ANALYTICAL FIELD CALCULATION OF HELICAL DIPOLE MAGNETS FOR RHIC

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Abstract

The analytical expression for the magnetic field of helical coils is presented. In particular, the multipole coefficients are introduced for helical dipole magnets, which are essential components of Siberian Snakes and spin rotators for polarized proton acceleration at RHIC. In addition, the comparison between analytical and numerical calculations is presented for simple helical dipole magnets.

1 INTRODUCTION

Using the expression of multipole expansion for a single helical current conductor[1], the magnetic field of helical dipole coil with an infinite length is derived as the summation of four(4) helical line currents with dipole symmetry, deriving the helical multipole coefficients. [2,3] The helical multipole coefficients are defined so that the non-twist helical multipole coefficients are equal to the 2-dimensional multipole coefficients. As a result, the dependence of helical multipole coefficients upon the twist parameter is derived. With the comparison between the analytical and numerical calculations, it is confirmed that the helical multipole coefficients derived from the analytically calculated field are consistent with those calculated numerically.

2 ANALYTICAL EXPRESSION FOR THE MAGNETIC FIELD OF HELICAL DIPOLES

Using the expression of the multipole expansion for a single helical current conductor, the magnetic field inside the helical coils with an infinite length can be derived as the summation of four(4) helical line currents with dipole symmetry as follows, [1,2,3]

$$
\left\{
\begin{aligned}
B_r(r,\theta,z) &= B_{ref}(k)\, r_0 \sum_{n=1}^{\infty} n! \left[\frac{2}{n\,k\,r_0}\right]^n k\, \acute{I}_n(n\,k\,r) \times \\
&\quad \left\{-a_n(k)\cos\left(n(\theta - k\,z)\right) + b_n(k)\sin\left(n(\theta - k\,z)\right)\right\} \\
B_\theta(r,\theta,z) &= B_{ref}(k)\, r_0 \sum_{n=1}^{\infty} n! \left[\frac{2}{n\,k\,r_0}\right]^n \frac{I_n(n\,k\,r)}{r} \times \\
&\quad \left\{a_n(k)\sin\left(n(\theta - k\,z)\right) + b_n(k)\cos\left(n(\theta - k\,z)\right)\right\} \\
B_z(r,\theta,z) &= B_{ref}(k)\, r_0 \sum_{n=1}^{\infty} (-k)\, n! \left[\frac{2}{n\,k\,r_0}\right]^n I_n(n\,k\,r) \times \\
&\quad \left\{a_n(k)\sin\left(n(\theta - k\,z)\right) + b_n(k)\cos\left(n(\theta - k\,z)\right)\right\},
\end{aligned}
\right. \tag{1}
$$

$$
\left\{
\begin{aligned}
&\lim_{k\to 0}\left[B_{r,\,h}(r,\theta,z)\right] = B_{r,\,2d}\,(r,\theta) \\
&= B_{ref}\sum_{n=1}^{\infty} \left(\frac{r}{r_0}\right)^{n-1}(-a_n\cos n\theta + b_n\sin n\theta) \\
&\lim_{k\to 0}\left[B_{\theta,\,h}(r,\theta,z)\right] = B_{\theta,\,2d}\,(r,\theta) \\
&= B_{ref}\sum_{n=1}^{\infty} \left(\frac{r}{r_0}\right)^{n-1}(b_n\cos n\theta + a_n\sin n\theta) \\
&\lim_{k\to 0}\left[B_{z,\,h}(r,\theta,z)\right] = B_{z,\,2d}\,(r,\theta) = 0
\end{aligned}
\right. \tag{2}
$$

Therefore, the normal and skew multipoles due to many helical line currents with current I_i, radius a_i, angle φ_i are,

$$
\left\{
\begin{aligned}
B_n(k) &= B_{ref}(k)\, b_n(k) = -\frac{\mu_0}{\pi}\frac{1}{2^n(n-1)!\, r_0}\,(n\,k\,r_0)^n \times \\
&\quad \sum_i I_i \left(k\,a_i\,K_{n-1}(n\,k\,a_i) + K_n(n\,k\,a_i)\right)\cos n\varphi_i \\
A_n(k) &= B_{ref}(k)\, a_n(k) = -\frac{\mu_0}{\pi}\frac{1}{2^n(n-1)!\, r_0}\,(n\,k\,r_0)^n \times \\
&\quad \sum_i I_i \left(k\,a_i\,K_{n-1}(n\,k\,a_i) + K_n(n\,k\,a_i)\right)\sin n\varphi_i
\end{aligned}
\right. \tag{3}
$$

where $k = 2\pi/L$, and $I_n(nkr)$ and $K_n(nkr)$ are the modified Bessel functions of the first and second kind of order n, respectively. With the definition of $b_1(k) = 1$ (=constant), naturally $B_{ref}(k) = B_1(k)$. Then, the asymptotic forms for the reference field $B_{ref}(k)$ and for these helical multipole coefficients $a_n(k)$ and $b_n(k)$ are given in the limit $k \to 0$ (or $L \to \infty$) by,

$$
\left\{
\begin{aligned}
&\lim_{k\to 0}\left[B_{ref}\,(k)\right] = B_{ref} \\
&\lim_{k\to 0}\left[a_n\,(k)\right] = a_n \\
&\lim_{k\to 0}\left[b_n\,(k)\right] = b_n
\end{aligned}
\right. \tag{4}
$$

3 COMPARISON BETWEEN ANALYTICAL AND NUMERICAL CALCULATIONS

3.1 Calculation for four helical line currents

Following numerical values are assumed in the calculation for four helical line currents with dipole symmetry.

Radius of helical line current: a = 50 mm,
Angle of helical line current: $\varphi = \pm\pi/6$, or $\pm 5\pi/6$,
Current: $I = -1 \times 10^5$ A,
Pitch length: L = 2 m,
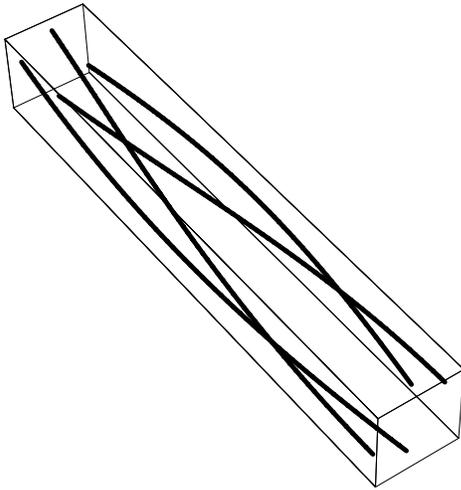$k = 2\pi/L = 1/(a\tan\alpha) = \pi$,

Fig. 1. 3D view of a four currents' helical dipole.

Table 1. Normal multipole coefficients for a infinitely long four line currents' helical dipole.

n	Pole	bn-theta	bn(k)
(Bref)		1.411	1.411
1	dipole	0.9998	1.
3	sextupole	-2.859×10^{-10}	0
5	decapole	-0.1239	-0.1243
7	14-pole	-0.04386	-0.04419
9	18-pole	-1.721×10^{-10}	0
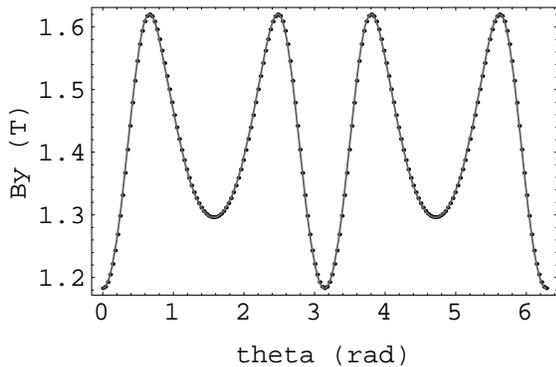
Fig. 3. Distribution B_y on the circle of r = 30 mm, with the comparison between the numerical (black dots) and analytical (gray line) calculations.

Pitch of the winding: $\alpha = \tan^{-1}(1/0.05\,\pi)$,
Reference radius for multipole: $r_0 = 30$ mm,

as shown in Fig.1. Both of the numerical and analytical calculations for the helical dipoles with the infinite length, are made, using Mathematica [4]. Both calculated helical multipole coefficients are listed in Table 1. The
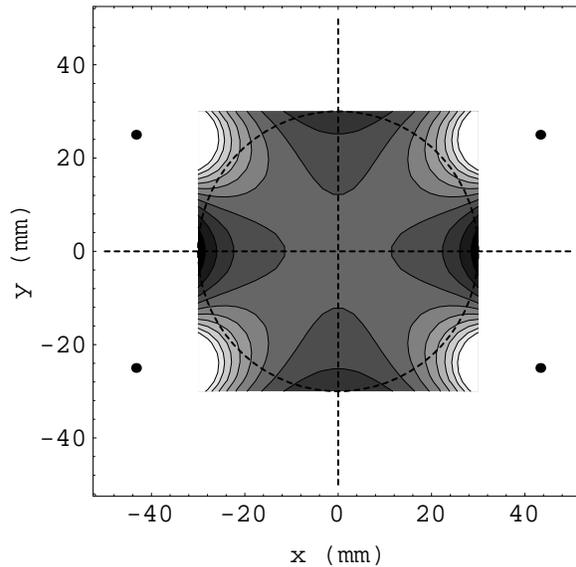

Fig. 2. Contour plot of dipole field B_y.

bn-theta and $b_n(k)$ correspond to the normal multiple coefficients derived from the angular component of numerically calculated field B_θ and analytically calculated helical coefficients, respectively. The analytically calculated contour plot of the dipole field B_y (r, θ, z=0) is shown in Fig.2. Both analytical (gray line) and numerical (black dots) calculations for the dipole field B_y (r=30 mm, θ, z=0) are also shown in Fig.3. As a result, it is confirmed that the agreement between the analytical and numerical calculations is quite good in the interior region of helical coils. The twist dependence of the dipole field B_{ref} is also shown in Fig.4.

3.2 Calculation for four helical current shells

Similarly, the magnetic fields are calculated for a four helical current shells with dipole symmetry of current density $\pm j_z$, radii a_1, a_2, limiting angles $\pm\varphi_1$, $\pm\varphi_2$, with,

Inner radius of helical line current: $a_1 = 50$ mm,
Outer radius of helical line current: $a_2 = 60$ mm,
Inner angle of helical line current: $\varphi_1 = 0$, or π,
Outer angle of helical line current: $\varphi_2 = \pi/3$, or $2\pi/3$,
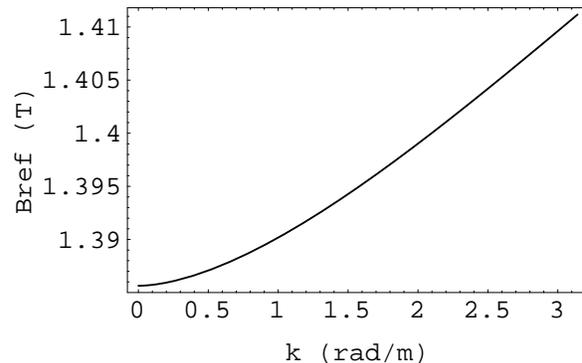Coil length: L = 2 m,

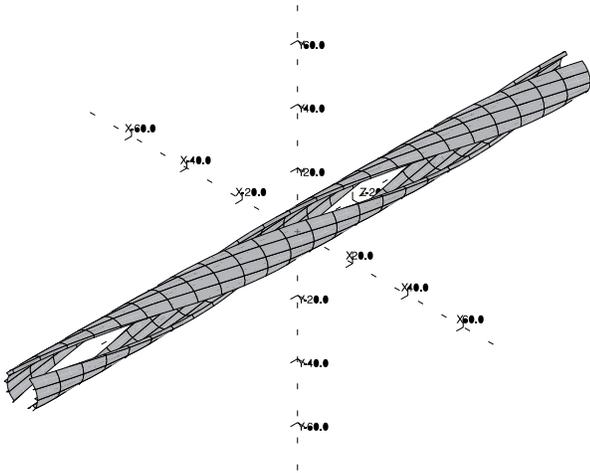Fig. 4. Twist dependence of the dipole field B_{ref}.

Fig.5. 3D view of a four helical current shells' dipole.

Table 2. Normal multipole coefficients for a 2 m long, 4 helical current shells' dipole.

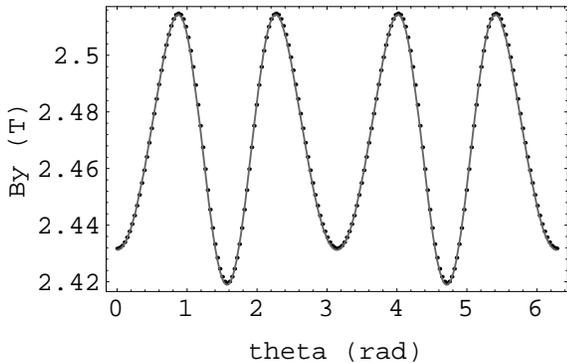| n	Pole	bn-theta	bn(k)
(Bref)		2.463	2.456
1	dipole	1.	1.
3	sextupole	$-1.001 \ 10^{-6}$	$1.369 \ 10^{-17}$
5	decapole	-0.01727	-0.01734
7	14-pole	0.003718	0.003743
9	18-pole	$-3.246 \ 10^{-6}$	$3.775 \ 10^{-19}$

Fig. 6. Distribution of By on the circle of r = 30 mm.

Current: $I = -2 \times 10^5$ A,

Current density: $j_z = 347$ A/mm^2

$k = 2\pi/L = 1/(a \tan\alpha) = \pi$,

Pitch of the winding: $\alpha = \tan^{-1}(1/0.05 \ \pi)$,

Reference radius for multipole: $r_0 = 30$ mm,

as shown in Fig.5. The numerical calculation for the helical coil with the finite length of one period is made, using the code 'OPERA-3d'. [5] Both analytical (gray line) and numerical (black dots) calculations for the dipole field B_y (r=30 mm, θ, z=0) is shown in Fig.6. The 3D
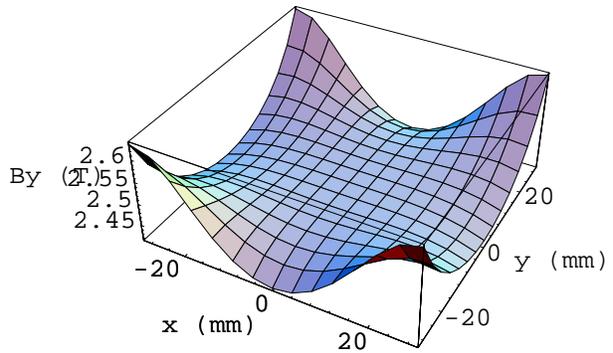

Fig. 7. 3D plot of dipole field By.

plot of the analytically calculated dipole field B_y (r, θ, z=0) is shown in Fig.7. Both of the analytically and numerically calculated helical multipole coefficients are also listed in Table 2. In addition, this analytical method was applied for two helical magnet of different type. [6,7,8]

4 CONCLUSION

As an extension of single helical current conductor, the magnetic fields of helical dipole magnet are derived and helical multipole coefficients are presented. In addition, it was confirmed that this analytical calculation is consistent with the numerical calculation by the 3D magnetic field numerical code, OPERA-3d/TOSCA

5 ACKNOWLEDGMENTS

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