LINEAR BEAM DYNAMICS IN THE SUPERCONDUCTING LINEAR ACCELERATOR OF THE ENERGY AMPLIFIER

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Abstract

For the design of the linear optics of the superconducting linear accelerator for a Full Scale Energy Amplifier, a computer code is under development for the 3-dimensional linear motion with space charge and transverse cavity effects included. A sequential matching routine allows an easy optimization of the optics for the long structure of the linear accelerator. The program is described and first results of the optics optimization for the linear accelerating structure are discussed.

1 INTRODUCTION

A possible scenario of accelerators to drive a full scale Energy Amplifier was proposed in [1] by C. Rubbia. Based on this proposal a preliminary study was performed in [2] for the overall concept and some of the accelerator systems.

The accelerating system proposed consists of a warm part for the low energy range with a final energy of 100 MeV, followed by a superconducting linear accelerator which is built up by three different types of low beta accelerating cavities and LEP type cavities in the final stage. Each of the different structures has to cover a wide range of particle velocity variation with a corresponding loss in efficiency. The optimization of the overall structure and the derivation of the cavity lengths, (i.e. β_o, since $L_{cell}=\beta_o\lambda/2$) is described in [3]. All cavities are built up by four cells. Several cavities are housed in the same cryostat, varying from 7 for the low energy structure, to 4 for the high energy end. Each cryostat is preceded by a normal conducting quadrupole doublet for transverse focusing.

2 COMPUTER CODE DESCRIPTION

2.1 Basic Concepts

In order to take the loss in efficiency in consideration for particle velocities βc which are largely deviating from the cavity β_o, the computer code LINAD was developed for linear optics calculation and tracking. Major parts of the code closely follow the concepts developed for TRANSPORT and TRACE3D [4,5]. LINAD calculates the envelopes of a bunched beam, represented by a 6*6 σ matrix, and tracks single particles along the linac structures. The drifts and the magnets are represented by a 6*6 matrix as in the program Transport [5], while the accelerating elements are treated in the thin element approximation. Each cavity can be subdivided in an arbitrary number of segments in order to handle the acceleration and the transverse cavity effects in a more accurate way. As in [4], the space charge effects are derived from an approximation of the electric field given by an uniformly charged ellipsoid as described by Lapostolle in [6].

The electromagnetic field in a 4-cell cavity (assuming radial deviations from the center which are small as compared to the cavity dimensions) can be analytically expressed by:

$$E_z = E_o \sin\left(\frac{2\pi z}{\beta_o\lambda}\right) \sin\left(\frac{2\pi z}{\beta\lambda}+\phi\right)$$

$$E_r = -\frac{E_o\pi r}{\beta_o\lambda} \cos\left(\frac{2\pi z}{\beta_o\lambda}\right) \sin\left(\frac{2\pi z}{\beta\lambda}+\phi\right)$$

$$B_\theta = \frac{E_o\pi r}{\lambda} \sin\left(\frac{2\pi z}{\beta_o\lambda}\right) \cos\left(\frac{2\pi z}{\beta\lambda}+\phi\right)$$

with E_o the amplitude of the axial field, r the distance from the symmetry axis, $\beta_o c$ the geometrical velocity of the cavity, βc the beam velocity and ϕ the synchronous phase (defined as the phase difference between the particle passing the center of the symmetric structure and the zero crossing of the accelerating wave).

2.2 Acceleration

To take care of the beam velocity change along the cavity, each cell of the cavity can be cut in an arbitrary number of segments. Corresponding to the energy gain in the cavity segment, the beam velocity is then updated before entering the subsequent one.

The voltage seen by the particle passing one cavity segment defined by the coordinates z_1 and z_2 is then given by [3]:

$$V = \int_{z_1}^{z_2} E(z)\,dz = \frac{E_o}{k_o^2-k^2}\Big[-k_o\cos(k_o z)\sin(kz+\phi) + k\sin(k_o z)\cos(kz+\phi)\Big]_{z_1}^{z_2}$$

where $k_0 = \frac{2\pi}{\beta_0\lambda}$, $k = \frac{2\pi}{\beta\lambda}$.

For a four cell structure with the length $2\beta_o\lambda$ we find for the total accelerating voltage:

$$V = \frac{E_o\lambda}{\pi}\cdot\frac{\beta^2\beta_o}{\beta_o^2-\beta^2}\sin\left(2\pi\frac{\beta_o}{\beta}\right)\cos\phi$$

Figure 1 shows the energy gain per segment (with 100 segments per half cavity) as a function of the longitudinal position in the cavity, for two different operation modes.

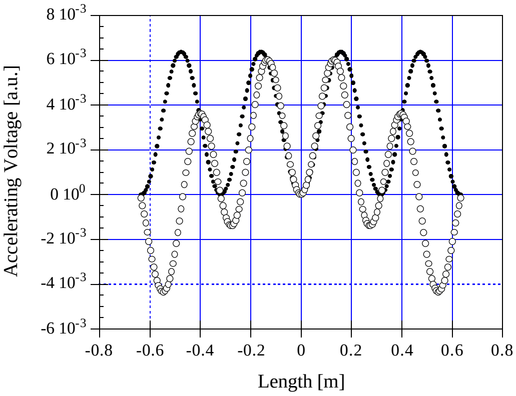

Figure : 1 Energy gain per segment (for zero synchronous phase) in a 4-cell cavity with β_0=0.75, as a function of longitudinal position. Two different particle velocities are shown, corresponding to β=0.75 (black dots) and β=0.55.

The possibility for a numerical integration of data points (representing the accelerating field) from a file, was also implemented. This allows an improvement of the sinusoidal approximation for the accelerating field and to include more realistic models, as for instance proposed in [3] or fields derived by numerical computations.

2.3 Transverse Cavity Effects

Transverse cavity effects can greatly alter the focusing in the linear structure. From the electromagnetic fields we can derive the transverse force for the general case of particles which are largely deviating in velocity from the one the cavity is optimized for (x stands for the horizontal and vertical plane):

$$F_x = q\left(E_r - \beta c\ \beta\right) = -\frac{E_o q\,\pi}{\beta_o\,\lambda}\Big\{ \cos\left(k_o z\right)\sin\left(kz+\phi\right) + \beta_o\beta\sin\left(k_o z\right)\cos\left(kz+\phi\right)\Big\}\cdot x$$

The transverse kick generated by a cavity segment is then obtain by integration over the segment length:

$$p_x = \frac{E_o q}{4c}\left\{\frac{1+\beta\beta_o}{\beta_o+\beta}\cos\left[\left(k+k_o\right)z+\phi\right] + \frac{1-\beta\beta_o}{\beta_o-\beta}\cos\left[\left(k-k_o\right)z+\phi\right]\right\}_{z_1}^{z_2}\cdot x$$

For a 4-cell structure with the length $2\beta_0\lambda$ we find for the total transverse momentum generated by the cavity (only valid if the change in velocity over the cavity length can be neglected):

$$p_x = -\frac{qE_o}{\gamma^2 c}\cdot x\cdot\frac{\beta_o}{\beta_o^2-\beta^2}\sin\left(2\pi\frac{\beta_o}{\beta}\right)\sin\phi$$

Figure 2 shows the transverse momentum generated per segment as a function of the longitudinal position in the cavity. Two different operation modes are considered.

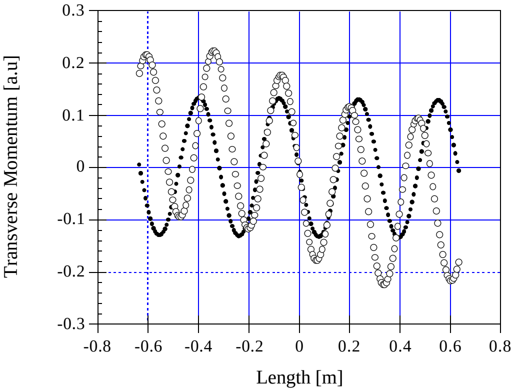

Figure : 2 Transverse momentum per segment (for zero synchronous phase) in a 4-cell cavity with β_0=0.75, as a function of longitudinal position. Two different particles velocities are shown, corresponding to β=0.75 (black dots) and β=0.55.

2.4 Matching Routines

Different types of matching routines are foreseen which allow a sequential matching of the transverse and longitudinal optics along the structure, together with the adjustment of beam energy. For the moment the following matching options are available:

- Energy at the exit of an element or energy gain at a specific acceleration element.
- Longitudinal and/or transverse phase advances between two elements.
- Structure end values of β_x, α_x or/and β_y, α_y or/and β_ϕ, α_ϕ.

At the moment the operation of the code is limited to the treatment of quadrupole magnets for transverse focusing and multicell cavities for acceleration. The matching of the longitudinal phase advance is at present functional for the multicell cavities. The code determines the synchronous phase ϕ for the multicell cavity if chosen as a tuning parameter and sets the desired phase advance for the selected range. In case of transverse phase matching, the program determines the gradient in the quadrupoles indicated by the user.

3 FIRST RESULTS FOR THE LINAC STRUCTURE OPTICS

Some initial optics calculations were performed for various parts of the superconducting linac structure. The following procedure was adopted for these calculations:

- Find the RF phase ϕ at the entrance of each cavity in order to get a good compromise between the energy gain the longitudinal phase advance and the longitudinal betatron function,
- Perform the transverse matching to get a correct transverse phase advance and a smooth variation of the transverse betatron functions,
- See the behaviour of the phase advances and the betatron functions in the presence of the space charge effects. If the tune depressions are too low or/and the betatron functions oscillate too much, rematch the phase advances.

Figure 3 shows the transverse and longitudinal β functions along 5 periods of a β-graded LEP structure. The 4-cell cavity is optimized for β_o=0.8 while the initial β of the particles is β=0.69. Zero current is assumed, which means that the optics is not affected by space charge forces (but just by transverse cavity effects).

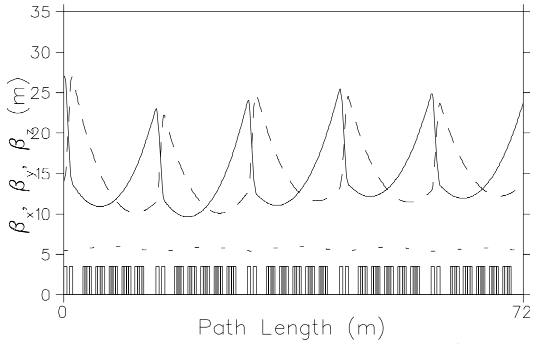

Figure : 3 Transverse and longitudinal β functions along 5 periods of the low beta structure with β_o=0.8; for zero beam current.

The same section is shown in figure 4 for a beam current of 20 mA, after re-matching of the structure. The corresponding phase advances and the tune depressions are shown in figures 5 and 6, respectively.

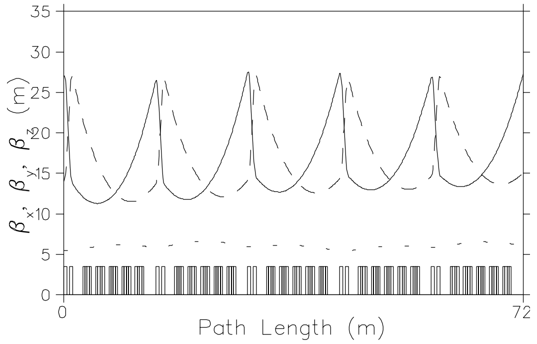

Figure : 4 The same parameters as figure 3 with 20 mA beam current (after rematching).

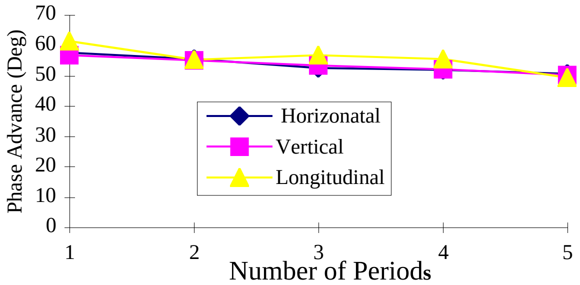

Figure : 5 Phase advances for a 20 mA beam current.

Figure 7 shows the energy gain over the structure length. The increase in effciency, i.e. energy gain with higher β can be clearly observed.

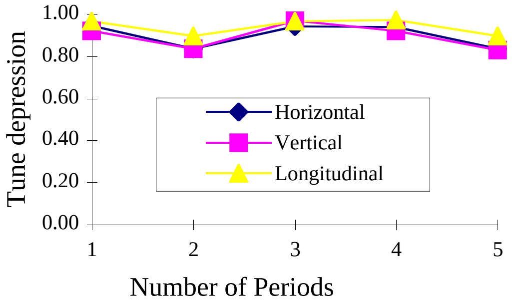

Figure : 6 Tune depression for a 20 mA beam current along 5 periods of the EA3 structure, β_o=0.8.

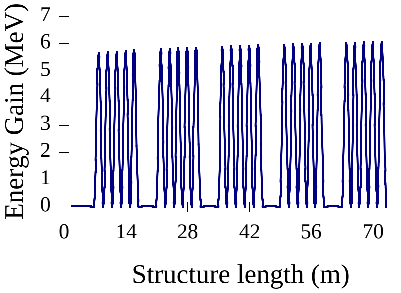

Figure7 : Energy gain over 5 periods of the low beta structure with β_o=0.8.

4 CONCLUSIONS

A new computer code has been developed which accurately treats the acceleration in multicell cavities for particles with velocities which deviate largely from the velocity for which the cavity is optimized. Each cavity can be subdivided in an arbitrary number of segments in order to take care of the energy increase in the structure. Acceleration. as well as transverse cavity effects can therefore be treated in an improved manner.

In addition sequential matching routines have been implemented which allow an easy matching of long linac structures with many periods.

First results of calculations for the superconducting linac proposed in [1,2] as a driver for a full scale Energy Amplifier were presented.

REFERENCES

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