OPTIMIZATION OF THE ACCELERATING STRUCTURE FOR THE SUPERCONDUCTING LINEAR ACCELERATOR OF THE ENERGY AMPLIFIER

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Abstract

The superconducting linear proton accelerator of the original proposal by C. Rubbia for a Full Scale Energy Amplifier is composed of three different low beta cavity sections for the low energy part and makes use of the LEP accelerating structures at the high energy end [1]. An optimization of the structure distribution is performed, based on an improved model for the accelerating field.

1 EFFECTIVE ACCELERATION FOR AN IDEAL FIELD

In order to judge the importance for an improved model of the accelerating field for the optimization of the accelerating structure we start with an ideal field which is approximated by a sine wave.

1.1 Accelerating Voltage for Arbitrary Particle Velocities

The space and time dependence of an idealized accelerating electric field in a cavity is given by:

$$E(z,t) = E_o \sin 2\pi \left(\frac{z}{\beta_o \lambda}\right) \sin \omega \left(t - t_o\right) \qquad (1)$$

with E_0 the longitudinal electric field amplitude, z the longitudinal coordinate, λ the RF wave length, β_0 the beta for which the cavity is optimized and ω the angular rf frequency.

For the temporal coordinate of a particle which is moving with speed βc, we have:

$$t = \frac{z}{\beta c} \qquad (2)$$

The voltage seen by the particle passing the cavity is then given by integration of (1) along the cavity, i.e.

$$V = E_o \int \sin 2\pi \left(\frac{z}{\beta_o \lambda}\right) \sin \frac{2\pi c}{\lambda} \left(\frac{z}{\beta c} - t_o\right) dz \qquad (3)$$

For a structure with 2n cells in total we then find for the accelerating voltage:

$$V = \frac{E_o \lambda}{\pi} \cdot \frac{\beta^2 \beta_o}{\beta_o^2 - \beta^2} \cos(n\pi) \ \sin\left(n\pi \frac{\beta_o}{\beta}\right) \cos\phi \qquad (4)$$

with $\phi = (2\pi c/\lambda)t_0$ the synchronous phase, which can be set to zero for the subsequent considerations without loss of generality.

In the special case of $\beta = \beta_0$ we find for the accelerating voltage:

$$V(\beta = \beta_o) = E_o \, n \left(\frac{\beta \lambda_o}{2}\right) \qquad (5)$$

As can be easily verified, the limits of the particle beta for which acceleration can be performed, i.e. whenever expression (4) is positive are given by:

$$\left[\frac{n}{(n+1)}\right] \beta_0 < \beta < \left[\frac{n}{(n-1)}\right] \beta_0$$

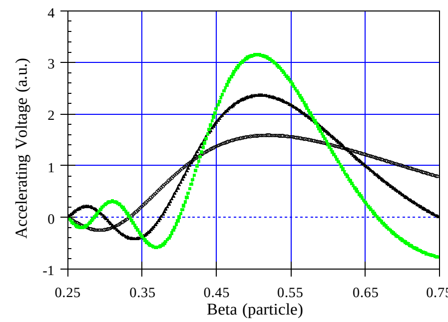

Figure : 1 Accelerating voltage (without the factor $E_0\lambda/\pi$ in equation 4) for 3 different structure lengths (4, 6 and 8 cells), for a structure optimized for β_0=0.5.

Figure 1 shows the accelerating voltage for different cell numbers for a structure optimized for β_0=0.5. For larger cell numbers the voltage decays more rapidly and the total acceleration range is narrowed down. There is also a slight shift of the maximum accelerating voltage to higher betas, which becomes more pronounced for smaller cell numbers. It is caused by an increase in average voltage of a single cell for particles with betas higher than the nominal one (it is assumed here that the particle is passing at zero phase the center of a cell structure with an even number of cells). The reduction of this shift with higher cell numbers is then due to the loss of synchronism.

1.2 Optimization of the Three Different Low Beta Structures

In a first step the accelerating voltage is kept constant for the low beta structures with different cavity betas. By using expression (4) we can optimize the three different structures to get the maximum overall acceleration efficiency.

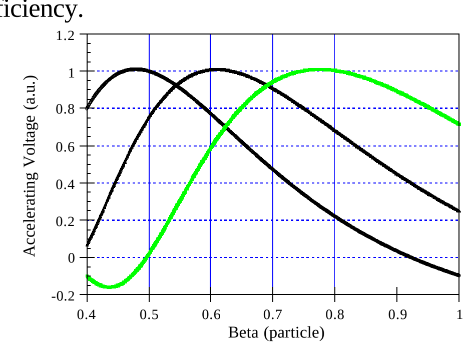

Figure : 2 Accelerating voltages for the three different low beta structures (optimized for β_0=0.46, 0.59, 0.75 from left to right) as a function of particle beta.

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Figure 2 shows the accelerating voltages for the three optimized low beta structures as a function of the particle beta, covering the energy range from 100 MeV to 1 GeV. The minimum efficiency at the intersection points between two different structures is 92.4%.

1.3 Optimization of Low Beta Structures with Variable Peak Fields

So far the optimization was performed by assuming that the maximum accelerating field achieved for LEP [2] can also be applied for the low beta structures. Numerical cavity calculations performed in [3] have revealed that by reducing the length of the cavity, the surface electric and magnetic peak fields are increase. In order to maintain the peak fields as for the LEP cavities, the accelerating field must be reduced with the length of the structure. For the initial cavity design an increase of the peak field for constant accelerating field, proportional to the inverse of the length was found. This in turn implies that the electric field in equation (4) has to be replaced by $E=\beta_o E_o$. In a further optimization step [3] a considerable reduction of the peak field could be achieved, leading to the modified condition for the accelerating field, $E = (0.33+0.66\ \beta_o)E_o$.

For the optimization of the overall efficiency we have to derive the maximum integrated voltage over the beta range β_1 to β_2, defined as:

$$V_t = \frac{E(\beta_0)\lambda}{\pi} \int_{\beta_1}^{\beta_2} \frac{\beta^2 \beta_o}{\beta_o^2 - \beta^2} \sin\left(2\pi \frac{\beta_o}{\beta}\right) d\beta$$

Figure 3 presents the results of the optimization procedure in a form showing the accelerating voltage/cavity as a function of the kinetic energy. E_o was chosen to be 12 MV/m which corresponds to the maximum gradient reached with the LEP cavities for β=1 (note: $V(\beta=\beta_o)L_{cav}=E_o\beta\lambda_o$).

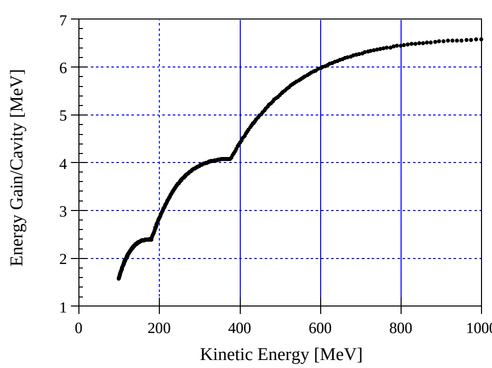

Figure : 3 Accelerating voltage/cavity as a function of kinetic energy (cavity betas β_o: 0.46, 0.59, 0.75, number of 4-cell cavities: 35, 54, 111)

It becomes evident from the graph, that the scaling of the accelerating voltage has created a shift of the structure beta towards higher energies. Only the lower branch of the quasi symmetric distribution is taken for the acceleration, since the accelerating voltage at the lower branch for larger β_o, is larger than in the upper branch for a symmetric arrangement.

2 EFFECTIVE ACCELERATION FOR A REALISTIC FIELD

In the following we introduce an analytical model for the accelerating field, based on results of different numerical calculations for cavities with different lengths.

2.1 Approximation of the Numerically Evaluated Field

The real cavity field has been divided in 3 distinct regions with separate analytical approximations:

I $\underline{0 < z < 3/4\ \beta_o\lambda}$: It is assumed that the field is undisturbed in this region.

II $\underline{3/4\ \beta_o\lambda < z < \beta_o\lambda}$: Is approximated by a cos-function with an increased period length

III $\underline{3/4\ \beta_o\lambda < z < \infty}$

Figure 4, shows a comparison between the approximation and the numerically evaluated field, and also indicates the three areas with different analytical approximations.

After matching the field values and the slopes at the connection points, we find the following expressions:

$$E_I = -E_o \sin\left[\frac{2\pi}{\beta_o\lambda} \cdot z\right]$$

$$E_{II} = E_o \cos\left[\frac{2\pi}{\beta_o\lambda} \cdot q\left(z - \frac{3}{4}\beta_o\lambda\right)\right] \qquad (6)$$

$$E_{III} = E_o \cos\left(\frac{\pi}{2} q\right) \exp\left[-\left(\frac{2\pi}{\beta_o\lambda}\right) q \tan\left(\frac{\pi}{2} q\right)(z - \beta_o\lambda)\right]$$

with q as a matching parameter, which is given by the ratio of a/b, as indicated in the drawing.

Based on the numerical data of the different numerical calculations for cavities with different lengths, a scaling for the matching parameter q has been derived, which for the various cavity types can be approximated by the linear relation: $q = 0.33 + 0.688\ \beta_o$.

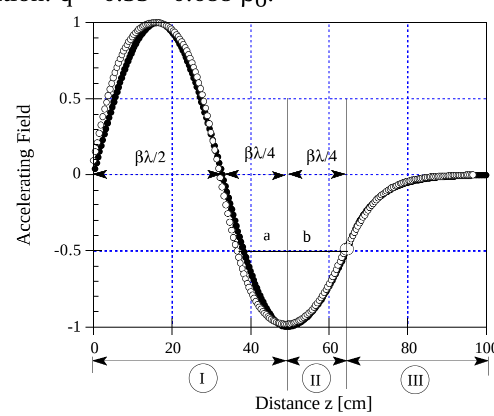

Figure : 4 Comparison of the numerically evaluated field with the analytical model (black dots), for q=0.625 and β_o=0.48.

In order to get the contributions of the 3 distinct field approximations to the accelerating voltage we have to perform the integration over the corresponding region:

$$V_i = \int E_i(z) \sin\left(\frac{2\pi}{\beta\lambda} \cdot z\right) dz \qquad (7)$$

which leads to:

$$V_I = \frac{E_o\lambda}{\pi}\cdot\frac{\beta_o^2\beta}{\beta^2-\beta_o^2}\cos\left(\frac{3}{2}\pi\frac{\beta_o}{\beta}\right)$$

$$V_{II} = \frac{E_o\lambda}{\pi}\cdot\frac{\beta_o\beta}{(q\beta)^2-\beta_o^2}\left\{\beta_0\left[\cos\left(2\pi\frac{\beta_0}{\beta}\right)\cos\left(\frac{\pi}{2}q\right) - \cos\left(\frac{3}{2}\pi\frac{\beta_0}{\beta}\right)\right] + q\beta\left[\sin\left(2\pi\frac{\beta_0}{\beta}\right)\sin\left(\frac{\pi}{2}q\right)\right]\right\}$$

$$V_{III} = \frac{E_o\lambda}{\pi}\cdot\frac{\beta\cos\left(\frac{\pi}{2}q\right)}{1+p^2}\left[\cos\left(2\pi\frac{\beta_o}{\beta}\right)+p\sin\left(2\pi\frac{\beta_o}{\beta}\right)\right]$$

with $p = \frac{\beta}{\beta_o}\, q\, \tan\left(\frac{\pi}{2}q\right)$

The contributions of the terms V_I, V_{II} and V_{III} are separately drawn in figures 5 for the 'Real Field' and the Ideal Field .

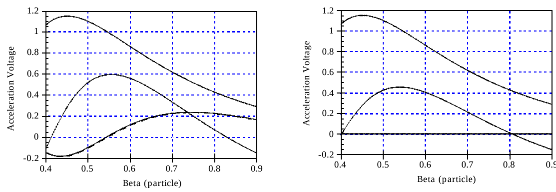

Figure : 5 Contributing terms to the accelerating voltage (without the factor $E_0\lambda/\pi$ in equations 7). Real Field (left) and Ideal Field (right), where V_I gives the largest and V_{III} the least contribution.

By comparing the two graphs we find an enhanced second term for the Real Field. On the other hand we get a negative contribution from the third term for betas lower than a certain threshold, since the particle is exposed to a decelerating field when it enters this region. This can easily be seen by looking at figure 4 for the Real Field. If the motion is synchronized to the structure beta, the particle finds an increased field in section II but sees then an out of phase field when it enters section III. For higher velocities also the third term contributes and we therefore get a shift of the maximum towards higher betas.

2.2 Optimization of the Three Low Beta Structures for a Realistic Field

Due to the fast decay of the accelerating voltage for the real field towards lower betas, the optimum structure betais here shifted back towards smaller betas [4,5]. The results of the optimization are presented in figure 6.

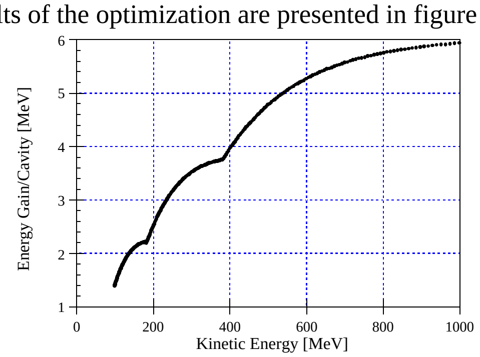

Figure : 6 Accelerating voltage/cavity as a function of kinetic energy (cavity betas β_o: 0.49, 0.64, 0.80, number of 4-cell cavities: 36, 63, 131)

The importance of an appropriate field model is visualized in figure 7, where the accelerating gradient/cavity is drawn as a function of kinetic energy, assuming the Real Field, for a structure optimized for the Ideal Field. A considerable mismatch at the transition between two structures occurs.

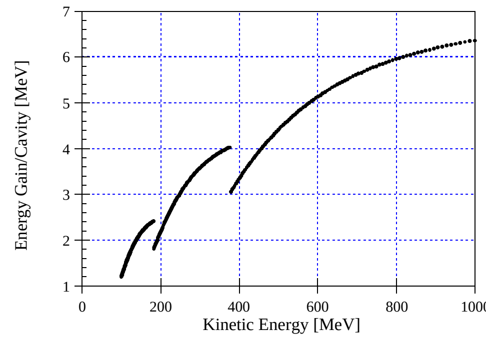

Figure : 7 Accelerating gradient/cavity vs. kinetic energy, computed with the realistic field for a structure optimized for the ideal field.

3 CONCLUSIONS

It was suggested in [1], that a sequence of 3 low beta structures be used for the Energy Amplifier to cover the energy range from 100 MeV to 1 GeV. The effective acceleration voltages have been optimized for the 3 different structures, using the ideal sinusoidal field and an improved analytical approximation of the Real Field derived by numerical calculations in [2] and [3].

An optimization of the overall accelerating structures was performed for two different cases. In a first step it was assumed that the achievable gradient is identical in all structures [4]. In a second step the limits given by the surface fields have been included in the optimization [5].

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