PERTURBATION OF RELEVANT RESONANCE FOR SLOW EXTRACTION EFFICIENCY INCREASE

N.N.Alexeev, S.L.Bereznitsky, A.E.Bolshakov, ITEP, Moscow, Russia

Abstract

In synchrotron, slow extraction of circulating beam when influenced by non-linear perturbation was analysed both analytically and by computer simulation. It was found that beam loss on the septum may be reduced two times and more if particles distribution in the cross section of kicker magnet is deformed with decapole perturbation of sextupole magnetic field using for resonance excitation. Similar effect is observed for gradient resonance extraction with octupole perturbation. Results of simulation for ITEP PS slow extraction system are presented.

1. INTRODUCTION

The resonance slow extraction of the beam circulating in the synchrotron is impossible without loss of a part of beam that jumping unsuccessfully over a septum. To increase the efficiency of the slow extraction, the septum width has to be constructed as thin as possible. We are considering another way of the beam loss minimisation which is base on the forming the particles distribution in the cross section of the kicker magnet to reduce a portion of scattered particles in the septum of given width. The required deformation of the conventional distribution may be obtained by the relevant resonance perturbation as it will be shown below.

2. FOUNDATIONS OF ANALYSIS

The equation of motion at resonance slow extraction is written usually in the form

$$\frac{d^2\eta}{d\phi^2} + \nu^2\eta = -a\eta^{n-1}\cos(m\phi) \qquad , \qquad (1)$$

where η, ϕ - Courant-Snyder variables, $\nu=\nu_r+\Delta\nu$ - tune value, n=2 - for gradient resonance, n=3 - for sextupole resonance, $m=n\cdot\nu_r$ - resonance harmonic number. Eq (1) may be rewritten in the co-ordinate system (X,Y) proposed by Krylov-Bogolubov-Metropolsky [1]

$$\eta = X(\phi)\cos(\nu_r\phi)-Y(\phi)\sin(\nu_r\phi)$$

$$\eta' = -(\nu_r+\Delta\nu)\{ X(\phi)\sin(\nu_r\phi)+Y(\phi)\cos(\nu_r\phi)\} \ (2)$$

in the form

$$X' + \Delta\nu Y = \frac{1}{\nu} a\eta(X,Y,\phi)^{n-1}\cos(m\phi)\sin(\nu_r\phi)$$

$$Y' - \Delta\nu X = \frac{1}{\nu} a\eta(X,Y,\phi)^{n-1}\cos(m\phi)\cos(\nu_r\phi) \qquad (3)$$

Removing from (3) harmonic addendums and keeping addendums responsible for slow changing of betatron oscillation amplitude we obtain for gradient resonance

$$X' + \Delta\nu Y = \frac{a}{4\nu} Y$$

$$Y' - \Delta\nu X = \frac{a}{4\nu} X \qquad (4)$$

and for sextupole resonance

$$X' + \Delta\nu Y = \frac{a}{4\nu} XY$$

$$Y' - \Delta\nu X = \frac{a}{8\nu}(X^2-Y^2) \qquad (5)$$

It follows from Eq.(4) that a rate of oscillation amplitude increases at gradient resonance as

$$R' = R\frac{a}{4\nu}\sin(2\varphi) \quad , \qquad (6)$$

where $X=R\cos(\varphi)$,$Y=R\sin(\varphi)$. For sextupole resonance, the rate dependence of amplitude is

$$R' = -\frac{a}{8\nu} R^2\cos(3\varphi) \ (7)$$

When oscillation amplitudes are big enough, the phase space trajectories are close to the linear separatrix and $\varphi\approx$const.

The particles distribution f(R) in the kicker magnet cross section may be obtained in assumption that the flow of particles to the transverse direction at the beam ejection process is stable and independent of R. In this case

$$R'\cdot f(R)\approx \text{const} \qquad (8)$$

and the particles density in the kicker magnet cross section for Eq.(7,8) conditions is decreased with R, so the maximum density value corresponds to the septum co-ordinates.

3. PERTURBED MOTION ANALYSIS

It follows from Eq.(8) that a distribution function f(R) with lower density value at the septum co-ordinates may be obtained by means of corresponding modification of R' function. It may be done for example through the relevant resonance perturbation according, to

$$\frac{d^2\eta}{d\phi^2} + \nu^2\eta = -a\eta^{n-1}(1 - k\eta^2)\cos(m\phi) \quad .(9)$$

where k - factor of resonance perturbation. The rate expressions of oscillation amplitude growth for Eq.(9) conditions are

$$R' = R\frac{a}{4\nu}\left(1-\frac{k}{2}R^2\right)\sin(2\varphi) \qquad (10)$$

and

$$R' = -\frac{a}{8\nu}R^2\left(1-\frac{3}{4}kR^2\right)\cos(3\varphi) \qquad (11)$$

Setting the k factor in Eq.(10-11) to optimum value, one can possible to obtain the required particles distribution f(R) with minimum density value at the septum co-ordinates.

4. RESULTS OF SIMULATION

Analytical analysis was verified by computer simulation for ITEP Proton Synchrotron slow extraction system constructed on simplest scheme with one sextupole lens for resonance excitation and one kicker magnet for particles extraction. Fig.1 shows layout of main components for this system.

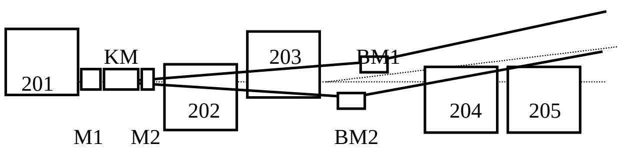

Fig.1 Schematic arrangement of beam ejection system, where K-is kicker magnet with septum, M12 - magnets for local orbit deviation, BM - bending magnets.

The effective thickness of septum is 2 mm and the width of magnetic gap is 4 mm. So the efficiency of beam ejection at conventional sextupole resonance conditions is near to 60%. Summary distribution diagram of particles in the cross section of kicker magnet is shown in fig.2.

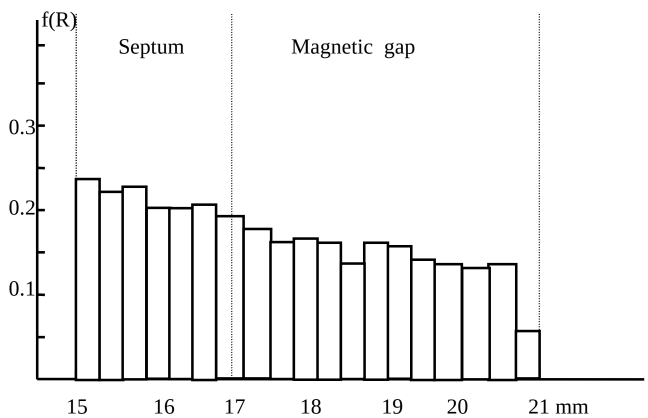

Fig.2. Conventional particles distribution in the cross section of kicker magnet at slow extraction

One can see in this picture that the density slope is close to the $1/R^2$ function as predicted by Eq. (8). and the ejection efficiency is 59%.

Small perturbation of sextupole resonance according to Eq.(9) causes deformation of distribution of interest in that way that the density descends at septum co-ordinates and raises at magnetic gap, but the distribution shortens, so the ejection efficiency goes down. Obvious way to high efficiency is to lengthen the distribution by extension of sextupole resonance. Result of this procedure optimisation is shown in fig.3.

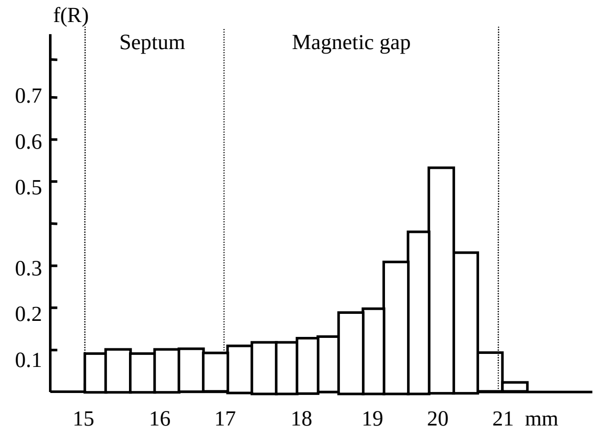

Fig.3. Optimised distribution of particles in kicker magnet cross section at decapole perturbation of sextupole resonance

The particles loss in septum was reduced two times and efficiency of the beam ejection has raised by 20%.

Deformation of distribution shown in fig.3 is a result of the rate amplitude moderation with approach to the septum. In the vicinity of septum the particles are jumping hardly over the obstacle to the distant edge of septum, but at the more distance from the septum, the jumps are further and particles hit the magnetic gap free space.

Fig.4 and Fig.5 show phase space trajectories near unperturbed and perturbed sextupole resonance

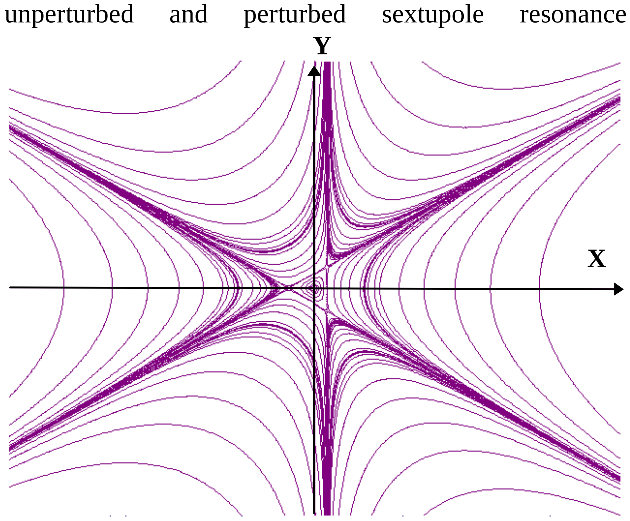

Fig.4. Phase-space trajectories near sextupole resonance

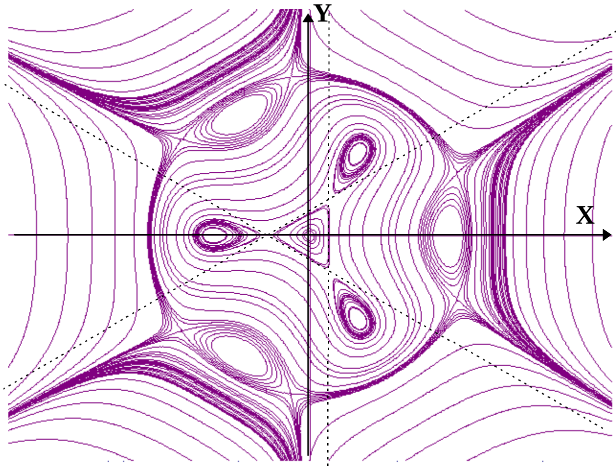

Fig.5. Phase-space trajectories near perturbed sextupole resonance

described by Hamiltonian

$$H=\frac{\Delta\nu}{2}(X^2+Y^2)+\frac{a}{24\nu}X[(X^2-3Y^2)-\frac{3}{4}k\{X^4-3Y^4-2X^2Y^2\}]$$

The open domain of unstable motion around a small triangle shown in Fig.4 is transformed to the ring domain of formally stable motion with fixed points of two levels, shown in Fig.5. The unexcited beam is located in the vicinity of origin, so trajectories streamlining three fixed points of the first level is only occupied. The kicker magnet phase space image is placed near the external side of a fixed point separatrix for the best interception of particle by particle of the whole beam.

Fig.6 and Fig.7 show co-ordinate/angle distribution of the beam in the cross section of kicker magnet for the cases of conventional and perturbed resonance. One can

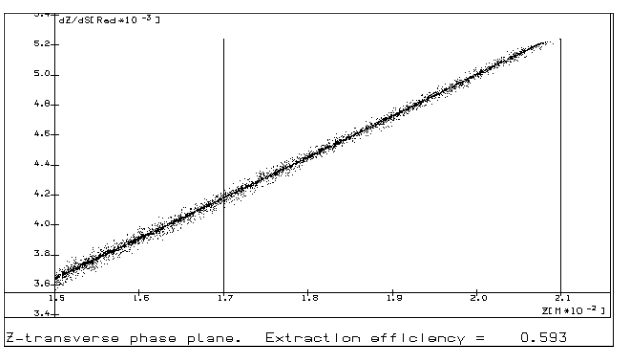

Fig.6. Beam phase portrait for ejection at conventional sextupole resonance

see that the growth of ejection efficiency is accompanied by some swelling of the transverse phase volume. This is a natural fee for the ejected beam intensity increase.

Emittance growth is a serious problem and disadvantage of this method if it can't be compensated by some way. We haven't try to seek minimum emittance conditions but it's clear that the value of emittance growth depends on a betatron oscillation phase shift that happens due to the additional non-linear field component. The value of this phase shift may be regulated by changing some parameters and conditions of slow extraction system. Though there are some complications in theoretical treatment and in the choice of the optimum configuration for ejection system, we expect to get answers for main questions in near future.

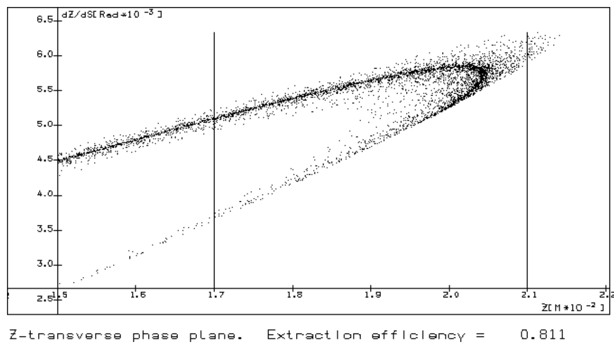

Fig.7. Beam phase portrait for ejection at perturbed sextupole resonance

CONCLUSION

The method of beam loss minimisation at slow extraction based on the relevant resonance perturbation was considered analytically and by computer simulation. It was shown that an ejection efficiency growth may be obtained with the n+2 order perturbation of the n order resonance that using for extraction. The optimum value of perturbation for the maximum gain of efficiency depends on the machine configuration and the initial parameters of the unexcited beam. The problem of ejected beam emittance growth is the subject for future research.

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