SELFACCELERATION OF ELECTRONS IN ONE-DIMENSIONAL BUNCHES,MOVING IN COLD PLASMA

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Abstract

Nonlinear dynamics of the one-dimensional ultrarelativistic bunch of electrons,moving in cold plasma,is considered in multiple scales perturbative approach.A square root of the inverse Lorentz factor of the bunch electrons is taken as a small parameter.Bunch electrons momenta is changed in the first approximation.In the underdense plasma and for the model example of the combined bunch the selfacceleration of the bunch electrons can be remarkable.

1 INTRODUCTION

Nonlinear wake waves exitation in overdense plasma by relativistic electron (positron) one-dimensional bunches,when it is possible to obtain an exact analytical solution,was considered in [1]-[7].The bunch assumed as a given one (rigid bunch approximation) in the most of these work.

The buck influence of the exited plasma wake on electron bunch was considered numericaly in [7],[8].Some attempts of the analytical treatment of the problem have been performed in [8]-[12].

Langmuir already noticed, that the beam, which has passed the plasma column, contains a significant portion of electrons with the energies higher than the initial energy of the beam. In the recent times various groups (see e.g. [11]-[13]) also observed experimentally the effect of the selfacceleration.

In the present work the problem of the plasma back nonlinear influence on the driving one-dimensional electron bunch is treated by the method of multiple scales [16].The bunch is ultrarelativistic and the square root of the inverse Lorentz factor of the bunch $\epsilon \equiv \gamma_0^{-1/2} \ll 1$ is taken as a small parameter.

It is assumed that the bunch-plasma interaction takes place by two stages. First one is the formation of the stationary plasma wake field,generated by the rigid bunch and the second stage is the influence by this field on the momenta of the bunch electrons and wake field itself.This assumption is valid,when $\gamma_0 \gg 1$.Indeed,the time τ_s needed for the formation of the stationary wake in a plasma with density n_0,generated by the electron bunch with the density $n_b, n_b/n_0 < 1/2 - \triangle, \frac{1}{8}\gamma_0^{-2} \ll \triangle < 1/2$,is $\tau_s \sim \omega_p^{-1}, \omega_p^2 = 4\pi e^2 n_0/m$.The time required for the change of the bunch electrons momenta p_0 is $\tau_p \sim p_0/eE_0$ where E_0 is the electric field inside the bunch.In the overdense ($\frac{n_b}{n_0} < 1/2$) plasma $E_0 \leq \frac{mc\omega_p}{e}$;in the underdense ($\frac{1}{2}n_0 \ll n_b$) plasma $E_0 \leq \frac{mc\omega_p}{e}(\frac{n_b}{n_0}\gamma_0)^{1/2}$,[5]. Hence in the overdense plasma $\tau_s/\tau_p \leq \gamma_0^{-1} \ll 1$,and in the underdense plasma $\tau_s/\tau_p \leq (\frac{n_b}{n_0})^{1/2}\gamma_0^{-1/2} \ll 1$.(For some special values of $\frac{n_b}{n_0} \simeq 1/2$ these conditions can change the form or even violate).As in [5] consider the flat electron bunch with the infinite transverse dimensions,longitudinal length d and initial homogenious charge density n_b,moving in the lab system with the initial velocity v_0 through neutral cold plasma with the immobile ions.

In the work [17] it was shown that when the beam is traversing the semiinfinite plasma after a few plasma wave lengths transient effects dissipate and stationary wake field regime is established in coinsideness with the abovementioned estimate. In what follows this moment is taken as an initial one,$t = 0$,and the future development for $t > 0$ of the bunch -plasma system is considered.

2 FORMULATION OF THE PROBLEM

The considered electron bunch -cold plasma system is described by the hydrodynamic equations of the motion,continuity equations for charge densities and currents for the bunch and plasma electrons,and Maxwell equation (Coulomb law) for the electric field.

Dimensionless variables and arguments are introduced:

$$\begin{aligned} t' &= \omega_p t, z' = k_p z, \omega_p^2 = \frac{4\pi e^2 n}{m}, k_p = \omega_p/c, \\ E &= \frac{mc\omega_p}{e}E', n'_e = \frac{n_e}{n}, n'_b = \frac{n_b}{n}, \beta_e = \frac{v_e}{c}, \\ \rho_e &= \frac{p_e}{mc}, \rho_b = \frac{p_b}{mc\gamma_0}, \beta_b = \frac{v_b}{c}, \gamma_0 = (1-\beta_0^2)^{-1/2} \end{aligned} \quad (1)$$

where $p_e, v_e, n_e; p_b, v_b, n_b$ are the momenta,velocity and density of the plasma and bunch electrons subsequently,n is a normalizing constant,which is suitable to choose $n = n_0$ for overdense plasma and $n = n_b$ for underdense plasma cases,$\beta_0 = v_0/c$ is the initial velocity of the bunch electrons in the laboratory system.

The equations,which describe the considered problem are:

$$\begin{aligned} \frac{\partial \rho_e}{\partial t} &+ \beta_e \frac{\partial \rho_e}{\partial z} = -E \\ \frac{\partial \rho_b}{\partial t} &+ \beta_b \frac{\partial \rho_b}{\partial z} = -\frac{1}{\gamma_0}E \\ \frac{\partial n_e}{\partial t} &+ \frac{\partial}{\partial z}(\beta_e n_e) = 0 \\ \frac{\partial n_b}{\partial t} &+ \frac{\partial}{\partial z}(\beta_b n_b) = 0 \\ \frac{\partial E}{\partial z} &= n_0 - n_e - n_b \end{aligned} \quad (2)$$

Eqs. (2) are written in dimensionless variables (1) and prime is omitted.Considering the ultrarelativistic

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bunch,introduce a small parameter $\epsilon^2 = 1/\gamma_0$,and all variables,entering in (2),let be a functions of the fast $\tilde{z} = z - \beta_0 t$ and slow $\tau = \epsilon t, \zeta = \epsilon z$ arguments.According to the multiple scale method [16],all variables,entered in (2),are developed in the following serieses:

$$\begin{aligned} \rho_e &= \rho_{e0}(\tilde{z}) + \epsilon\rho_{e1}(\tilde{z},\zeta,\tau) + \cdots, \\ \beta_e &= \beta_{e0}(\tilde{z}) + \epsilon\beta_{e1}(\tilde{z},\zeta,\tau) + \cdots, \\ \rho_b &= \rho_{b0} + \epsilon\rho_{b1}(\tilde{z},\zeta,\tau) + \cdots, \\ \beta_b &= \beta_{b0} + \epsilon\beta_{b1}(\tilde{z},\zeta,\tau) + \cdots, \\ n_e &= n_{e0}(\tilde{z}) + \epsilon n_{e1}(\tilde{z},\zeta,\tau) + \cdots, \\ n_b &= n_{b0} + \epsilon n_{b1}(\tilde{z},\zeta,\tau) + \cdots, \\ E &= E_0(\tilde{z}) + \epsilon E_1(\tilde{z},\zeta,\tau) + \cdots. \end{aligned} \tag{3}$$

In (3) $\rho_{b0}, \beta_0, n_{b0}$ are the constants connected with the initialy rigid electron bunch and differs from zero, when $0 \leq \tilde{z} = z - \beta_0 t \leq d$. The functions $\rho_{e0}(\tilde{z}), \beta_{e0}(\tilde{z}), n_{e0}(\tilde{z}), E_0(\tilde{z})$ are the solutions of the steady state (stationary) problem,and are obtained in [1],[3],[5].Derivatives in (2),according to the multiple scales method are given by

$$\frac{\partial}{\partial t} = -v_0\frac{\partial}{\partial \tilde{z}} + \epsilon\frac{\partial}{\partial \tau}, \quad \frac{\partial}{\partial z} = \frac{\partial}{\partial \tilde{z}} + \epsilon\frac{\partial}{\partial \zeta} \tag{4}$$

Decompositions (3) correspond to the above mentioned main assumption,which,as it was seen,is valid for the ultra-relativistic bunch.

The following steps are evident– decomposition of the eqs. (2),using (3) provides the sequence of the guasilinear equations,describing the steady state regime as a zero approximation and in the next approximations-the back influence of the generated electric field on initialy rigid electron bunch and on wake wave itself.

3 SOME RESULTS

1. The change of the bunch electron momenta (in usual units) arise in the first approximation and is

$$\rho_{b1} = -eE_0(\tilde{z})t\gamma_0^{1/2} \tag{5}$$

i.e. bunch electrons traversing underdense plasma decelerate $(E_0(z) \geq 0)$; in the bunch traversing overdense plasma electrons from the head part of the bunch decelerated, electrons from the rear part-accelerated ($E_0(z) < 0$). Hence the bunch traversing overdense plasma contracted around the point $z_0 = d - \frac{\lambda}{2}$ (λ - plasma nonlinear wave length), and bunch traversing underdense plasma-expands. The front of the bunch moves with the velocity v_0 in lab system. The end of the bunch d_0, which initialy ($t = 0$) was at $\tilde{z} = 0$ in overdense plasma case (contruction) for $t \geq 0$ is

$$d_0(t) = -\frac{t^2}{2\gamma_0^3}E(0) - \frac{t^3}{2\gamma_0^4}E^2(0) + \frac{t^4}{2\gamma_0^5}E^3(0) + \cdots \tag{6}$$

i.e. the change of the bunch length is by the order of magnitude $\sim \gamma_0^{-3}$

2. Only forth order corrections to plasma electrons momenta, plasma electron density and electric field are differ from zero and are proportional to γ_0^{-2}. It means that rigid bunch approximation for plasma wake wave characteristics is valid for ultrarelativistic case up to terms proportional to γ_0^{-2} for the time interval $t \leq \omega_p^{-1}\gamma_0^{1/2}$.

3. The large enough accelerating fields can be obtained in combined bunch, which consists from the first part with charge density $n_b^{(1)} \gg 1/2n_0$ (underdense regime) and second part with charge density $n_b^{(2)} \ll 1/2n_0$ (overdense regime). Then first bunch generates large (but decelerating) electric field, second bunch invert the sign of this field on the back side of the bunch, and selfacceleration of electrons from rear side of the bunch can take place, with acceleration gradient $G \sim \gamma_0^{1/2}$.

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