MEASUREMENT AND ANALYSIS OF LONGITUDINAL BUNCHED BEAM ECHOES IN THE FERMILAB TEVATRON

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Abstract

Bunched beam echoes have been generated in the Fermilab Tevatron using a simple impulse kick technique involving the rf system. The echoes occur as a result of the nonlinear mixing of quadrupole and dipole components of successive impulse kicks, and may be readily observed using a bunch phase measurement system. The echo contains information on the synchrotron tune spread, as well as on various nonlinear phenomena, including the nonlinear tune spread and the effects of noise on the incoherent synchrotron motion. By measuring the detailed form of the echo response as well as its overall envelope as a function of the echo delay time, it is possible, in principle, to extract some of this information. In this paper we present experimental results along with a perturbation model which includes the effects of a nonlinear tune spread. In particular, we wish to explain the observed rapid decay of the echo envelope in terms of relevant beam parameters.

1 INTRODUCTION

Echoes have been studied in a variety of media for many years as a means of experimentally determining aspects of the dynamics of constituent particles (refs. [1] [2] [3]). In recent years, echoes have also been studied in high energy synchrotrons, and have been used to measure diffusion rates, although only in coasting beams [4] [5].

In this work we report the observation of bunched beam echoes in a high energy synchrotron and present a model which describes aspects of the experimental observations. At issue is whether such echoes can be used as diagnostic tools for determining experimentally various beam parameters.

Echoes are formally a nonlinear wave phenomenon whereby two harmonically related waves mix to provide a disturbance separated in time or space from the original excitation. Since echoes can occur at times much longer than a decoherence time, they may contain information about the long-term incoherent particle dynamics of a given system. Although echoes have often been viewed as a consequence of wave mixing, they can also occur in systems with negligible self-fields as a superposition of the motion of free-streaming particles, which is the view taken in this paper. We are interested in determining which aspects of the beam parameters can be determined from the properties of the echo response.

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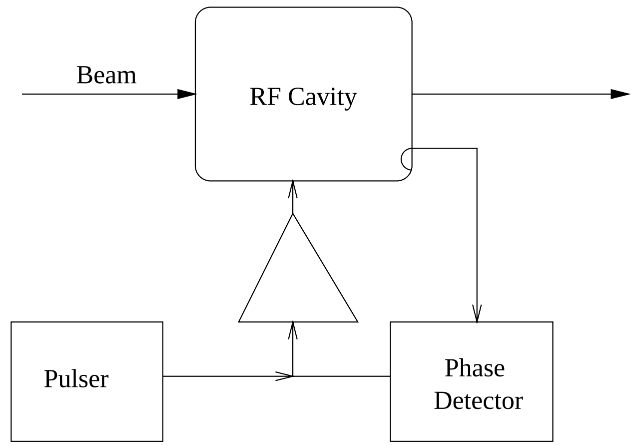

Figure 1: Sketch of the experimental setup. Successive impulses are applied with a pulser to the low-level phase feedback loop of the rf system. The resulting loop response applies a kick to the bunch energy.

2 EXPERIMENTAL RESULTS

Bunched beam echoes can be readily generated by two successive kicks in either energy or phase to a single stored bunch. The impulse must be much shorter than a synchrotron period, and sufficiently small so that the energy gain during a single kick is small compared to the bunch height. In the Tevatron, such a kick may be easily applied by introducing an impulse into the low-level rf phase loop, as shown in Fig. 1.

Two such kicks are applied separated by a variable delay, resulting in a response as shown in Fig. 2. Depicted is the bunch arrival phase relative to the synchronous phase established by the rf frequency. It is noted that the initial kicks decay in a time determined by the inverse frequency spread of the beam. The echo occurs as a temporally localized response at the synchrotron frequency at a time approximately equal to the delay between the two excitation impulses.

As the delay between the two excitation impulses is varied, the envelope of the echo maps out a characteristic curve, as shown in Fig. 3. Based on the experience with echoes in unbunched beams, it may be conjectured that the decay of the envelope may be due to stochastic processes affecting the beam dynamics, but it also may be due to the nonlinearity of the tune spread [6]. However, the observed echo decay is much faster than that seen in unbunched beams. It is our goal in the remainder of this paper to develop a model which would help determine the relative importance of these two effects.

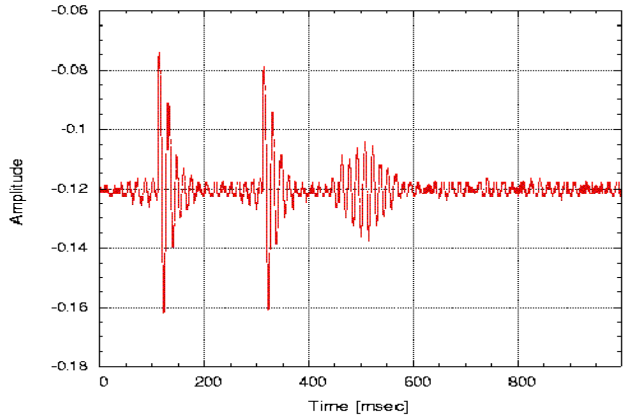

Figure 2: Bunched-beam echo generation in the Tevatron following two impulse kicks applied to a bunch through the rf system. A phase detector is used to detect the synchrotron oscillations associated with the impulse kicks. In this case the quadrupole portion of the second kick combines with the dipole portion of the first kick to produce a dipolar echo response. The phase amplitude is shown in arbitrary units.

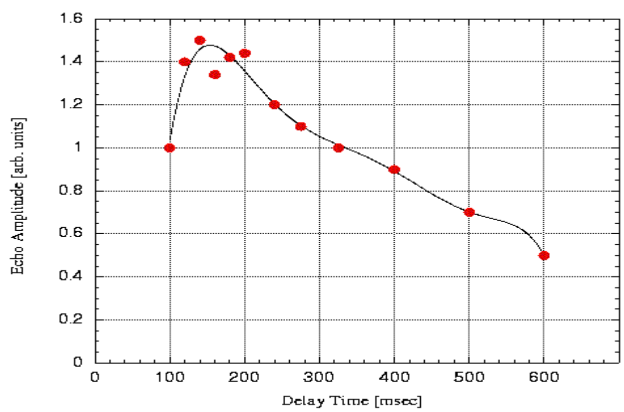

Figure 3: Decay of the echo envelope as a function of the delay time between impulse kicks. The decay rate indicates the combined influence of noise and nonlinear decoherence on the beam response.

3 PERTURBATION MODEL

A simplified expression for the dipole echo can be derived, assuming the absence of wakefields and external noise, using the same expansion of the Vlasov equation to second order used to describe three-wave coupling [6]. The analysis is carried out, however, in action-angle coordinates since the characteristics of the motion form level curves in this space. The transformation is given by

$$J = \tau^2 + (\frac{\dot{\tau}}{\omega_s})^2$$

$$\tau = \sqrt{J} cos\,\phi$$

$$\frac{\dot{\tau}}{\omega_s} = \sqrt{J} sin\,\phi$$

where τ is the arrival time of a particle relative to the synchronous time, and ω_s is the synchrotron frequency. We assume a Gaussian distribution of particles, which in normalized form is given as

$$\psi_o = \frac{1}{2\pi J_o} e^{\frac{-J}{J_o}}$$

After a kick of magnitude Δ_1 at t = 0, the perturbed particle distribution can be written

$$\psi_1 = \psi_o + \frac{\partial \psi_o}{\partial \frac{\dot{\tau}}{\omega_s}} \Delta_1 + \frac{1}{2} \frac{\partial^2 \psi_o}{\partial \frac{\dot{\tau}}{\omega_s}^2} \Delta_1^2 + \cdots$$

which, in action-angle coordinates is given by

$$\psi_1 = \psi_o + 2\sqrt{J} sin(\phi + \omega_s t) \frac{\partial \psi_o}{\partial J} \Delta_1$$
$$+ {\Delta_1}^2 (2\sqrt{J} sin[\phi + \omega_s t] \frac{\partial}{\partial J} [2\sqrt{J} sin\{\phi + \omega_s t\} \frac{\partial \psi_o}{\partial J}]$$
$$+ \frac{cos[\phi + \omega_s t]}{\sqrt{J}} \frac{\partial}{\partial \phi} [2\sqrt{J} sin\{\phi + \omega_s t\} \frac{\partial \psi_o}{\partial J}])$$

We are only concerned with the dipolar part of the response, which is given by

$$< \tau > = \frac{1}{2} \int_0^\infty dJ J \int_0^{2\pi} d\phi \, cos\,\phi \, \psi_1(\phi, J, t)$$

Evaluation of this expression for a Gaussian distribution leads to the familiar result that the perturbation decoheres in a Landau damping time, given approximately by the inverse tune spread over the bunch radius, J_o. If now a second kick is made at $t = t_1$, the new particle distribution can be written as

$$\psi_2 = \psi_1 + 2\sqrt{J} sin(\phi + \omega_s [t - t_1]) \frac{\partial \psi_1}{\partial J} \Delta_2$$
$$+ \frac{cos(\phi + \omega_s [t - t_1])}{\sqrt{J}} \frac{\partial \psi_1}{\partial \phi} \Delta_2$$
$$+ \frac{{\Delta_2}^2}{2} (2\sqrt{J} sin[\phi + \omega_s \{t - t_1\}]$$
$$\frac{\partial}{\partial J} [2\sqrt{J} sin\{\phi + \omega_s t\} \frac{\partial \psi_1}{\partial J} + \frac{cos[\phi + \omega_s t]}{\sqrt{J}} \frac{\partial \psi_1}{\partial \phi}]$$
$$+ \frac{cos[\phi + \omega_s t]}{\sqrt{J}} \frac{\partial}{\partial \phi} [2\sqrt{J} sin\{\phi + \omega_s t\} \frac{\partial \psi_1}{\partial J}$$
$$+ \frac{cos(\phi + \omega_s t)}{\sqrt{J}} \frac{\partial \psi_1}{\partial \phi}])$$

We may neglect the Δ_1 and Δ_2 terms, as they only describe the decoherence associated with the original kicks, as noted above. Similarly, the ${\Delta_1}^2$ and ${\Delta_2}^2$ terms do not produce echoes. Examination of the $\Delta_1 \Delta_2$ terms shows they do not produce a dipole moment. Hence only the $\Delta_1 {\Delta_2}^2$ and ${\Delta_1}^2 \Delta_2$, which are the mixing of dipole and quadrupole excitations, are associated with the echo response. After considerable algebra, carrying out the ϕ integration, we arrive at the following expression for the dipole echo, which arises from the beating of the dipole and quadrupole oscillations of the successive impulses.

$$
\begin{aligned}
<\tau>_{echo} = \frac{\pi}{8}\int_0^\infty & dJ\{(JC)'\cos(\omega_s[t-t_1]) \\
& + (J[B-A])'\sin(\omega_s[t-t_1]) \\
& + 2J(A'+B')\sin(\omega_s[t-t_1])\}
\end{aligned}
$$

where primes correspond to $\frac{\partial}{\partial J}$ and

$$
\begin{aligned}
A(J) &= 2\cos(\omega_s t_1)\frac{\partial\psi_o}{\partial J} \\
&+4J\frac{\partial^2\psi_o}{\partial J^2}\cos(\omega_s t)\cos(\omega_s[t-t_1]) \\
&-4Jt\frac{\partial\omega_s}{\partial J}\frac{\partial\psi_o}{\partial J}\sin(\omega_s t)\cos(\omega_s[t-t_1])
\end{aligned}
$$

$$
\begin{aligned}
B(J) &= 2\cos(\omega_s t_1)\frac{\partial\psi_o}{\partial J} \\
&+4J\frac{\partial^2\psi_o}{\partial J^2}\sin(\omega_s t)\sin(\omega_s[t-t_1]) \\
&+4Jt\frac{\partial\omega_s}{\partial J}\frac{\partial\psi_o}{\partial J}\cos(\omega_s t)\sin(\omega_s[t-t_1])
\end{aligned}
$$

$$
\begin{aligned}
C(J) &= 4Jt\frac{\partial\omega_s}{\partial J}\frac{\partial\psi_o}{\partial J}\cos(\omega_s[2t-t_1]) \\
&+4J\frac{\partial^2\psi_o}{\partial J^2}\sin(\omega_s[2t-t_1])
\end{aligned}
$$

Finally, dropping further terms which do not lead to a delayed response, we obtain an expression for the echo response, including nonlinear synchrotron tune spread contributions, to second order.

$$
\begin{aligned}
&<\tau>_{echo}= \\
&\tfrac{\pi}{2}\int_0^\infty dJ\{(J^2\psi_o''' + 2J\psi_o'' - J^2{\omega_s'}^2 tt_1\psi_o') \\
&\quad\times\sin(\omega_s[t-2t_1]) \\
&\quad-(2J\psi_o'\omega_s't + 2J^2\omega_s't\psi_o'' + J^2\omega_s''t\psi_o') \\
&\quad\times\cos(\omega_s[t-2t_1])\}
\end{aligned}
$$

where

$$
\omega_s = \omega_{so} + \omega_{so}'J + \frac{1}{2}\omega_{so}''J^2
$$

4 DISCUSSION AND CONCLUSIONS

While this expression can be evaluated further analytically, we are running out of space, so we will simply show the results in Fig.4. It is seen that the model reproduces the measured echo form, which differs from the unbunched beam case through the influence of the nonlinearity of the synchrotron frequency; namely it is responsible for the Gaussian echo shape. However, the experimental results indicate a rapid characteristic decay of the echo envelope with t_1 and we find that the model cannot reproduce this effect at any reasonable value of synchrotron tune nonlinearity.

Therefore, we find that the tune nonlinearity must not be responsible for the echo envelope decay, at least not as included it in this model (to second order). One possibility is the fact that our pertubation approach breaks down at sufficiently long times, as evidenced by the continuously growing amplitude in Fig. 4. However, the observed decay occurs at very short times where our expansion should still be valid. The implication is that other mechanisms, such as noise, must be playing a role. This leaves open the possibility that the echoes may be used to directly measure this noise. The analysis of this possibility will be left for a future work.

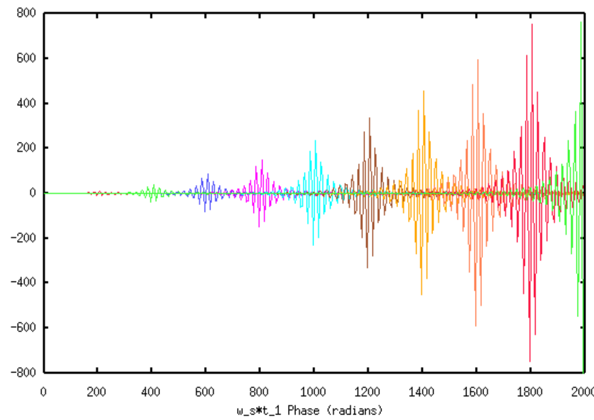

Figure 4: Theoretical echo response as a function of the delay time between excitation impulses. Multiple echoes are shown corresponding to equal increments of the impulse delay time, t_1. The echo amplitude increases nonlinearly with the delay time and the echo decoherence is determined by the inverse tune spread of the bunch.

5 REFERENCES

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