

# ENHANCEMENTS TO THE LONGITUDINAL DYNAMICS CODE ESME

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## Abstract

ESME is a program developed at Fermilab for simulating both single particle and multi-particle dynamics in proton synchrotrons. The code has evolved incrementally for more than fifteen years, accumulating many useful features and some internal inconsistency in the process. In the latest revision (8.2), a significant effort has been made to eliminate inconsistency and ambiguity in the determination of phases for multiple RF systems. The use of frequency and phase curves is now more transparent. Other additions or improvements include additional features for time domain calculation, low noise distributions to extend multi-particle capability, run-time memory allocation and portable graphics. A Web page has been established to facilitate the distribution of the source code and documentation. Further information, bug reports and fixes will be made available through this resource.

## 1 INTRODUCTION

ESME is a computer program which follows the evolution of a distribution of particles in energy and azimuth as it is acted upon by the radio frequency system(s) of a proton synchrotron. The code was initially developed during the years 1981-82 for the design of the Tevatron Antiproton Source and documented for general use in 1984. In 1986, provisions were made for longitudinal coupling and space charge to investigate the usefulness of a  $\gamma_T$ -jump in the Fermilab Booster. Since then, various incremental improvements have been made. Recently, due to users interest and anticipation of the use of the code to help model longitudinal phase space manipulations required for operation of the new Recycler Ring and future Tevatron luminosity upgrades, a number of improvements were made including better support for user-defined frequency curves, new distributions with low numerical noise, dynamic memory allocation and portable graphics taking better advantage of color displays.

## 2 CODE DESCRIPTION

### 2.1 Single Particle Dynamics

At the heart of ESME is a pair of single particle difference equations

$$\theta_{i,n} = \left[ \frac{\tau_{s,n-1}}{\tau_{s,n}} \theta_{i,n-1} + 2\pi \left( \frac{\tau_{i,n}}{\tau_{s,n}} - 1 \right) \right] \quad (1)$$

$$E_{i,n} = E_{i,n-1} + eV(\phi_{s,n} + h\theta_{i,n}) - eV(\phi_{s,n}) \quad (2)$$

\* Operated by the Universities Research Association, Inc., under contract with the U.S. Department of Energy.

where  $h$  is the harmonic number and the subscripts  $s$  and  $i$  denote respectively quantities related to the synchronous particle and to particle  $i$ . Thus,  $\tau_{s,n}$  is the synchronous period at turn  $n$ ,  $\tau_{i,n}$  is the period for particle  $i$  at turn  $n$  and  $E_{i,n}$  and  $\theta_{i,n}$  are respectively the energy and azimuth of particle  $i$  at turn  $n$ . Coupling between the longitudinal and transverse dynamics enters into this system of equations through the relation  $\tau_{i,n} = L_{i,n}/(c\beta_{i,n})$ . The dependence of the orbit path length  $L$  on the momentum is set by externally specified coefficients of the series expansion of  $L$  in powers of  $\Delta p_i/p_0$ .

### 2.2 Collective Effects

In the context of this paper, we define collective effects as any effect in which the distribution influences single particle motion. ESME can model the following types of collective effects: (1) reaction of the beam environment, (2) space charge and (3) feedback systems (e.g. from bunch centroid to RF phase or bunch width to RF amplitude). In all cases, only the effect on longitudinal dynamics is modeled.

In its normal mode of operation, ESME treats space-charge and the coupling to the beam environment in the frequency domain. The frequency representation is a natural one for high energy proton synchrotrons where collective effects are often dominated by a few narrow cavity resonances. Longitudinal impedances can be of two types: a simple resonance for which it is sufficient to specify three parameters i.e. frequency, strength and width, or a user-specified table of the complex longitudinal impedance  $Z_{||}(\omega)$  at different frequencies.

The charge distribution  $\lambda(s)$  is obtained by projecting the phase space distribution. The relatively small number of particles ( $10^4 - 10^7$ ) used for a typical simulation results in small scale spatial density fluctuations. For relativistic particles in a smooth cylindrical beam pipe, one can show that the longitudinal electric space charge field is given by [1]

$$E_s(s) = -[1 + 2 \log(b/a)] \frac{\lambda'(s)}{4\pi\epsilon_0\gamma^2} \quad (3)$$

where  $\lambda'(s)$  is the derivative of the longitudinal line charge distribution and  $b/a$  is the ratio of vacuum chamber to beam radii. Note that the presence of a derivative in equation (3) indicates that small scale fluctuations will produce high electric fields and "spurious" emittance growth.

ESME implicitly assumes that the RF frequency *does not change significantly over a turn*. This allows the beam current to be treated as a periodic function and its spectral contents to be efficiently calculated using the FFT. The finite number of harmonics used for the FFT acts as a low-pass filter that mitigates spurious emittance growth due to

loca fluctuations. Optionally, the distribution can also be smoothed using a variety of methods, e.g. high order polynomial fitting.

In certain situations, the frequency domain representation may be inappropriate. The canonical example is the situation where the revolution period changes significantly during a period smaller than the fill time of the cavities. To deal with this situation, ESME can model simple resonances in the time-domain. However, more complicated environment responses must still be handled through  $Z_{||}(\omega)$ .

### 3 NEW FEATURES AND ENHANCEMENTS

#### 3.1 Frequency and Phase for Multiple RF Systems

ESME provides for several independent RF voltages representing either Fourier components of a complex waveform or the output of physically independent RF systems. Voltage, frequency, and phase may have arbitrary programs. The phase of each RF system consists of a programmed phase with a default of zero, an optional synchronizing phase, and an optional phase feedback term. In the simplest case of a single RF voltage with a programmed phase of zero, the synchronizing phase  $\vartheta_s = \varphi_s/h$ , where  $h$  is the harmonic number and  $\varphi_s$  is the conventionally defined synchronous phase. The synchronous trajectory can be moved freely with respect to the central orbit by using phase curves, frequency curves, or specified momentum offset for such processes as momentum stacking and phase displacement acceleration. The resulting complexity of defining the phase variables correctly prompts more questions than any other general feature; fortunately it is hidden in many typical applications.

#### 3.2 Low Noise Distributions

Assuming a rotation period  $T$  and  $M$  identical evenly spaced bunches, the Fourier spectrum of the beam current consists a train of impulses of period  $MT^{-1}$  modulated by the transform  $B(\omega)$  of the bunch profile. At high frequencies,  $B(\omega)$  has a tail which is approximately constant and proportional to the inverse square root of the number of particles in the beam,  $N_p^{-1/2}$ . The bunch-lengthening and bucket distortion effects resulting from the lower frequency part of the spectrum can be simulated with a modest number of macroparticles  $n_p$  sufficing to produce the general form of  $B(\omega)$ . However, if effects such as microwave or negative mass instabilities are to be included with the intention of generating meaningful quantitative information, the noise in the spectrum of the starting distribution should be the Schottky noise of the real beam not the numerical noise corresponding to  $n_p$ . Since  $N_p/n_p \simeq 10^{13}/10^7$  the numerical noise must be reduced by a few orders of magnitude. ESME can now produce quieter distributions without increasing the number of particles by populating the phase space using quasi-random distributions derived from Sobol [2] sequences of appropriate dimensionality.

The high frequency tail for quasi-random distributions is proportional to  $n_p^{-1}$  rather than  $n_p^{-1/2}$ . Unfortunately, the mapping (1-2), even in the absence of collective effects, reintroduces high frequency noise. Nevertheless, the distribution will generally remain quieter than one generated by pseudo-random numbers. Thus, the technique is useful but getting reliable results requires caution.

#### 3.3 Memory Management

ESME is written in Fortran 77. The code utilizes a number of large arrays to store phase space coordinates, longitudinal distributions, etc. Communication between various modules relies heavily on large COMMON blocks. With the availability of inexpensive memory, users have become eager to run simulations with increasingly large number of particles. The old structure of the code required recompilation resulting in a proliferation of executables. To avoid this situation, the memory management has been extensively modified to support dynamic memory allocation. This was accomplished without making dramatic structural changes by passing pointers through COMMON blocks rather than actual arrays. CRAY style rather than "standard" Fortran 90 syntax was used. While the former is a de-facto standard under most UNIX f77 compilers, Fortran 90 is still not widely used. Porting the code to Fortran 90 should be a straightforward task if this ever becomes an issue.

#### 3.4 Graphics

Over the years, ESME has been modified many times to support various graphic packages. Prior to the current version, ESME relied on HIGZ, a library developed at CERN. Although support for HIGZ remains available, we have opted to migrate to the PGPLOT library and to use it for future development. PGPLOT is compact, supports a wide variety of devices, is free for non-commercial use, distributed in source form and currently runs under every flavor of UNIX. A Windows NT port exists and should make it possible to support that platform. The new graphics code makes better use of color, in particular to distinguish among different classes of particles. New types of plots are supported including more informative phase space density contour plots and mountain range plots of the beam spectrum evolution.

#### 3.5 Internet Resources

A Web page has been established to enable users to report bugs and new releases in a timely manner. The URL is <http://www-ap.fnal.gov/ESME>. At the time this article was written, the current version was 8.2. Source code, documentation and binaries for various platforms should be available from the above address within a few days of this conference.

## 4 CONCLUSIONS AND FUTURE PROJECTS

ESME does not claim to be taking advantage of the most recent advances in software technology. However, as it stands it is a mature well-proven tool that occupies a specialized niche. Our objectives for the near future are (1) fix problems that are reported (2) to the extent that fundamental changes in the structure of the code are not required, add new features that will allow users to solve practical problems. In an upcoming release, we intend to provide a mode where all calculations will be performed consistently in the time-domain; the beam environment reaction will be specified through a calculated or measured wake potential. This should be useful in situations where the Fourier representation is not economical e.g. a ring populated with a large number of short intense bunches.

## 5 ACKNOWLEDGEMENTS

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## 6 REFERENCES

- [1] A. M. Sessler and V. K. Neil, "Longitudinal Resistive Instabilities of Intense Coasting Beams", NIM 36, No. 4, pp 429 - 436 (April 1965)
- [2] Sobol' I.M. USSR Computational mathematics and mathematical Physics, 7,4, 86-122.
- [3] Information about the PGPLOT library can be found at <http://astro.caltech.edu/~tjp/pgplot>.

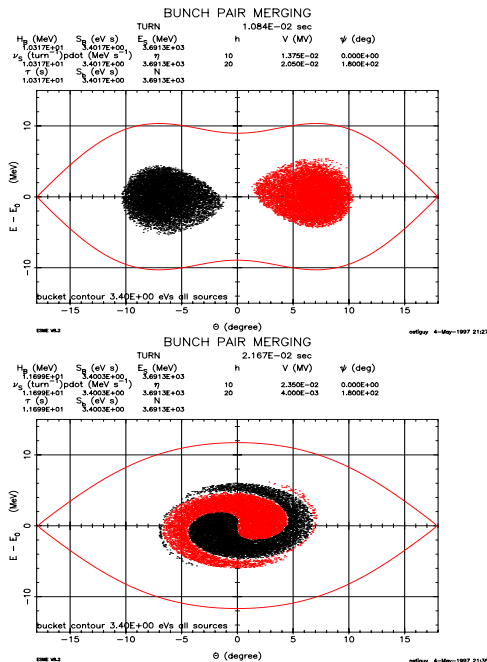


Figure 1: Adiabatic merging of h=20 bunches into h=10 bunch in CERN PS

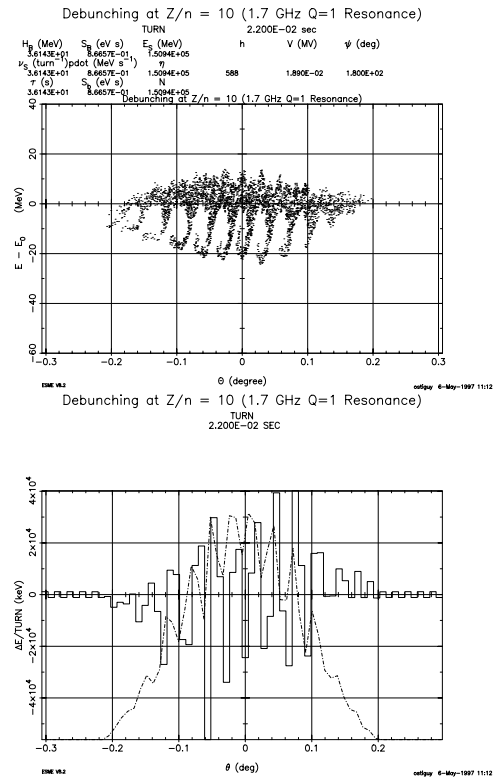


Figure 2: Well developed microwave instability. Lower plot shows azimuthal charge distribution ( ... ) and longitudinal electric field ( — ).

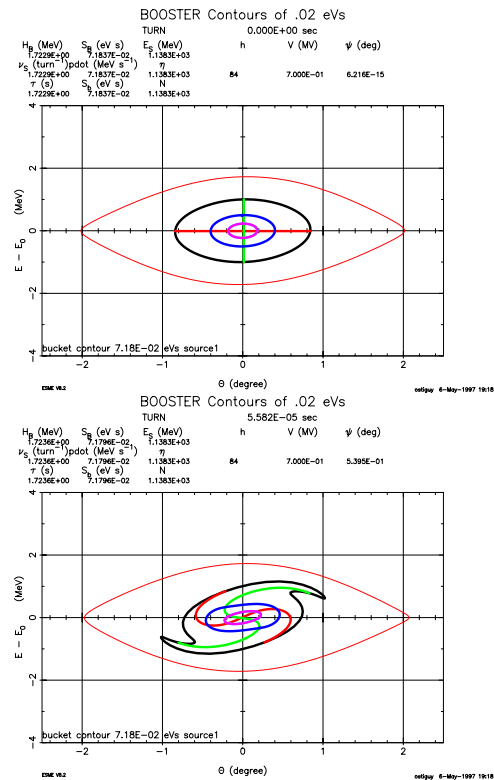


Figure 3: Distortion of .02, .005, and .002 eVs contours caused by locating all cavities at one place in rapid cycling booster ring