# APPLICATION OF THE GSM METHOD AND TD COMPUTATION OF THE LONG RANGE WAKE IN LINEAR ACCELERATOR STRUCTURES

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*Abstract*

For a proper design of linear colliders it is important to know the transverse long range wake which can either be computed directly in TD or alternatively from the corresponding higher order resonant modes. In this contribution, the resonant modes of the first, third and sixth dipole passband of the $180$-cell accelerating structure used for the S-band linear collider at DESY have been analyzed using an accurate and numerically efficient generalized scattering matrix (GSM) method. Furthermore, the MAFIA program package has been applied to calculate the wake function in time domain (TD). The agreement of both methods turns out to be excellent. From the results one can predict that the sixth dipole passband significantly contributes to the transverse wakefield.

## 1 INTRODUCTION

At several high energy physics laboratories around the world strong efforts are currently made to design an $e^+e^-$ linear collider with an initial center of mass energy of about $500\,\mathrm{GeV}$ [1]. In most of the designs trains of bunches are accelerated by long tapered multi-cell structures which are similar to disc-loaded circular waveguides. In order to avoid deflections of the beam which decrease the efficiency of the collider or even may lead to cumulative beam instabilities, one has to control the wakefield excited by previous bunches in the train. Higher order modes corresponding to the first and sixth dipole passband are mainly responsible for this phenomenon.

For a closed cavity the wake function can be computed from its resonant modes [2]. The contribution of each mode to the wakefield is characterized by the so-called loss parameter. In reality accelerator cavities are not closed because the beam pipe is open. Nevertheless, the computation of the wake function presented in [2] can also be used to approximate the spectral components of the wake below the cutoff frequency of the beam pipe. Therefore the beam pipe has to be modeled sufficiently long so that the cavity modes do not depend on the position of the short.

The GSM method has been proved accurate and numerically efficient for the investigation of a large variety of waveguide and cavity problems [3], [4]. In the field of linear accelerators, this method has also successfully been applied. E.g., the beam loading in a tapered X-band accelerating structure driven by monopole modes has been analyzed using the GSM method [5].

Dipole modes corresponding to the first passband of a $180$-cell structure which has been designed for the S-band linear collider at DESY [6] have been studied in [7] and [8] using the GSM method and a discrete network model, respectively. However, in [7] only the first dipole passband has been considered. Furthermore, instead of the original structure a simplified model has been assumed which consists of $30$ packages containing $6$ identical cells each. The numerical results have demonstrated that this assumption leads to a wide scattering of the loss parameters which is however an artifact of the calculation model.

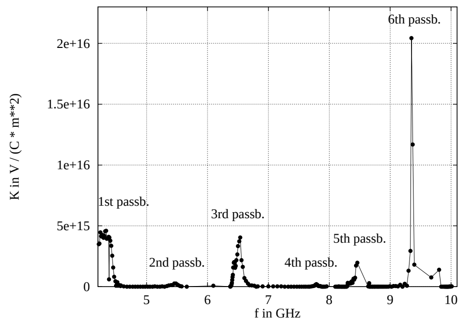


Figure 1: Normalized longitudinal loss parameters corresponding to dipole modes of the $30$-cell structure.

In this contribution, the GSM method is applied to determine the loss parameters corresponding to the resonant modes of the first, third and sixth dipole passband of the S-band linear collider structure. As a reference method the MAFIA program package which is based on a grid discretization of Maxwell's equations is used to compute the wake function as it develops in time. The contributions of the individual resonant modes to the wakefield are then obtained by means of Fourier transform.

## 2 LOSS PARAMETER COMPUTATION OF DIPOLE MODES

For a qualitative estimate of the loss parameters covering several higher order dipole passbands we have analyzed a $30$-cell structure which consists of every sixth cell of the original structure. The results presented in Fig. 1 underline the importance of the sixth dipole passband which contains the modes with the highest loss parameters. Furthermore Fig. 1 shows that besides the first, third and sixth passband the resonant modes corresponding to other passbands can be neglected.

In Fig. 2 the loss parameters of the $180$-cell structure corresponding to the first passband are presented. This passband contains more than $100$ modes with considerably high loss parameters.

It is worth noting that in the frequency range from about

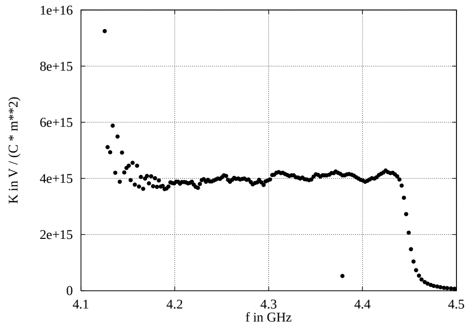


Figure 2: Normalized longitudinal loss parameters corresponding to the first dipole passband of the 180-cell structure.

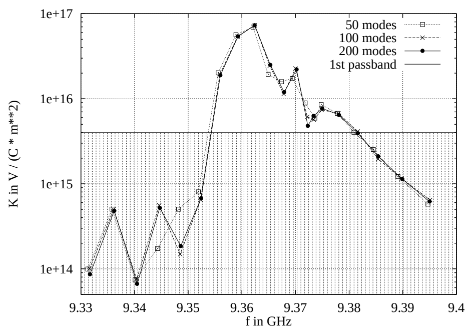


Figure 3: Normalized longitudinal loss parameters corresponding to the sixth dipole passband of the 180-cell structure.

$4.15$ GHz to $4.44$ GHz all modes (despite of one single mode) have nearly the same loss parameter. The odd mode at $f = 4.3785$ GHz is not an intrinsic mode of the tapered periodic structure. It is rather characterized by the interaction of the first few cells with the beam pipe. Note that this mode is also observed in the loss parameter distribution of the $30$-cell structure, see Fig. 1.

Fig. 3 shows the results of a detailed investigation of the sixth dipole passband of the original structure in the immediate vicinity of the mode with the highest loss parameter. An extensive study of convergence has been carried out in order to demonstrate the accuracy of the method. It has turned out that the loss parameters are stable if more than $100$ waveguide modes are used in the GSM method.

The level of the loss parameters corresponding to the first dipole passband ($\approx 4 \cdot 10^{15}$ V/(C m$^2$)), which is also given in Fig. 3, is about $20$ times less than the peak loss parameter observed in the sixth dipole passband ($\approx 8 \cdot 10^{16}$ V/(C m$^2$)).

The loss parameters belonging to the third dipole passband, which have also been calculated, are in the same order of magnitude as those corresponding to the first dipole passband. However, the contribution of the third dipole passband to the wake function is very small which will be shown below.

Figs. 4 and 5 present the loss parameter of the dipole

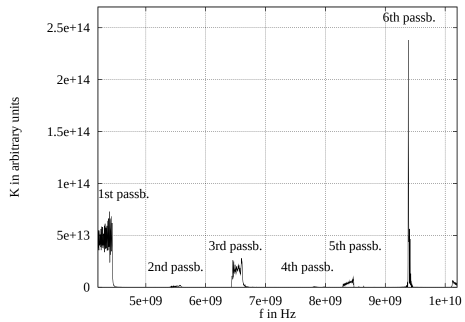


Figure 4: Fourier transform of the transverse wake function calculated in TD.

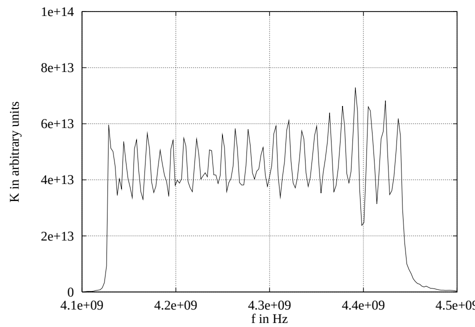


Figure 5: Detailed representation of the Fourier transform shown in Fig. 4 in the frequency range corresponding to the first dipole passband.

modes in the frequency range from $4.2$ GHz to $10.2$ GHz and those which correspond to the first dipole passband, respectively, as a result from the MAFIA calculations. In both Figs., the frequency resolution of the Fourier transform is $6$ MHz so that individual modes cannot be recognized. Nevertheless the MAFIA results agree very well with those of Figs. 1 and 2 which have been computed using the GSM method. Nevertheless one has to keep in mind that the results which are shown in Fig. 1 correspond to a $30$-cell structure.

The sum of all loss parameters $\sum k_\nu$ corresponding to a particular passband can be obtained directly from the mode computation or by the peak value of the envelope of the filtered normalized longitudinal wake function. The sums corresponding to the first, third and sixth passband obtained by the GSM method, MAFIA calculations and a discrete network model [8] are compared in Table 1. From this table it can be concluded that the three methods are in good agreement.

## 3 COMPUTATION OF THE TRANSVERSE LONG RANGE WAKE FUNCTION

Figs. 6, 7 and 8 present the long range wake functions corresponding to the first, third and the sixth dipole passband, respectively. These curves can either be obtained by superimposing the corresponding resonant modes (GSM method) or by applying

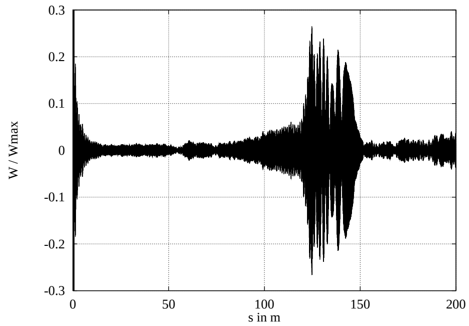


Figure 6: Transverse long range wake corresponding to the first dipole passband of the 180-cell structure.

Table 1: Sums of the loss parameters corresponding to the individual passbands.

| | $\sum k_\nu$ in $10^{17}$ V/(Cm$^2$) | | |
|---|---|---|---|
| | band 1 | band 3 | band 6 |
| GSM | 5.85 | 2.41 | 2.09 |
| MAFIA | 5.84 | 2.30 | 1.98 |
| [8] | 5.69 | — | — |

bandpass filters with appropriate passbands to the wake functions (MAFIA).

From the results it can be concluded that wakefield corresponding to the third dipole passband is negligible. The recoherence lengths of the first and sixth dipole passband are 120 m and 100 m, respectively. These values are consistent with the densities of the corresponding mode spectra which are 2.5 MHz and 3.0 MHz, respectively.

The recoherence phenomenon can effectively be suppressed by artificially reducing the quality factor of the dipole modes down to 3000 [8]. On the other hand, keeping in mind that the bunch to bunch distance in the S-band linear collider is 8 m, the wake function corresponding to the first *and* sixth dipole passband is not sufficiently decohered when the following bunch enters the structure. Consequently these modes have to be taken into account in beam dynamics simulations.

## 4 CONCLUSIONS

The GSM method has been applied to the computation of various dipole passbands corresponding to multi-cell linear accelerator structures. A detailed study of the first, third and sixth dipole passband of the 180-cell structure used for the S-band linear collider at DESY has shown that the maximum loss parameter is observed in the sixth dipole passband. The validity of the results which have been obtained using the GSM method has been confirmed by the MAFIA program package. The computation of the wake functions corresponding to the individual passbands has shown that the sixth dipole passband has to be considered in beam dynamics simulation whereas the contribution of the third dipole passband can be neglected.

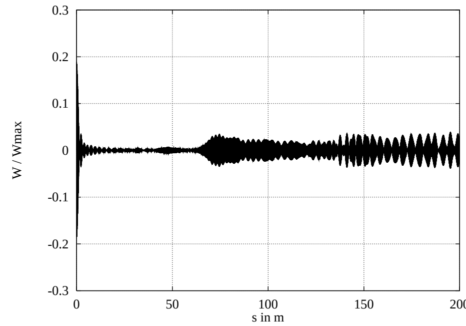


Figure 7: Transverse long range wake corresponding to the third dipole passband of the 180-cell structure.

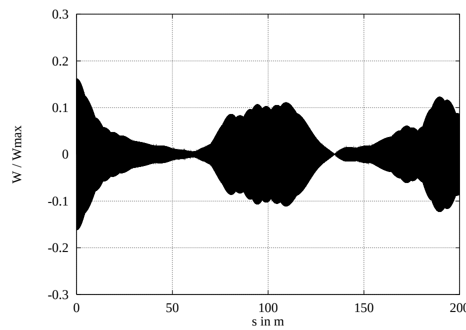


Figure 8: Transverse long range wake corresponding to the sixth dipole passband of the 180-cell structure.

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