PERTURBATION OF THE PERIODIC DISPERSION UNDER BEAM CROSSING OPTICS IN LHC

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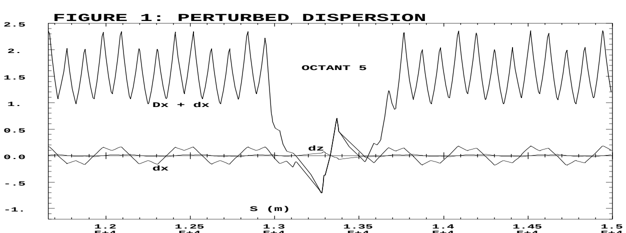
Abstract

Beam crossing and separation schemes in the LHC interaction regions impose non-zero closed orbit in the low-β triplets. The related perturbative dispersion is derived; propagation, multi-crossing interference, perturbative effects around the ring are investigated and quantified. Horizontal and vertical compensation schemes are presented.

1 INTRODUCTION

Crossing angle and orbit off-centering schemes at the interaction points (IP) in the LHC ring are foreseen [1][2], for the purpose of early separation of the beams so as to reduce harmful effects related to beam-beam interactions in that region where they share a common vacuum pipe. Such closed orbit (c.o.) geometry imposes horizontal and vertical off-centering in the low-β triplets, which has sensible effect on dispersion in collision optics when betatron functions reach very large values. This report provides an understanding and study of the building-up and effects of the anomalous dispersion in the LHC ring (Version 4.2), and investigates compensation schemes.

2 ANOMALOUS DISPERSION



2.1 Equation of the anomalous dispersion

The perturbative dispersion $d_y(s)$ due to $y_{co}(s)$ c.o. in the low-β triplets is the closed solution of [3]

$$d^2d_y/ds^2 + K(s)d_y = -\Delta B(s)/B\rho + K(s)y_{co} \quad (1)$$

with $y \equiv x \; or \; z$, $B\rho$ = particle rigidity, K(s) = quadrupole strength, and the field term $\Delta B(s)/B\rho$ is introduced by the c.o. dipoles. Eq. (1) can be solved in the elementary kick approximation $K(s)y_{co}(s) = \int K(s)y_{co}(s)\delta(s-s_q)ds_q$ which yields the periodic solution (Fig. 1)

$$d_y(s) + y_{co}(s) = \frac{\sqrt{\beta(s)}}{2\sin\pi\nu}\sum(KL)_q y_{co}(s_q)\cdot \sqrt{\beta(s_q)}\cos\nu[\pi - |\phi(s)-\phi(s_q)|] \quad (2)$$

where $\phi(s) = 1/\nu\int ds/\beta$ = normalized betatron phase, $\phi(s_q)$ = phase at the kick, β = betatron function, ν = machine tune. The closed orbit $y_{co}(s_q)$ at the kick can be expressed in terms of its transport from the IP (optical functions β^*, ϕ^* while $\alpha^* \equiv 0$ is assumed). This yields

$$d_y(s) = -y_{co}(s) + \{\frac{\sqrt{\beta(s)/\beta^*}}{2\sin\pi\nu} \quad (3)$$
$$+y^*\sum(KL)_q\beta(s_q)\cos\nu[\phi(s_q)-\phi^*]\cos\nu[\pi-|\phi(s)-\phi(s_q)|]$$
$$+y'^*\beta^*\sum(KL)_q\beta(s_q)\sin\nu[\phi(s_q)-\phi^*]\cos\nu[\pi-|\phi(s)-\phi(s_q)|]\;\}$$

2.2 Upper limits of the perturbation

Beyond the low-β triplets associated with the non-zero c.o. Eq. (2) can be written under the form $d_y(s)/\sqrt{\beta(s)} = -y_{co}(s)/\sqrt{\beta(s)} + \bar{D}_y\cos\nu[\phi(s)+\Omega]$, with

$$\bar{D}_y = \{[\sum(KL)_q y_{co}(s_q)\sqrt{\beta(s_q)}\cos\nu(\pi+\epsilon\phi(s_q))]^2 \quad (4)$$
$$+[\sum(KL)_q y_{co}(s_q)\sqrt{\beta(s_q)}\sin\nu(\pi+\epsilon\phi(s_q))]^2\}^{1/2}/(2\sin\pi\nu)$$

($\epsilon = \pm1$ for $\phi(s)\gtrless\phi(s_q)$, $\forall q$). Numerical calculation of the sums from first order optics yields [3]-[6]

$$\frac{\bar{D}_x|_{x^*=0}}{x'^*} \approx \frac{\bar{D}_z|_{z^*=0}}{z'^*} \approx 170,\; \frac{\bar{D}_x|_{x'^*=0}}{x^*} \approx \frac{\bar{D}_z|_{z'^*=0}}{z^*} \approx 2 \quad (5)$$

Since β_x and β_z have similar shapes Eq. (5) tells that the perturbation due to $10^{-4}rad$ c.o. angle ("d_x" plot in Fig. 1) is about ten times that due to $10^{-3}m$ c.o. off-centering ("d_z" plot in Fig. 1) . Extrema of $d_y(s) = \bar{D}_y\sqrt{\beta(s)}$ can be derived, this is studied in more details in Section 3.

2.3 Comparison with the effects of $D1/D2$ dipoles

Dispersive effects due to crossing can be compared to those due to the separator/recombiner dipoles $D1/D2$, in particular in view of simultaneous compensation by an optical assembly such as proposed in [7]. A single dipole (D1 or D2) with bend Θ_D excites a dispersion of closed form

$$\frac{^D d_x(s)}{\sqrt{\beta(s)}} = \frac{\Theta_D}{2\sin\pi\nu} <\sqrt{\beta(s_D)}> \cos\nu[\pi-|\phi(s)-\phi(s_D)|] \quad (6)$$

with $<\sqrt{\beta(s_D)}>$ = mean value of $\sqrt{\beta(s_D)}$ and assuming $\phi(s_D) \approx C^{ste}$, over a dipole. The overall perturbation is obtained by superposing the effects of the two pairs $D1/D2$, which, with $\phi(s_{D1}) \approx \phi(s_{D2})$, leads to

$$\frac{^{D1/D2}d_x(s)}{\sqrt{\beta(s)}} = {}^{D1/D2}\bar{D}_x\cos\nu[\phi(s)+\tau] \quad (7)$$
$$\approx \frac{\Theta_D}{2\sin\pi\nu}[<\sqrt{\beta(s_{D1})}> - <\sqrt{\beta(s_{D2})}>]$$

Given $<\sqrt{\beta(s_{D1})}> \gg\approx \sqrt{<\beta(s_{D1})>}$ and $\Theta_D = 2.17\,10^{-3}rad$, it comes out $^{D1/D2}\bar{D}_x \approx 460\,10^{-4}$, which yields about $\pm0.6\,m$ modulation at $\beta(s) = 180\,m$. This can be readily compared to the analogous coefficients due to $x'^* = 10^{-4}rad$ c.o. angle (Eq. 5), namely

$$^{Crossing}\bar{D}_x \;/\; ^{D1/D2}\bar{D}_x \approx 170/460 \approx 35\% \quad (8)$$

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In other words, the modulation in the arcs due to x'^* is $\pm 0.35 \times 0.6 \approx \pm 0.2\,m$ (Fig. 1). It also means that a correction scheme intended to compensate the dispersion due to $D1/D2$ can take care in addition of $10^{-4} rad$ c.o. angle by changing its strength (increase or decrease depending on the crossing sign) by about 35%.

3 TYPICAL EFFECTS OF CROSSING ANGLE GEOMETRY

We consider the sole crossing scheme ($y^* = 0, y'^* \neq 0$), which has the major perturbative effect as shown above (Eq. 5). Beyond the crossing region Eq. (3) leads to [3]

$$d_y(s < s_{q_{Left}}, s > s_{q_{Right}}) = -y_{co}(s) \tag{9}$$
$$\pm y'^* \sqrt{\beta(s)\beta^*}/(2\sin\pi\nu)\sin\nu[\pi - |\phi(s) - \phi^*|] \sum (KL)_q \beta(s_q)$$
$$(\pm 1 \text{ for resp. } \phi(s) \gtrless \phi(s_q), \forall q)$$

Extrema in the arcs

These are attained when $\sin\nu[\pi - |\phi(s) - \phi^*|] \approx 1$. Considering that $\beta_{max}(s) \approx 180\,m$ while $\sum (KL)_q \beta(s_q) \approx 370$ in odd-type IR's [3], it comes for x'^* *or* $z'^* = 10^{-4} rad$ c.o. angle, $d_{x,extr} < 0.228\,m$ ($\nu_x = 63.28$), i.e., about 10% of the first order dispersion ; or $d_{z,extr} < 0.212\,m$ ($\nu_z = 63.31$).

Extrema in low-β triplets

The phase in triplets is $\phi(s) \approx \phi(IP) \pm \pi/2\nu$ while $\beta_{max} \approx 4430\,m$ at $IP1/5$, $\approx 4020\,m$ at $IP2/8$. Given the c.o. angle $x'^* = 10^{-4} rad$ at $IP5$ and betatron phases $\phi(IP1) = 0, \phi(IP2) = 2\pi 8.985/\nu_x, \phi(IP5) = \phi^* = \pi, \phi(IP8) = 2\pi 55.745/\nu_x$, $\nu \equiv \nu_x = 63.28$ and $\beta^* = 0.5\,m$, Eq.(9) yields

$$d_{x,extr} \text{ at } IR1, 2, 5, 8 = 1.13\,m,\ 1.07\,m,\ -0.71\,m,\ -1.05\,m \tag{10}$$

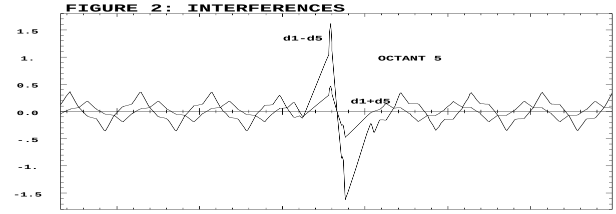
Dispersion at IP's

Eq. (9) with $\phi(s) = \phi(IP)$ and phase values above yield

$$d_x(IP1, 2, 5, 8) = 0, 1.08\,10^{-3} m, 1.38\,10^{-3} m, -2.58\,10^{-3} m \tag{11}$$

which gives negligible beam size increase for $\delta p/p = 10^{-4}$.

FIGURE 2: INTERFERENCES



Interferences

By virtue of the superposition principle interferences occur under crossings at several IP's. Two-IP interference for instance, in the case of a pair of inclined crossing geometries, or in the case of four alternating crossings [8]. Consider $z'^* = \pm 10^{-4} rad$ vertical c.o. angle at $IP1$ and $IP5$. Given $\phi(IP1) = 0$, $\phi(IP5) = \pi, \nu \equiv \nu_z = 63.31$, the resulting extremum in $IP5$ low-β triplets is (Eq. 9)

$$d_{z,extr} = \pm z'^* \frac{\sqrt{\beta_{max}\beta^*}}{2\sin\pi\nu}(1 + \epsilon\cos\pi\nu) \sum (KL)_q \beta(s_q)\ (\epsilon = \pm 1) \tag{12}$$

yielding, $d_{z,extr} \approx 0.46\,m$ for identical sign crossings ($\epsilon = 1$), $d_{z,extr} \approx 1.64\,m$ for opposite signs ($\epsilon = -1$) (Fig. 2).

Strong effects may arise from four-IP interference (non-alternating crossing configuration [8]). Consider c.o. angles $x'^* = \epsilon_{IP}\, 10^{-4} rad$ with signs either identical, $\epsilon_1 = \epsilon_2 = \epsilon_5 = \epsilon_8 = 1$ or alternate, $\epsilon_1 = \epsilon_2 = 1$ and $\epsilon_5 = \epsilon_8 = -1$. The perturbation at $IP5$ low-β triplet reaches

$$d_{x,extr} = \pm x'^* \sqrt{\beta_{max}\beta^*}/(2\sin\pi\nu) \tag{13}$$
$$\sum_{IP=1-8} \epsilon_{IP} \cos\nu[\pi - |\phi(IP) - \phi(IP5)|] \sum (KL)_q \beta(s_q)$$

yielding, $d_{x,extr} \approx 0.42\,m$ if all crossings have identical signs, $d_{x,extr} \approx 4.1\,m$ in the second case.

4 CORRECTION SCHEMES

4.1 Self-absorption within regular IR tuning procedures

The simplest way to compensate the anomalous dispersion is by re-tuning the IR. As expected from $d_x(s) \approx 10\% D_x(s)$ under $\pm 10^{-4} rad$ c.o. angle (after Eq. 5), doing so leads to very limited changes in the Q1-Q10 IR quadrupole strengths. As to the optical functions, there is no meaningful difference with the unperturbed ones [3].

4.2 Quadrupole correction of the horizontal dispersion

Corrector strength

Quadrupole correctors excite a perturbative dispersion which superposes with that due to c.o. in the low-β triplets. This translates to additional term $\int K_Q(s) d_x(s) \delta(s - s_Q) ds_Q$ in Eq. (1) (index Q stands for the correctors). Besides, minimizing the corrector strength imposes on the one hand $\phi(s_Q) = \phi(s_q) + \pi/\nu[modulo\,\pi/\nu]$, on the other hand maximizing $D_x(s_Q)\sqrt{\beta_x(s_Q)}$ (which also minimizes effects on the orthogonal plane). Considering that $\phi(s_q)$ and $D_x(s_Q)\sqrt{\beta_x(s_Q)} \approx C^{ste}$ the correction strength writes

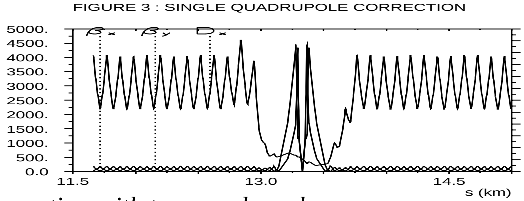
$$\sum_Q (KL)_Q = \pm \sum_q (KL)_q x_{co}(s_q) \sqrt{\beta_x(s_q)}\ /\ D_x(s_Q)\sqrt{\beta_x(s_Q)} \tag{14}$$

Numerical calculations for odd IR give $\sum (KL)_q\, x_{co}(s_q)\, \sqrt{\beta_x(s_q)}|_{Left/Right} = -1.12\,10^{-2}$ / $1.50\,10^{-2}$ for respectively the left and right low-β triplets. Hence the integrated strengths that independently close the left and right dispersion bumps : $|(KL)_Q|_{Left/Right}| \approx 3.9\,10^{-4}$ / $5.2\,10^{-4} m^{-1}$.

Correction with a single quadrupole

A single quadrupole with strength $9\,10^{-4} m^{-1}$ (after Eq. above) is sufficient to cure the anomalous dispersion, since the two π/ν apart low-β triplets sources of the defect excite independent perturbations that add in phase. It may be placed close to MSCBH multipole and would excite a defect in phase opposition thus canceling the anomalous dispersion beyond the local chromatic bump so determined. Fig. (3) shows the resulting second order dispersion at Octant 5, prior to any re-tuning of the IR, to be compared to the uncorrected situation (curve "($D_x + d_x$)" in Fig. 1). Yet a single quadrupole has sensible effect on the tune and β mismatch, namely, $\Delta\nu_x = \beta_x(s_Q)(KL)_Q/4\pi \approx 1.3\,10^{-2}$ ($\beta_x(s_Q) = 178\,m$), $\Delta\nu_z = 0.23\,10^{-2}$ ($\beta_z(s_Q) = 32\,m$), and $\Delta\beta_x/\beta_x < \beta_x(s_Q)|(KL)_Q|/2\sin(2\pi\nu_x) \approx \pm 8.5\%$, $\Delta\beta_z/\beta_z \approx \pm 1.5\%$ (with $\nu_x,\ \nu_z \approx 63.3$).

FIGURE 3 : SINGLE QUADRUPOLE CORRECTION



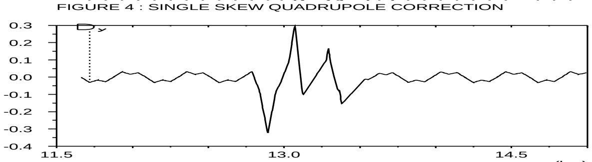
Correction with two quadrupoles

These effects can be taken care of to good level ($<$ 1% dispersion beating, $< 3\%$ β-beat and at worst .0018 tune shift, prior to any re-tuning of the IR) by using two quadrupoles ; this could constitute a minimal correction scheme, yet there are several possibilities more or less beneficial w.r.t. residual dispersion, tune shift and β-beat : the two quadrupoles can be placed one at each end of the IR, or both at the same end, with each one half the strength $|(KL)_Q/2| \approx 4.5\,10^{-4}m^{-1}$; this has the effect of avoiding tune-shift and β-beats. They can be placed one at each end of the IR, with strengths $3.9\,10^{-4}/5.2\,10^{-4}m^{-1}$ to balance the opposite low-β triplet ; this brings quasi-zero dispersion and derivative at the IP.

Correction with four interlaced quadrupole pairs

Following a correction scheme proposed for SSC [9], the method above has been extended to four pairs of quadrupoles. Such correction scheme is also assimilable within the modular LHC IR tuning scheme [7] and other Q-shift system [10]. As expected from the discussions above, the correction is very efficient in terms of tune-shift, β-beat and dispersion. More details can be found in [3].

FIGURE 4 : SINGLE SKEW QUADRUPOLE CORRECTION



4.3 Correction of the vertical dispersion

The vertical anomalous dispersion can be compensated by skew quadrupoles (as proposed at SSC [9]) located at arc ends close to MSCBV correctors and maxima of $D_x\sqrt{\beta_z}$ and low β_x. Their role is to couple the horizontal dispersion into the vertical plane.

Corrector strength

The vertical dispersion verifies $d^2d_z/ds^2 + K(s)d_z = R(s)D_z$. The closed solution is (after Eq. 2)

$$d_z(s) = \frac{\sqrt{\beta_z(s)}}{2\sin\pi\nu_z}\cdot$$

$$\sum(RL)_{SQ}D_x(s_{SQ})\sqrt{\beta_z(s_{SQ})}\cos\nu[\pi-|\phi(s)-\phi(s_{SQ})|] \quad (15)$$

where index SQ denotes the correctors, R = skew quad strength. Taking $\phi(s_{SQ}) = \phi(s_q) + \pi/\nu[\pi/\nu]$ while $\phi(s_q)$ and $D_x(s_{SQ})\sqrt{\beta_z(s_{SQ})}\approx C^{ste}$ gives the correction strength

$$\sum(RL)_{SQ}=\sum(KL)_q z_{co}(s_q)\sqrt{\beta_z(s_q)}/D_x(s_{SQ})\sqrt{\beta_z(s_{SQ})}$$

$(RL)_{SQ}|_{Left/Right} \approx 10.6\,10^{-4}/7.9\,10^{-4}m^{-1}$ is necessary for balancing the effects of the left and right low-β triplets under $z'^* = 10^{-4}rad$ c.o. angle at IP [3]-[6].

Correction with a single skew quadrupole

The corrector is placed at an arc end next to a MSCBV multipoles with the strength $18.2\,10^{-4}m^{-1}$ (Eq. above). Dispersion does not exceed 0.32 m in the crossing octant (Fig. 4), it is less than 0.05 m everywhere else in the ring (see the uncorrected situation, curve "d_x" in Fig. (1)).

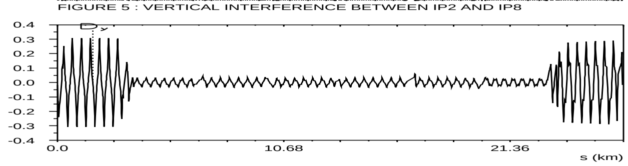
Interlaced correction scheme

Residual effects on the first order focusing are weak ; however they can be improved by using quadrupole pairs ; doing so damps the dispersion to 0.2 m in the crossing low-β triplet. The philosophy is the same as above, for the horizontal plane ; more details can be found in [3].

Interferences

If no correction of the vertical dispersion is foreseen, yet some benefit may be drawn from interference, as long as adequate phase relation is fulfilled between IP's of concern. Fig. 5 shows such self-cancellation in the range IR2/IR8 when setting $z'^* = 10^{-4}rad$ c.o. angle at IP2 and IP8 simultaneously. This plot can be readily compared to the situation due to a single crossing (curve "dx" in Fig. 1, and extrema at all IP's, Eq. (10)).

FIGURE 5 : VERTICAL INTERFERENCE BETWEEN IP2 AND IP8



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