FLAT BEAMS IN A 50 TeV HADRON COLLIDER

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1 INTRODUCTION

The basic beam dynamics of a next generation 50 x 50 TeV hadron collider based on a high field magnet approach have been outlined over the past several years [1, 2, 3]. Radiation damping not only produces small emittances, but also flat beams, just as in electron machines. Based on "Snowmass 96" parameters, we investigate the issues associated with flat beams in very high energy hadron colliders.

2 FLAT BEAMS

The energy loss per particle per turn is

$$U_s = \frac{C_g E_s^4}{2\pi} C < G^2 > \tag{1}$$

where E_s is the nominal storage energy, C is the total circumference, and $G = 1/\rho$ is the dipole bending strength. Angle brackets $<>$ denote an average over the entire design trajectory circumference. The constant C_g is

$$C_g = 8.846 \times 10^{-5} \ \mathrm{m\,GeV^{-3}} \ \ \text{(electron)} \tag{2}$$
$$= 7.783 \times 10^{-18} \ \mathrm{m\,GeV^{-3}} \ \ \text{(proton)} \tag{3}$$

The exponential damping times for the amplitudes of horizontal, vertical, and longitudinal oscillations are given by

$$\tau_{x,y,s} = \tau_0 / J_{x,y,s} \tag{4}$$

where the characteristic time $\tau_0 = 2T(E_s/U_s)$ is simply related to the revolution period, T. Natural partition number values $(J_x, J_y, J_s) = (1, 1, 2)$ are assumed. Note that $U_s \sim B\gamma^3$ and $\tau_0 \sim 1/(B^2\gamma)$, where B is the dipole field and γ is the Lorentz factor, independent of the lattice optics structure. The equilibrium rms momentum width is

$$\left(\frac{\sigma_p}{p}\right)^2 = \frac{C_q \gamma^2}{J_s} \frac{< G^3 >}{< G^2 >} \tag{5}$$

while the natural *normalized* rms horizontal emittance is

$$\epsilon_x \equiv \gamma \frac{\sigma^2}{\beta} = \frac{C_q \gamma^3}{J_x} \frac{< G^3 H >}{< G^2 >} \tag{6}$$

In these equations the constant C_q is

$$C_q = 3.832 \times 10^{-13} \ \mathrm{m} \ \ \text{(electron)} \tag{7}$$
$$= 2.087 \times 10^{-16} \ \mathrm{m} \ \ \text{(proton)} \tag{8}$$

while H is a property of the FODO cell optics

$$H = \gamma\eta^2 + 2\alpha\eta\eta' + \beta\eta'^2 \tag{9}$$

* Operated by Associated Universities Incorporated, under contract with the U.S. Department of Energy.

Energy, E_s	50.0	TeV
Peak luminosity, L	10^{34}	$\mathrm{cm^{-2}s^{-1}}$
Circumference, C	89.0	km
Dipole field, B	12.5	T
Number of bunches, M	20,000	
Initial bunch intensity, N	$12.5\,\mathrm{x}10^9$	
Half cell length, L	260	m
Number of collision points	2	
Collision betas, $\beta_x^*,\ \beta_y^*$	$5.0,\ 0.5$	m
Natural emittance ratio, κ	0.1	
Full crossing angle, $\alpha/\sigma_y'^*$	10.0	
Separation distance, L_{sep}	50.0	
Bunch spacing	4.45	m
Stored energy	2.00	GJ
Synchrotron radiation power	492	kW
Dipole heat load	5.87	W/m
Damping time, τ_0	2.26	hr
Norm. rms H emittance, ϵ_x	0.59	μm
Natural mmtm. spread, σ_p/p	$5\,\mathrm{x}10^{-6}$	

Table 1: Independent and dependent collider parameters.

(where α, β, γ, and η are Twiss functions). The momentum width is also independent of the lattice structure, and scales like $\sigma_p/p \sim \sqrt{B\gamma}$. By contrast, the natural horizontal emittance depends strongly on the lattice, scaling like $\epsilon_x \sim B^3 L^3$, where L is the half length of a FODO cell. The *normalized* emittance is independent of energy!

Figure 1 shows how the natural horizontal emittance may be tuned, for the primary parameters of Table 1, by adjusting the half cell length around its nominal value. The beam needs to be heated longitudinally to about $\sigma_p/p \simeq 10^{-4}$, to avoid significant intra beam scattering (IBS) consequences. If the linear coupling and the vertical dispersion in the arcs are both well controlled, the equilibrium vertical emittance will be much smaller the horizontal emittance, $\kappa \equiv (\epsilon_y/\epsilon_x) << 1$. There is no reason why hadron storage rings should not be able to achieve $\kappa \approx 0.1$ or less, in common with conventional electron storage rings.

3 BEAM-BEAM INTERACTIONS

The head on tune shift parameters are given by

$$\xi_{x,y} = \frac{r}{2\pi\gamma} \frac{N\beta_{x,y}^*}{\sigma_{x,y}^* \left(\sigma_x^* + \sigma_y^*\right)} \tag{10}$$

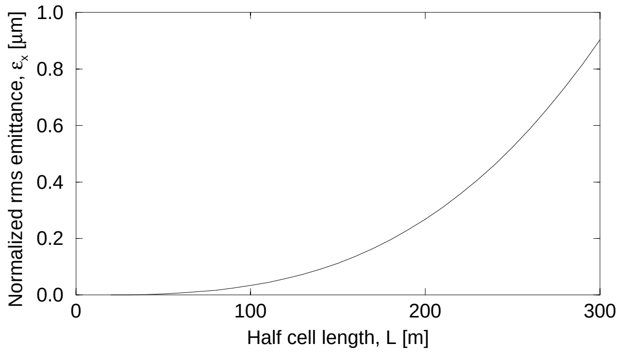

Figure 1: Equilibrium emittance versus half cell length

where N is the single bunch intensity, $r = 1.535 \times 10^{-18}\mathrm{m}$ is the classical radius of the proton, and values at the interaction point (IP) are denoted by a superscript asterisk*. The two tune shift parameters are made equal for flat beams by asserting that the β^* ratio is the same as the emittance ratio, so that

$$\kappa \;\equiv\; \frac{\epsilon_y}{\epsilon_x} \;=\; \frac{\beta_y^*}{\beta_x^*} \;=\; \frac{\sigma_y^*}{\sigma_x^*} \;<<\; 1 \qquad (11)$$

The vertical tune shift due to n_{LR} long range beam-beam interactions near a single IP is approximately

$$\Delta Q_y \;\simeq\; \frac{n_{LR}}{\kappa}\,\frac{\xi_y}{(\alpha/\sigma_y'^*)^2} \qquad (12)$$

where α is the full vertical crossing angle, and $\sigma_y'^*$ is the vertical angular size at the IP. This expression is valid if the phase of the collisions is approximately ± 90 degrees, whether or not they occur in or beyond the first quadrupole.

When valid, this last expression is equivalent to the more general (but less convenient) expression

$$\Delta Q_{x,y} \;=\; \mp\frac{r}{2\pi\gamma}\,\left\langle\frac{\beta_{x,y}}{\Delta^2}\right\rangle\, MN\,\frac{4L_{sep}}{C} \qquad (13)$$

where M is the total number of bunches, Δ is the full vertical beam separation, and angle brackets <> indicate an average over the parasitic collision region, within $\pm L_{sep}$ of the IP. The horizontal tune shift is negative. This shows that, for fixed values of (MN), C, and Δ, the only two ways to ameliorate the long range tune shift are to reduce L_{sep}, and to reduce β values at the parasitic collisions.

If most of the parasitic collisions occur in the IP drift, the total horizontal tune shift is much smaller than the vertical,

$$\Delta Q_x \;\simeq\; -\kappa\,\Delta Q_y \qquad (14)$$

since $\beta_x/\beta_y \simeq \beta_y^*/\beta_x^* = \kappa$ in the drift. Even when there are many collisions in and beyond the first quadrupole, it is still reasonable to expect the horizontal long range tune shift to be less of a problem than the vertical.

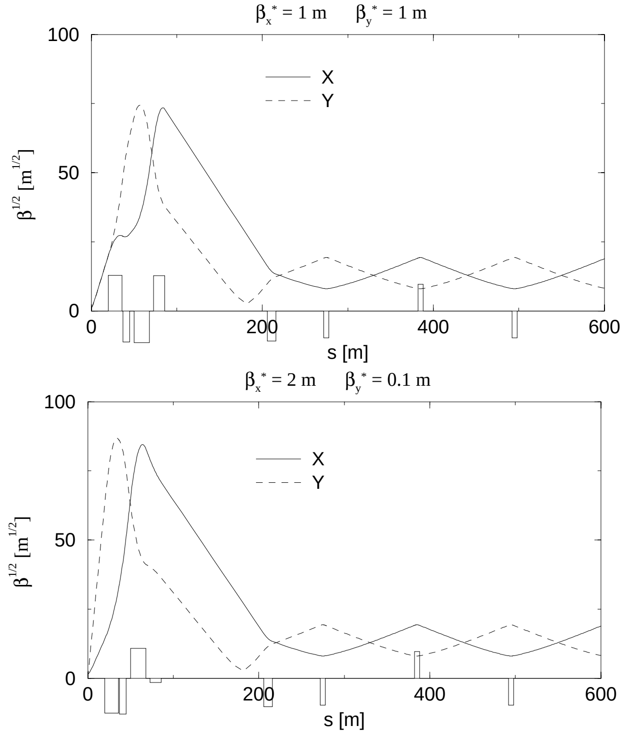

Figure 2: Round and flat ($\kappa = 0.05$) IR optics, with a common magnet layout.

4 INTERACTION REGION OPTICS

Flat beams allow the use of quadrupole doublets in the interaction region (IR), with many advantages over conventional triplet layouts. Figure 2 shows the same IR quadruplet layout accommodating both round and flat ($\kappa = .05$) optics. The height of each quadrupole rectangle is proportional to its strength, up to a maximum of about 500 T/m. While the round optics solution emulates a triplet, the flat optics solution emulates a shorter weaker doublet, with the fourth quad almost turned off. Because the "center of gravity" of the doublet is closer to the IP, maximum beta values $\widehat{\beta}$ with round and flat optics are comparable, even though the flat β_y^* value is reduced by a factor of 10. This is conveniently parameterized by an effective length, $L_{eff}^2 \equiv \beta^*\widehat{\beta}$, since L_{eff} is almost independent of β^* (at fixed κ). The horizontal and vertical effective lengths are approximately 74 m for the round optics, but are only 119 m and 27.4 m, respectively, in the flat optics. A true IR doublet layout would abandon the conservative round optics option, but would allow a beam splitting dipole between the two quadrupoles, justifying the nominal value $L_{sep} \approx 50\mathrm{m}$.

The instantaneous luminosity may be written

$$L \;=\; \frac{M}{4\pi T}\,\frac{N^2\gamma}{\kappa\epsilon_x\beta_x^*} \qquad (15)$$

where T is the revolution period, showing that β_x^* may be increased as κ is decreased, with other parameters (including β_y^*) held constant. This explains why the collision betas in Table 1, $(\beta_x^*, \beta_y^*) = (5.0, 0.5)\mathrm{m}$, are unusually large, and is probably the most significant optical advantage of flat beams. Both the long range tune shifts and also $\widehat{\beta}$, and all the associated nonlinear problems, may be reduced by an order of magnitude.

5 STORE PERFORMANCE

A distinguishing characteristic of the strong emittance damping regime is the independence of integrated luminosity on the initial beam emittances. This allows the production of very dense beams and high luminosities with relatively weak bunch intensities. Particle burn-off then limits integrated luminosities and results in short store lengths. The Table 1 parameter set manipulates both the number and intensity of the bunches to produce ~ 20 hr store lengths. The instantaneous luminosity shown in Fig. 3 increases rapidly at the beginning of the store as the emittances damp, rising to a peak of $10^{34}\mathrm{cm}^{-2}\mathrm{s}^{-1}$ when the beams have achieved equilibrium after 6 hr. The luminosity then falls to $4\times10^{33}\mathrm{cm}^{-2}\mathrm{s}^{-1}$ after 14 more hours, as the average bunch intensity falls.

Fig. 4 shows that the horizontal emittance achieves an equilibrium value determined by the cell length in 5 hr. The equilibrium vertical emittance, due to linear coupling and vertical dispersion, is assumed to result in a 10:1 emittance ratio. The bunch intensity shown in Fig. 5 falls by a factor of ~ 2 during the 20 hr store, while the beam-beam tune shift parameters, determined by the bunch density and shape, peak at a relatively benign 0.003. Of more concern is the long range tune shift. With the IR outlined above, there are ~ 40 parasitic crossings before the beams are fully separated. Fig. 6 shows the (vertical) crossing angle needed to restrict the vertical tune shift to 0.03 or less. Physical beam separations of only several mm are needed in the nearest quadrupoles. Enhancing the optimum store length by adding more particles increases the synchrotron power which must be absorbed by the cryogenic system, nominally 6 W/m emitted into the dipoles.

6 CONCLUSIONS

Flat beams produce denser bunches, allowing larger β^* values and leaving room for further performance enhancement. Luminosities in the range of $10^{35}\mathrm{cm}^{-2}\mathrm{s}^{-1}$ may be considered by reducing β^* values.

Flat beams also permit doublet IR optics, with more modest magnetic strength demands, lower maximum betas, easier beam separation, and reduced long range tune shifts.

7 REFERENCES

[1] "Proceedings of the Workshop on Future Hadron Facilities in the US", Fermilab-TM-1907, 1994.

[2] M. Syphers et al, Dallas, PAC Proc., 1995, p.431

[3] S. Peggs et al, Barcelona, EPAC Proc., 1996, p.377

Figure 3: Luminosity evolution during a 20 hour store.

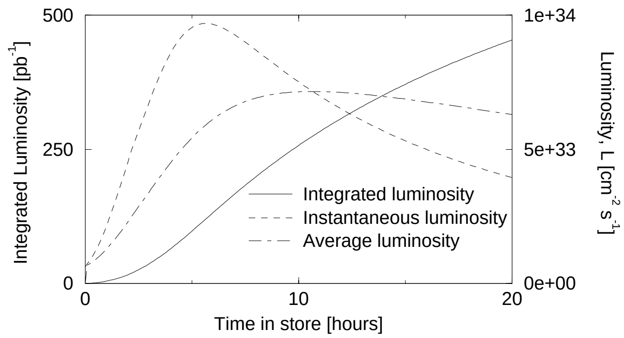

Figure 4: Emittance evolution, with radiation and IBS.

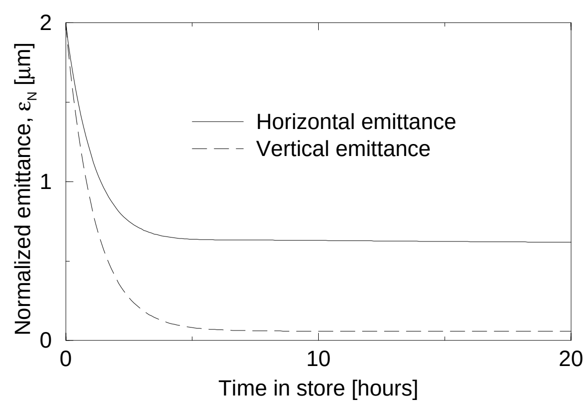

Figure 5: Beam-beam tune shift parameter evolution.

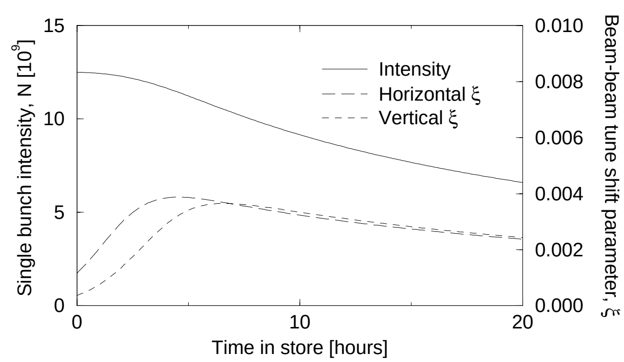

Figure 6: Long range tune shift evolution, per IR.