $\Delta\nu_{12} \neq 0$.

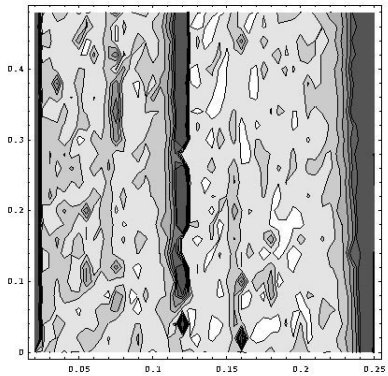

Figure 4: Contour plot of the maximum betatron amplitude vs. tune $\nu$ (horizontal axis and the tune difference between two IPs $\Delta\nu_{12}$ (vertical axis) for the round beams. $\Delta\vartheta = 0.002$, 100,000 turns.

## 3 CONCLUSION

In this article we studied new ways to improve single particle stability in colliders. From the simulations we conclude that in the presence of the beam-beam interaction, the round beams show better particle stability and slower transverse diffusion rates than not-round beams. We also performed a search for optimum bunch length and investigated the "inverse beta-function" longitudinal distribution and found a qualitative agreement with theoretical predictions.

The model we used in our simulations is not quite adequate to the Tevatron due to some evident reasons, and for further investigations of beam-beam effects we plan to study the influence of non-linearities outside the IP, consequences of the RCBs implementation for intrabeam scattering issues and for the effects of the parasitic interactions.

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