

WAKE FIELDS OF A BUNCH ON A GENERAL TRAJECTORY DUE TO COHERENT SYNCHROTRON RADIATION

M. Dohlus, A. Kabel, T. Limberg
DESY, Notkestr. 85, 22603 Hamburg, Germany

Abstract

If short bunches travel along trajectories with small bending radii a simple geometrical condition permits strong longitudinal and radial wake fields to act: electromagnetic fields emitted by a particle can 'overtake' on a shorter straight trajectory and interact with particles ahead. The bunch then starts to radiate coherently. The electromagnetic fields along the bunch have strong gradients and in general increase emittance. Investigations for the Tesla Test Facility Free Electron Laser have shown that the wrong choice of compressor parameters can increase the emittance by more than a factor of ten when compressing down to 50 μ m bunch length.

We present a formalism and numerical code to calculate the resulting electromagnetic fields on a general trajectory. Shielding effects of the vacuum chamber have been included. We present first results for the TTF-FEL undulator magnet and calculations of the influence of transition regions (bunch entering and leaving a homogeneous magnetic field).

1 INTRODUCTION

In [1] it has been shown:

- 1) The 'overtaking length' $L_o = (24\sigma_s R^2)^{1/3}$ is a measure for the interaction length inside of a bending magnet (with the bending radius R and the bunch length σ_s). Usually the overtaking length is large compared to σ_s but not very small compared to the dimensions of the configuration (e.g. R).
- 2) The longitudinal wake (per length) in the center of the bunch scales as $W_{||} = 1 / \left(\varepsilon (2\pi)^{3/2} 3^{1/3} \sigma_s^{4/3} R^{2/3} \right)$. This leads e. g. to a wake in the order of 1 MV/m for an InC bunch with $\sigma = 0.1$ mm on a curvature $R=1$ m. Additionally in [2] it has been demonstrated:
- 3) Shielding by horizontal conducting plates in the distance h starts to be effective if the 'shielding length' $L_s = (\sigma_s^2 + h^2) / (2\sigma_s)$ is of the same order as the 'overtaking length'. L_s is the path length for the electromagnetic field emitted by a particle and reflected from the wall to reach the particle one sigma behind it.
- 4) The length of transient regions (e.g. bunch entering and leaving a homogeneous magnetic field) is of the order of L_o . As the transient lengths may be comparable to magnet lengths their effects have to be taken into account for beam dynamic simulations. An analytical formulation for the longitudinal wake of 1D bunches in an arc has been derived in [3].
- 5) The wake due to coherent synchrotron radiation (including transient effects) has been taken into ac-

count in the calculation of emittance increase for the case of a bunch compressor in the TESLA Test Facility FEL [4].

This approach [2] is not self-consistent: the wake fields are calculated for a source distribution in pre-defined motion $\vec{r}_{s,i}(s)$ (s is length parameter of path, i is index of sub-bunch). The source bunch has no transversal dimensions (1D bunch) and does not change its longitudinal profile (rigid bunch). Two- or three-dimensional bunches are composed from one-dimensional line charges. The simulation of the shielding effects due to infinitely conducting plates in the horizontal plane is taken into account by mirror charges.

Our *new formulation*, described in chapter two, takes into account:

- 1) 2D bunch distributions with longitudinal profile $\lambda(s, t) = \lambda(s - v_s t)$ and profile $\eta(\tilde{z})$ in \vec{e}_z direction. \vec{e}_z is a constant unity vector that is essentially perpendicular to the plane of motion (the motion is not restricted to be exactly in a plane!), $\tilde{z} = z - \vec{r}(s) \cdot \vec{e}_z$ describes the offset to the central path $\vec{r}(s)$ in \vec{e}_z direction;
- 2) The 2D approach avoids the field singularities of the 1D approach [2] and is therefore better capable of the simulation of fully three dimensional source distributions (using an array of 2D bunches). So called 'space charge forces' are taken into account.
- 3) The new formulation is better suited for numerical integration (even for 1D bunches).

Chapter three describes the calculation of energy spread increase for the case of a single bending magnet and following drift space for different bunch lengths and bending radii. In chapter four, first results for a micro-bunch entering the TTF-FEL are presented.

2 CALCULATION OF THE LORENTZ FORCE

The action of the electromagnetic field caused by a source (index s) to a charged test particle (index t) is described by the *Lorentz force equation*:

$$\frac{1}{q_t} \vec{F}_t = \vec{E}_s + \vec{v}_t \times \vec{B}_s . \quad (1)$$

Using the scalar and vector potentials $\vec{E}_s = -\nabla V_s - \dot{\vec{A}}_s$, $\vec{B}_s = \nabla \times \vec{A}_s$ this force can be split into two terms:

$$\frac{1}{q_t} \vec{F}_t = \underbrace{\nabla \left(\vec{v}_t \cdot \vec{A}_s - V_s \right)}_{\frac{1}{q_t} \vec{F}_{At}} + \underbrace{\left(-\dot{\vec{A}}_s - \vec{A}_s (\vec{v}_t \cdot \nabla) \right)}_{\frac{1}{q_t} \vec{F}_{Bt}} . \quad (2)$$

The first term is a gradient field, the second term vanishes for bunches in linear motion. As the space charge and

current density distributions are determined, the scalar and vector potentials can be directly obtained from the *retarded potential equation*. For a two dimensional bunch with the density $\lambda(s - v_s t)\eta(\tilde{z})$ the scalar and vector potential integrations lead to:

$$\frac{4\pi\epsilon}{q_t}\vec{F}_{At} = \int \left\{ G_1(s')\vec{R}_{xy} + G_2(s')\vec{e}_z \right\} \left(1 - \vec{\beta}_t \cdot \vec{\beta}_s \right) ds', \quad (3)$$

$$\begin{aligned} \frac{4\pi\epsilon}{q_t}\vec{F}_{Bt} = & \beta_t\beta_s \int \left(\left\{ G_1\vec{R}_{xy} + G_2\vec{e}_z \right\} \cdot (\vec{e}_t - \vec{e}_s) \right) \vec{e}_s ds' \\ & - \beta_t\beta_s \int G_3(s') \frac{\partial \vec{e}_s}{\partial s'} ds' \\ & + (\beta_s - \beta_t) \int G_4(s') \vec{e}_s ds' \end{aligned} \quad (4)$$

with

$$\begin{aligned} G_1(s) &= \int \left\{ \lambda/R^3 + \dot{\lambda}/(c_0 R^2) \right\} \eta(\tilde{z}) d\tilde{z} \\ G_2(s) &= \int \left\{ \lambda/R^3 + \dot{\lambda}/(c_0 R^2) \right\} \tilde{z} \eta(\tilde{z}) d\tilde{z} \\ G_3(s) &= \int \lambda/R \eta(\tilde{z}) d\tilde{z} \\ G_4(s) &= \int -\dot{\lambda}/(c_0 R) \eta(\tilde{z}) \tilde{z} \end{aligned}$$

and $\vec{R} = \vec{r}_t - \vec{r}_s(s') - \tilde{z}\vec{e}_z$, $\vec{R}_{xy} = \vec{R} - (\vec{e}_z \cdot \vec{R})\vec{e}_z$, $\vec{\beta}_s = \beta_s\vec{e}_s = \vec{v}_s/c_0$, $\vec{\beta}_t = \beta_t\vec{e}_t = \vec{v}_t/c_0$, and $\lambda = \lambda(s' + \beta_s R - v_s t)$.

For this derivation we used the chain rule with the boundary conditions $\|\vec{r}_s(s = \pm\infty)\|^{-1} = 0$. The main contribution to the longitudinal force is given by the integral (3). The last term in (4) vanishes if source and observer particle have the same velocity. The first term vanishes for particles in circular motion if source and observer are on the same orbit. The remaining part (proportional to the curvature $\|\partial\vec{e}_s/\partial s\|$) gives the so called 'centrifugal space charge force'. In the following calculations, all integrals have been solved numerically without further restrictions.

3 TRANSITIONS FOR LONGITUDINAL WAKES AND ENERGY SPREAD GROWTH IN BENDING MAGNETS

The field calculation simulates the bunch as a set of gaussian sub bunches—each with a different energy, different initial conditions and an individual path. The trajectories of these sub-bunches are not effected by the calculated fields.

An independent set of bunch slices, initially lined up, is traced through the bending magnet and experiences the energy variations due to the longitudinal fields. Due to the strong longitudinal variation of the fields, the slice centroids follow different paths and a 'centroid emittance' develops.

For the tracking, the bending magnets and drift spaces are cut into slices and the wake fields are calculated and

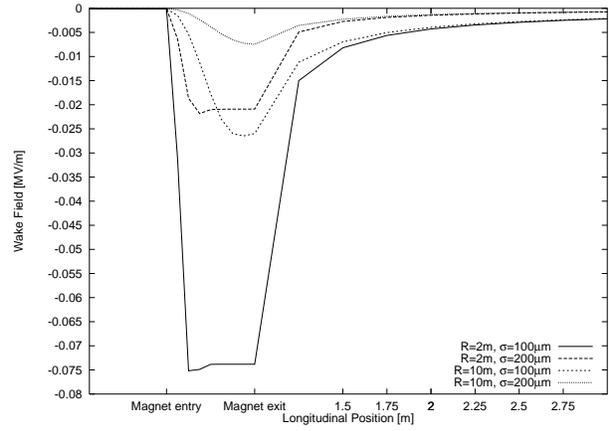


Figure 1: Longitudinal wake field at bunch center vs. longitudinal position.

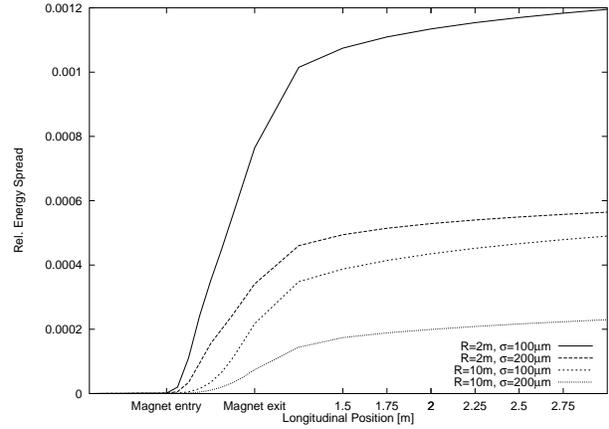


Figure 2: Energy spread vs. longitudinal position.

the consequent energy changes are applied at the end of each slice.

Fig. 1 and 2 show the longitudinal wake field at the bunch center and the resulting energy spread growth for different bunch lengths and bending radii vs. longitudinal position for a very simple set-up, comprising a bending magnet of length .5m and a drift space of length 2.0m. It can be seen that there is no steady-state region for the larger bending radius. The asymptotic behavior of the field past the magnet's exit is independent of the bunch's history and can be shown to be $\lambda/(2\pi\epsilon r)$, where λ is the 1D charge density and r is the distance from the magnet's exit.

4 FIRST CALCULATIONS OF AN UNDULATOR

The longitudinal charge distribution of a bunch traversing an FEL undulator changes drastically due to the micro-bunching of the SASE process. In the TTF-FEL case, the incoming bunch has a homogeneous peak-shaped distribution with an RMS-length of about $50\mu\text{m}$. Micro-bunching along the undulator then produces a chain of micro-bunches with a periodicity of the wavelength (6.4nm). As first results of undulator field calculations, we present here

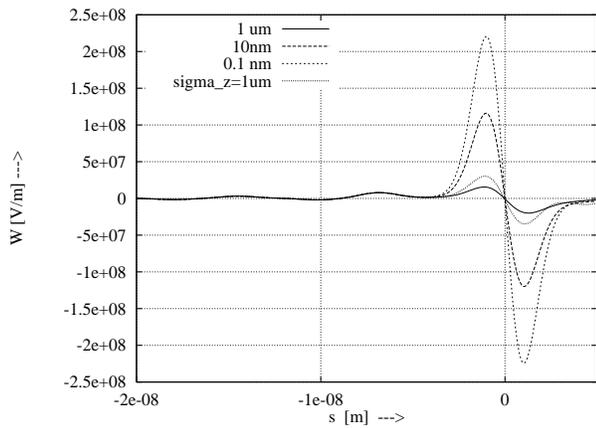


Figure 3: Longitudinal wake for a micro-bunch (3 nm FWHM) in the TTF-FEL undulator for different transverse offsets of the observation point vs. longitudinal position.

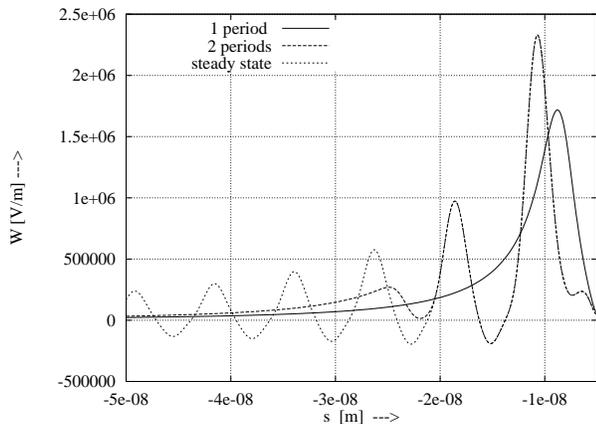


Figure 4: Longitudinal wake for a micro-bunch (3 nm FWHM) in the TTF-FEL undulator for different numbers of periods into the undulator vs. longitudinal position.

calculations of the longitudinal wake of a microbunch, modeled as a gaussian distribution with a FWHM of 3.2 nm, half the distance between micro-bunches. The undulator has a period length of 2.7cm and a field strength of 0.5 Tesla. Fig. 3 shows the longitudinal wake field for different transverse offsets in a range between 0.1nm and $1\mu\text{m}$. The curve labeled ' $\sigma_z = 1\mu\text{m}$ ' results from a centered bunch with a non-zero transversal extension. The fields are calculated at a position 20 undulator periods into the undulator, where a steady state regime is reached. The strong logarithmic dependence indicates a space charge like force to be the cause of the peak around the bunch center ($s=0$). For an evaluation of bunch energy loss, the transverse bunch distribution has to be folded with these curves.

Fig. 4 shows with higher resolution the region ahead of the bunch, (negative s in these plots means ahead) starting with the second peak, for the bunch being at different numbers of periods into the undulator. There is no significant dependance on vertical offsets in this region.

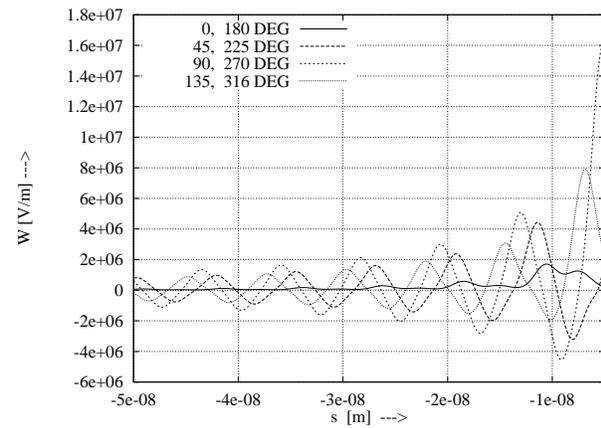


Figure 5: Longitudinal wake for a micro-bunch (3 nm FWHM) in the TTF-FEL undulator for different bunch positions within an undulator period in the steady state regime vs. longitudinal position.

Fig. 5 then varies the bunch position within an undulator period in the steady state region, the bunch position expressed in degrees of that period. At the zero crossings (90 and 270 degrees), the strength of the focussing field has its maximum.

5 SUMMARY

A new formalism for the calculation of wake fields acting on bunches travelling on arbitrarily curved trajectories makes the proper treatment of magnets possible where, unlike in bending magnets, the particle trajectory is not a circle: undulators, quadrupoles etc.. First results for the case of a micro-bunch in the TTF-FEL undulator are shown.

The longitudinal wake fields and the energy spread growth in the transition regimes of a bending magnet are calculated for different bunch lengths and magnet strengths. It can be seen that for bends whose length is considerably shorter than the bending radius no steady-state regime region exists.

6 REFERENCES

- [1] Y. Derbenev, J. Rossbach, E. Saldin, V. Shiltsev: 'Microbunch Radiative Tail-Head Interaction', TESLA-FEL 95-05, September 95.
- [2] M. Dohlus, T. Limberg: 'Emittance Growth due to Wake Fields on Curved Bunch Trajectories', presented at XVIII International Free Electron Laser Conference (Rome, 1996); DESY print TESLA-FEL 96-13.
- [3] E. L. Saldin, E. A. Schneidmiller, M. V. Yurkov: 'On The Coherent Radiation of an Electron Bunch Moving in an Arc of a Circle', DESY Print October 1996, TESLA-FEL 96-14.
- [4] A VUV Free Electron Laser at the TESLA Test Facility at DESY. Conceptual Design Report. DESY Print June 1995, TESLA-FEL 95-03