LONGITUDINAL BEAM TRANSFER FUNCTION DIAGNOSTICS IN THE ALS *

John Byrd
Lawrence Berkeley Laboratory, One Cyclotron Road, Berkeley, California 94720

Abstract

We describe the technique of longitudinal beam transfer function (BTF) for measuring several properties of longitudinal oscillations in a electron storage ring such as the synchrotron frequency, radiation damping time, and bunch length. The technique takes advantage of the Gaussian distribution of longitudinal oscillation amplitudes within the bunch at very low current and the dependence of synchrotron frequency on amplitude. Results of measurements made at the Advanced Light Source (ALS) are presented.

1 INTRODUCTION

Beam transfer function (BTF) diagnostics are used in almost all storage rings for measuring the betatron and synchtrotron frequencies. In the simplest case, a swept frequency drive excites either betatron or synchrotron oscillations while a beam signal is observed on a spectrum analyzer. In other applications, BTF techniques have been used[1, 2, 3] for measuring beam impedance and feedback loop stability. We describe in this paper an application of the BTF technique for use in an electron storage ring for making measurements of the distribution of synchrotron frequencies within a single bunch at low beam current. By taking advantage of the Gaussian distribution in the energy spread within the bunch resulting from the quantum nature of the emission of synchrotron radiation and the sinusoidal RF voltage, we can use the measurements to derive a relatively precise measure of the nominal synchrotron frequency, the longitudinal radiation damping rate, and the bunch length. Although these parameters can be measured using other techniques, the BTF method has the advantage of being relatively simple and inexpensive, and typically uses equipment that is either already available in the control room or easily assembled. The BTF technique can potentially be used to study the effects of short range wakefields and the longitudinal beam dynamics of more complicated situations such as double RF systems and low momentum compaction.

In general, the BTF is defined as the ratio of the driven beam response to the external excitation at a given frequency. In practice, either an network or FFT analyzer is used to supply a swept frequency or noise excitation to the beam via either a stripline kicker or RF cavity and the beam response is measured through a pickup. For the cases discussed in this paper, we excite longitudinal oscillations by phase modulating the voltage in the fundamental mode of an RF cavity and measure the synchrotron oscillations by detecting the phase of a beam pickup signal relative to a fixed reference phase or by measuring energy oscillations at a point of dispersion in the lattice. It is also assumed throughout the paper that the amplitude of synchrotron oscillations is small enough that the motion can be described as linear.

Section 2 discusses the longitudinal beam distribution and gives a physical interpretation of the transfer function for a Gaussian distribution. Section 3 presents the measurements performed at the ALS.

2 BEAM TRANSFER FUNCTION FOR A GAUSSIAN DISTRIBUTION

The energy distribution in an electron bunch in a storage ring is Gaussian, resulting from the balance of the quantum excitation from synchrotron radiation emission and radiation damping[4]. It can be shown that the phase space density distribution to second order is given by

$$\psi_0(\hat{\tau}) = \frac{1}{2\pi\sigma_\tau^2} e^{-(\hat{\tau}^2/2\sigma_\tau^2)} \tag{1}$$

where $\hat{\tau}$ is the oscillation amplitude and σ_τ is the RMS bunch length, both in units of time. The nonlinearity of the sinusoidal RF voltage results in a synchrotron frequency dependent of the amplitude of phase oscillation of the form

$$\omega_s(\hat{\tau}) = \omega_{s0}(1 - \mu\hat{\tau}^2) \tag{2}$$

where

$$\mu = \frac{\omega_{RF}^2(1 + \frac{5}{3}\tan^2\phi_s)}{16} \approx \frac{\omega_{RF}^2}{16} \tag{3}$$

where ϕ_s is the synchronous phase angle. For most cases, the effect of ϕ_s can be ignored.

The combination of the distribution of oscillation amplitudes and synchrotron frequency as a function of amplitude yields a unique distribution of synchrotron frequencies within the bunch for a given bunch length. As described below, the natural spread in frequencies leads to Landau damping of coherent oscillations.

Given the distribution, the BTF can be found by solving the Vlasov equation for a small perturbation of the phase space distribution at the excitation frequency and integrating to find the first moment of the distribution. This approach is identical to that used in finding the stability conditions of coherent instabilities. Equivalently, the BTF can be found by calculating moments of the distribution in response to an impulse excitation in the time domain and making a Fourier transform on the result. The time domain

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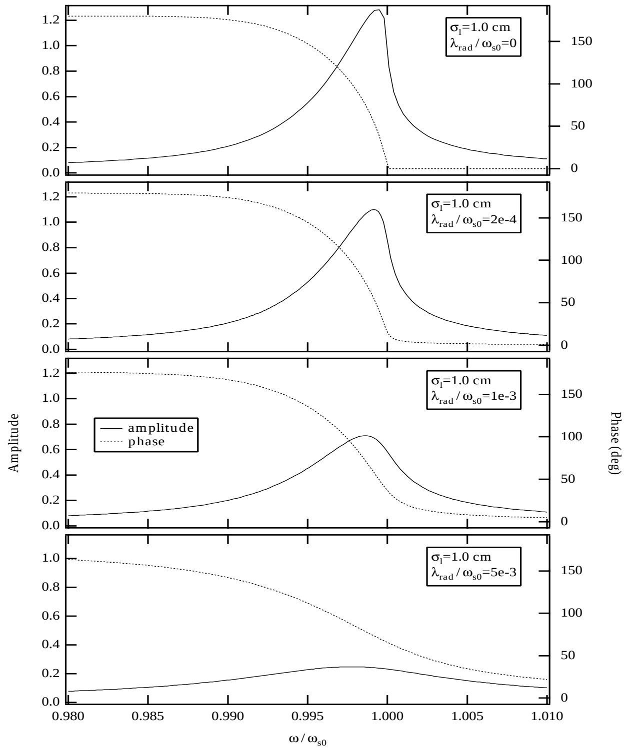

Figure 1: Calculated BTF for a 1 cm RMS bunch length as a function of increasing radiation damping.

impulse response has been calculated[5] and can be shown to give the same result as the Vlasov approach for infinitesimal excitation. The BTF can be expressed in terms of a dispersion integral given by[6]

$$I(\omega_m) \propto \int_0^\infty \frac{\hat{\tau}^2 d\hat{\tau}}{\omega_m - \omega_s(\hat{\tau})} \frac{\partial \psi_0}{\partial \hat{\tau}}. \tag{4}$$

where ω_m is the angular modulation frequency. Radiation damping is included by making the synchrotron frequency complex as given by

$$\tilde{\omega}_s = \omega_s + j\lambda_{rad} \tag{5}$$

For a Gaussian distribution in oscillation amplitude, the dispersion can be expressed in terms of the exponential integral[7], given by

$$I(x) \propto 1 - xE_1(x)e^{-x} \tag{6}$$

where $x = \frac{8(\omega_{s0} - \omega - j\lambda_{rad})}{\omega_{s0}\omega_{rf}^2\sigma_\tau^2}$.

The amplitude and phase of the BTF is plotted for several values of the bunch length as a function of increasing radiation damping in Figures 1a-d. For convenience, we have used the convention that the phase goes from 180 to 0 degrees passing from far below the nominal synchrotron frequency to above. Consider the case of no radiation damping ($\lambda_{rad} = 0$) shown in Figure 1a. When the external excitation frequency is greater than the zero–amplitude synchrotron frequency, the phase of the beam response does

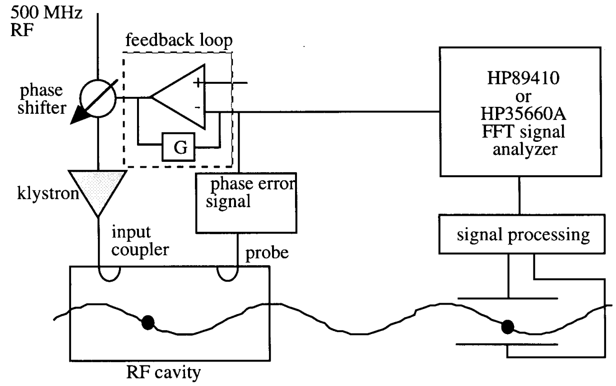

Figure 2: General setup for longitudinal BTF measurements.

not vary with frequency because all of the electrons within the bunch are being driven above their resonant frequency. When $\omega_m < \omega_{s0}$ and the external excitation frequency is within the spread of incoherent frequencies, some of the particles in the bunch are being driven resonantly, some below resonance, and some above resonance and thus there is a net phase shift between the drive and the response. When the external excitation frequency is far below the synchrotron frequency, all electrons have again the same phase response. Neither the amplitude and phase response are symmetric in frequency because the distribution of synchrotron frequencies with oscillation amplitude is not symmetric. Also note that the peak amplitude response does not occur at the nominal synchrotron frequency but slightly below.

For nonzero radiation damping as shown in Figures 1b-d, the response of individual electrons now have a natural width. This tends to smear the response of the distribution of synchrotron frequencies. As the width due to radiation damping approaches the width of the distribution of synchrotron frequencies, the response becomes much more like the Lorentzian shape expected from a damped harmonic oscillator.

3 BEAM MEASUREMENTS

The setup for measurement of the BTF is shown in Figure 2. We excited longitudinal oscillations by phase modulating (PM) the RF voltage. This was achieved by injecting the modulation signal as an error signal in the RF phase control feedback loop.

Synchrotron oscillations were detected using the phase detector for an existing longitudinal coupled–bunch feedback system[8]. This detector passes the sum of the signal from four capacitive button BPMs (beam position monitors) located at one point in the ring through a 4–tap comb filter with a center frequency of 3 GHz ($6 \times F_{rf}$). The sum of the four button signals is not sensitive to the transverse position of the beam. The signal is demodulated to baseband through a double balanced mixer using a 3 GHz local oscillator derived from the 500 MHz master oscillator. Detection at the sixth RF harmonic increases the sensitivity to phase oscillations compared to detection at the RF frequency and is also near the frequency of maximum pickup impedance of the BPMs. Because of the relatively short bunch length in the ALS, the 3 GHz component of the beam signal is reduced very little compared to the 500 MHz component.

The sensitivity required for the detection of the synchrotron oscillations depends somewhat on the storage ring parameters. For example, to obtain the most accurate results, we found that it was important to excite the synchrotron oscillations with an amplitude at least two orders of magnitude less than the natural bunch length in order that the oscillations remained quasilinear. Therefore, measurements with shorter bunch length require greater sensitivity. We indepently verified the amplitude of PM in the cavity by measuring the spectrum of PM sidebands present on a cavity probe signal with no beam. Also, to avoid conversion of PM to amplitude modulation, the frequency of the RF cavity was tuned to on resonance with the RF drive frequency for all measurements. For the conditions of these measurements, the variation of the cavity response over the bandwidth of the measurement was negligible.

An HP89410 FFT signal analyzer was used as the source and receiver for the signal for both measurements. We found that for a given level of excitation, a better signal/noise ratio could be achieved using a bandwidth limited noise source than for a swept frequency excitation. In principle both approaches yield the same results. In order to avoid the influence of collective effects on the bunch shape, all of the measurements presented here were made at the lowest single bunch current possible which still gave a reasonable signal level. We used a bunch current of 100-300 μA, well below the threshold for any instabilities and low enough that potential well distortion is negligible.

Shown in Figure 3 are BTFs measured at various beam energies and synchrotron frequencies. Each result is fit to the functional form given in Eq.6. The width of the amplitude response is due in roughly equal parts to the radiation damping and the spread of synchrotron frequencies and thus the BTF is much more symmetric. We independently measured the bunch length with a streak camera with values shown in parentheses in the figure for comparison. Although the agreement for the values of the bunch length is good, it is less precise for the case when the decoherence rate and the radiation damping rate are comparable. If the interest is in measuring bunch length, the BTF technique would be limited to relatively long bunches.

4 CONCLUSIONS

The BTF is a relatively simple method for measuring the incoherent spread in synchrotron frequencies in a single bunch. With a careful analysis, the small amplitude synchrotron frequency, the longitudinal radiation damping rate, and the bunch length can all be inferred from this measurement. The precision of the measurement depends primarily of the value of the radiation damping rate compared with the incoherent spread in synchrotron frequencies. The measurement of the bunch length is more precise when $\lambda_{rad} < \Delta\omega_s$ and vice versa for measurement of the radiation damping time.

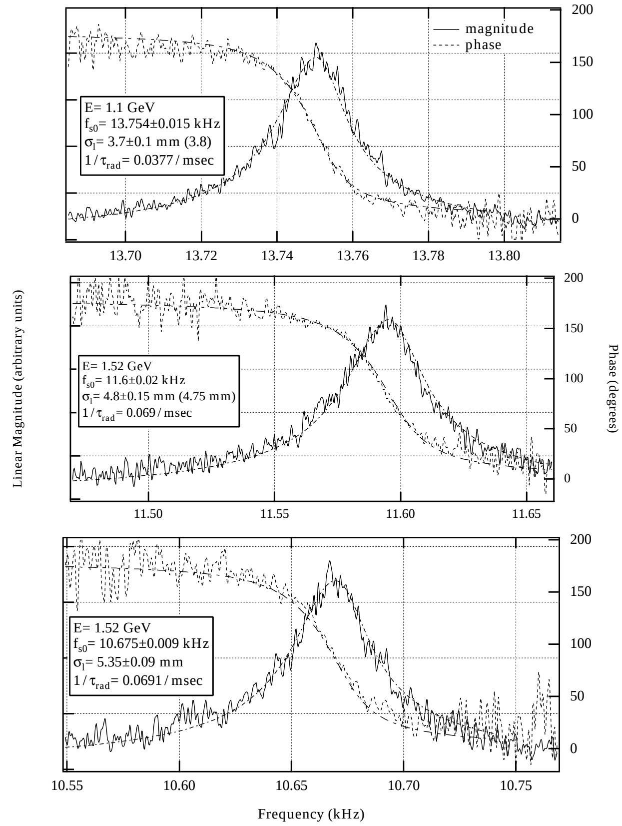

Figure 3: Results of BTF measurements at ALS for several energies and nominal bunch lengths. The fit values for the synchrotron frequency, bunch length, and damping rate are shown for each case. Bunch lengths measured using a streak camera are shown in parentheses for comparison.

We are hopeful that this technique can be used for measuring the effects of distortions of the phase space distribution due to instabilities or other effects. We would like to thank the the ALS operations group for assistance with the measurements.

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