INSTABILITY ANALYSIS OF AN ACTIVE HIGHER-HARMONIC CAVITY

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Abstract

A radiofrequency system with an active higher-harmonic ("Landau") cavity may prevent coupled-bunch instabilities and increase the bunch lifetime of an electron storage ring. However, the equilibrium phase and Robinson instabilities must be avoided. An algorithm is presented for evaluating an active Landau cavity, and applied to the electron storage ring, Aladdin.

1 INTRODUCTION

An active radiofrequency (RF) cavity with resonant frequency near a harmonic of the fundamental RF cavity may increase Landau damping of synchrotron oscillations and increase the bunchlength [1, 2], thereby suppressing coupled-bunch instabilities and increasing the Touschek lifetime. However, the equilibrium phase and Robinson instabilities must be avoided. To evaluate an active Landau cavity, we present an algorithm similar to that for a passive (unpowered) Landau cavity [3]. The algorithm is applied to the electron storage ring, Aladdin [4]. We use the notation of Sands [5].

2 ANALYSIS ALGORITHM

Consider an electron storage ring where the fundamental RF cavity and active Landau cavity are each operated in the "compensated condition" [5] with the generator current in phase with the voltage. We consider a Landau cavity which is operated to maximize the bunchlength by eliminating the quadratic and cubic terms of the synchrotron potential. The control systems which maintain the compensated condition and maximum bunchlength are assumed to be slow compared to the phase oscillation period [5].

We assume that the coupling between each cavity and its RF amplifier, as well as any fast RF feedback to compensate beam loading, may be represented by a resistor in parallel with the cavity impedance. The effect of this resistance is characterized by an equivalent RF-coupling coefficient [6, 7].

The following values must be input to the algorithm: V_{T1}: peak RF voltage in Cavity 1; Q_1^o: unloaded quality factor of Cavity 1; R_1^o: unloaded impedance of Cavity 1 at resonance (one-half of the "accelerator definition" of shunt impedance); β_1: RF-coupling coefficient for Cavity 1; α: momentum compaction; T_o: revolution period; ω_g: generator angular frequency; E: electron energy; σ_E: electron energy spread; I: average beam current magnitude; V_s: synchronous voltage; ν: harmonic number of Cavity 2; Q_2^o: unloaded quality factor of Cavity 2; R_2^o: unloaded resonant impedance of Cavity 2; β_2: RF-coupling coefficient for Cavity 2; τ_L: longitudinal radiation damping time; $Z(\omega_{C.B.})$: parasitic impedance driving coupled-bunch oscillations; and $\omega_{C.B.}$: parasitic mode angular frequency.

Let ω_1 be the resonant frequency of Cavity 1, $Q_1 = Q_1^o/(1+\beta_1)$ the loaded quality factor, $R_1 = R_1^o/(1+\beta_1)$ the impedance at resonance, and ϕ_1 the tuning angle, defined by $\tan\phi_1 = 2Q_1(\omega_g - \omega_1)/\omega_1$. This tuning angle is the same as that used by Sands [5], and the negative of that used by Wilson [6]. Robinson oscillations depend upon the angles $\phi_{1\pm}$ which obey $\tan\phi_{1\pm} = 2Q_1(\omega_g \pm \Omega - \omega_1)/\omega_1$, where Ω is the Robinson angular frequency.

Cavity 2 has resonant frequency ω_2 near $\nu\omega_g$, where ν is its harmonic number. $Q_2 = Q_2^o/(1+\beta_2)$ is the loaded quality factor, $R_2 = R_2^o/(1+\beta_2)$ is the impedance at resonance, and ϕ_2 is its tuning angle, given by $\tan\phi_2 = 2Q_2(\nu\omega_g - \omega_2)/\omega_2$. Robinson oscillations involve the angles $\phi_{2\pm}$ which obey $\tan\phi_{2\pm} = 2Q_2(\nu\omega_g \pm \Omega - \omega_2)/\omega_2$. For both cavities, the tuning angles are defined with the loaded value of Q.

Let Ω denote the real Robinson angular frequency, α_R the Robinson damping rate (negative for growth), and e > 0 the electron charge magnitude. Our algorithm proceeds as follows:

1. Calculate ψ_1 and ψ_2, the equilibrium phase angles of the bunch center in Cavities 1 and 2, and V_{T2}, the peak voltage in Cavity 2. By our convention, a phase angle equals zero for a bunch at the voltage peak. Neglecting the small difference between the synchronous phase and that of the bunch center, the assumption that the quadratic and cubic synchrotron potential terms vanish gives [3]:

$$V_{T1}\sin\psi_1 + \nu V_{T2}\sin\psi_2 = 0 \tag{1}$$

$$V_{T1}\cos\psi_1 + \nu^2 V_{T2}\cos\psi_2 = 0. \tag{2}$$

The energy of an electron at the synchronous phase is unchanged by a revolution around the ring:

$$V_s = V_{T1}\cos\psi_1 + V_{T2}\cos\psi_2 \tag{3}$$

Simultaneous solution of eqs. (1) – (3) yields:

$$\psi_1 = \cos^{-1}\left(\frac{V_s}{(1-\frac{1}{\nu^2})V_{T1}}\right) \tag{4}$$

$$\psi_2 = \tan^{-1}(\nu\tan\psi_1) - 180^{\mathrm{o}} \tag{5}$$

$$V_{T2} = -V_{T1}\frac{\sin\psi_1}{\nu\sin\psi_2} \tag{6}$$

If eq. (4) cannnot be solved with ψ_1 between 0 and 90 degrees, then there is no equilibrium phase.

2. Calculate the quartic coefficient of the effective synchrotron potential, $U(t) = at^2 + bt^3 + ct^4 + ...$, where t is the time displacement from the synchronous time. Neglecting the difference between the synchronous phase and that of the bunch center gives [3]:

$$c = -\frac{\alpha e \omega_g^3}{24ET_o}(V_{T1}\sin\psi_1 + \nu^3 V_{T2}\sin\psi_2) \qquad (7)$$

3. Calculate the bunchlength. In a quartic confining potential, the charge distribution is [8, 9]: $q(t) = k\exp(-t^4/t_o^4)$. The rms bunchlength, $\sigma_t = t_o/1.72$, obeys [8, 9]:

$$\sigma_t = 0.69(\frac{U_o}{c})^{1/4} \qquad (8)$$

where $U_o = \frac{\alpha^2}{2}(\frac{\sigma_E}{E})^2$ is the "filling height."

4. Determine the form factors. In a quartic confining potential, the Cavity 1 form factor obeys:

$$F_1 = \frac{\int_{-\infty}^{\infty}\cos(\omega_g t)\exp(-t^4/t_o^4)dt}{\int_{-\infty}^{\infty}\exp(-t^4/t_o^4)dt} \qquad (9)$$

The Cavity 2 form factor, F_2, obeys the same formula with $\omega_g \to \nu\omega_g$.

5. Calculate the tuning angles of Cavities 1 and 2 for operation in the "compensated condition" [5]:

$$\phi_1 = \tan^{-1}(\frac{2F_1 I R_1}{V_{T1}}\sin\psi_1) \qquad (10)$$

$$\phi_2 = \tan^{-1}(\frac{2F_2 I R_2}{V_{T2}}\sin\psi_2) \qquad (11)$$

6. Determine if the dipole longitudinal coupled-bunch instability may be expected. Neglecting Landau and radiation damping, the coherent frequency of the dipole coupled-bunch mode in a quartic synchrotron potential obeys [9]:

$$\Omega_{C.B.}^2 = i(\delta\Omega_o)^2 \qquad (12)$$

where, for resonant interaction with a cavity mode:

$$(\delta\Omega_o)^2 = \frac{eI\alpha}{ET_o}F_{\omega_{C.B}}^2 \omega_{C.B.} Z(\omega_{C.B.}). \qquad (13)$$

Here, $F_{\omega_{C.B.}}$ is the form factor at $\omega_{C.B.}$, given by eq. (9) with $\omega_g \to \omega_{C.B.}$. Landau damping is overcome provided that [9]:

$$\mid \Omega_{C.B.} \mid > 0.6\Delta\omega_s \qquad (14)$$

where, $\Delta\omega_s \equiv \omega_s(t_o)$ is a measure of synchrotron frequency spread, and $\omega_s(t_o)$ is the synchrotron frequency for oscillations of amplitude $t_o = 1.72\sigma_t$. Because the frequency is proportional to amplitude in a quartic potential:

$$\Delta\omega_s \equiv \omega_s(t_o) = \omega_s(1.72\sigma_t) = 1.72\omega_s(\sigma_t) \qquad (15)$$

where [8, 9]:

$$\omega_s(\sigma_t) = 1.17(U_o c)^{1/4} \qquad (16)$$

Therefore, Landau damping is overcome when:

$$\mid \Omega_{C.B.} \mid > (0.6)(1.72)(1.17)(U_o c)^{1/4} \qquad (17)$$

To include the effect of radiation damping, we subtract the damping rate τ_L^{-1} from $\mathrm{Im}(\Omega_{\mathrm{C.B.}})$:

$$\Omega_{C.B.} = [i(\delta\Omega_o)^2]^{1/2} - i\tau_L^{-1} = \frac{\delta\Omega_o}{\sqrt{2}} + i(\frac{\delta\Omega_o}{\sqrt{2}} - \tau_L^{-1}) \qquad (18)$$

We use the following criteria for the coupled-bunch instability: First, $\delta\Omega_o/\sqrt{2} > \tau_L^{-1}$ is required to overcome radiation damping. Second, Landau damping is overcome provided that eq. (17) is obeyed, where radiation damping is included in $\Omega_{C.B.}$ by using eq. (18).

7. Determine if the equilibrium phase instability will occur. Stability is assured if [3]:

$$F_1 V_{T1}\sin\psi_1 + \nu F_2 V_{T2}\sin\psi_2 >$$

$$R_1 F_1^2 I\sin 2\phi_1 + \nu R_2 F_2^2 I \sin 2\phi_2 \qquad (19)$$

8. If the previous inequality is satisfied, calculate the Robinson frequency, Ω, which obeys [3]:

$$\Omega^2 = \frac{e\alpha\omega_g}{T_o E}\{F_1 V_{T1}\sin\psi_1 - \frac{R_1 F_1^2 I}{2}(\sin 2\phi_{1-} + \sin 2\phi_{1+})$$

$$+\nu F_2 V_{T2}\sin\psi_2 - \frac{\nu R_2 F_2^2 I}{2}(\sin 2\phi_{2-} + \sin 2\phi_{2+})\} \qquad (20)$$

This calculation requires iteration; we start by evaluating the RHS with zero beam current.

9. Neglecting radiation damping, the Robinson damping rate obeys [3]:

$$\alpha_R = \frac{4\alpha e I}{ET_o}[F_1^2 R_1 Q_1 \tan\phi_1 \cos^2\phi_{1+}\cos^2\phi_{1-}$$

$$+F_2^2 R_2 Q_2 \tan\phi_2 \cos^2\phi_{2+}\cos^2\phi_{2-}]. \qquad (21)$$

To include radiation damping, we add the quantity τ_L^{-1} to the above value of α_R. If the result is positive, the Robinson mode is stable. If not, the Robinson mode will be unstable provided that Landau damping is overcome, for which we use a criterion appropriate for a quartic synchrotron potential:

$$(\Omega^2 + \alpha_R^2)^{1/2} > 0.6\Delta\omega_s. \qquad (22)$$

We evaluate a Landau cavity by performing the above algorithm for a sequence of values of ring current (I) and Cavity 2 RF-coupling (β_2). Steps 1–4 are independent of I and β_2, so only Steps 5–9 need be repeated.

3 APPLICATION

The above algorithm was applied to the fourth harmonic cavity at the electron storage ring, Aladdin. For analysis of the coupled-bunch instability, we considered a parasitic mode impedance of $Z(\omega_{C.B.})$ = 0.01 MΩ at $\omega_{C.B.}$ = 6.28 GHz. Input parameters are: V_{T1}: 80 kV, Q_1^o: 10000, R_1^o: 0.65 MΩ, α: 0.0335, T_o: 2.96x10^{-7} s, ω_g: 318 MHz, E: 800 MeV, σ_E/E: 4.8x10^{-4}, V_s: 17.4 kV, ν: 4, Q_2^o: 22700, R_2^o: 1.4 MΩ, and τ_L: 13.8 ms.

For all values of I, β_1, and β_2, the Landau cavity peak voltage, V_{T2}, is 19.5 kV, $\psi_1 = 76.6^o$, and $\psi_2 = -93.4^o$. The bunchlength, σ_t, is 603 ps, versus 271 ps in the absence of a Landau cavity. Because the Touschek lifetime is approximately proportional to the bunchlength, we expect the Touschek lifetime to increase by a factor of $\sim$ 2.2 with the active Landau cavity.

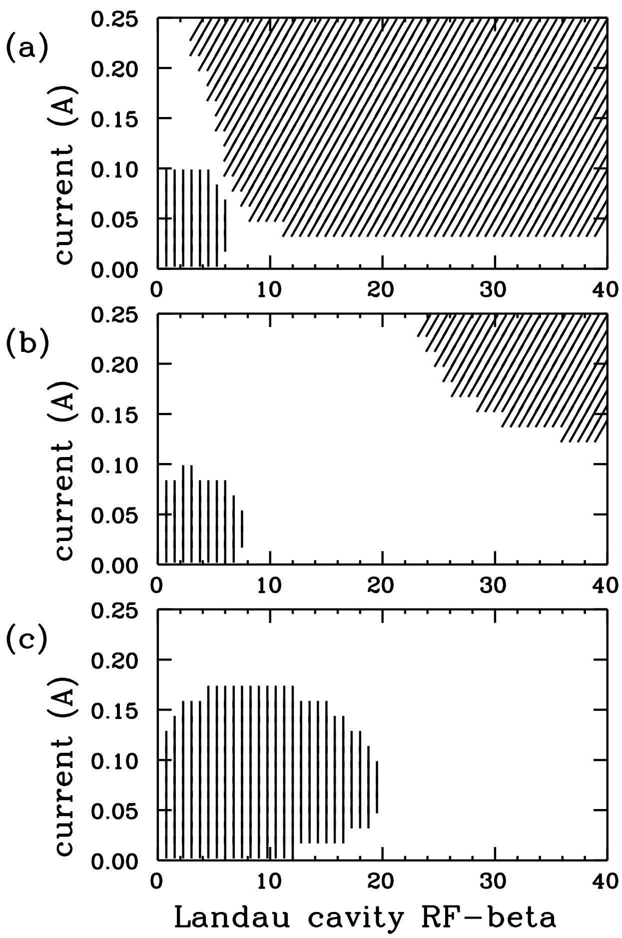

Figure 1: Instabilities are predicted for a range of ring currents (I) and active Landau cavity RF-coupling values (β_2), for the electron storage ring, Aladdin. $|$: Robinson instability. $/$: equilibrium phase instability. (a) $\beta_1 = 0$. (b) $\beta_1 = 2$. (c) $\beta_1 = 8$.

Stability plots are shown for three cases: $\beta_1 = 0$ in Fig. 1(a), $\beta_1 = 2$ in Fig. 1(b), and $\beta_1 = 8$ in Fig. 1(c). The coupled-bunch instability is suppressed for the range of ring current (I) and Landau cavity RF-coupling (β_2) shown: $0 < I < 0.25$ A, $0 < \beta_2 < 40$. For small values of I and β_2, the Robinson instability occurs, while the equilibrium phase instability occurs at large values of I and β_2. For $\beta_1 = 0$, there is no value of β_2 giving stable operation for the range of ring current shown. For $\beta_1 = 2$, values of β_2 between 8 and 23 ensure stable operation for $0 < I < 0.25$ A, while for $\beta_1 = 8$, values of β_2 exceeding 20 give stable operation.

4 SUMMARY

An algorithm has been developed to evaluate instabilities in an electron storage ring with an active higher-harmonic cavity. For the electron storage ring, Aladdin, an active fourth-harmonic cavity with appropriate RF feedback is expected to suppress coupled-bunch instabilities and to increase the bunchlength and Touschek lifetime by a factor of $\sim$ 2.2.

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