EFFECTS OF THE LANDAU CAVITY ON THE ELECTRON BEAM

L.H. Chang, Ch. Wang, W.K. Lau, C.C. Kuo
Synchrotron Radiation Research Center, Hsinchu, Taiwan

Abstract

This paper presents the procedure and the formulas to analyze the effects of Landau cavity on the electron beam while the main RF system is operated on the compensated condition. It is shown that the maximal available current, determined by the phase instability limit, is reduced with Landau cavity. The model of " potential energy " [1] is used for the calculations of the bunch length and the synchrotron frequency of the storage ring with double RF system. It is shown that the bunch length can be manipulated by tuning the resonance frequency of the passive Landau cavity, and the spread of synchrotron frequency can be induced by the addition of a Landau cavity.

1 INTRODUCTION

The beam life-time is always the concern of synchrotron radiation users. Especially for the third generation storage ring , it is operated with low beam emittance and short bunch length. The Touschek scattering is usually a limit of the beam life-time for the low energy machine. It can be improved either by increasing the energy acceptance or by decreasing the charge density of a bunched beam. The Landau cavity is one of the solutions to decrease the charge density by increasing the bunch length.

The Landau cavity can be operated either in active mode or in passive mode. In active mode, the gap voltage and the RF phase are adjusted to manipulate the slop of the accelerating voltage, which affects the bunch length. The optimized conditions for bunch lengthening and the beam dynamic theory were discussed by Hofmann and S. Myers in 1980 [2]. In passive mode, the bunch length is current dependent, and is manipulated by adjustment of the tuning angle of the Landau cavity. The implementation of the Landau cavity enhances the phase instability [3], which reduces the maximal available beam current. Moreover, the RF power generated by beam current may be enough to cause the damage of the passive Landau cavity. It is important to estimate these effects in the design of the harmonic Landau cavity.

2 PHASE INSTABILITY LIMIT

In a double RF system composed of a main RF system with accelerating frequency ω_{RF} and a nth harmonic RF system, The total RF voltage V_T seen by the electron with time displacement τ can be written as

$$V_T(\tau) = V_M \cos(\phi_M + \omega_{RF}\tau) + V_L \cos(\phi_L + n\omega_{RF}\tau) \tag{1}$$

where V_M and ϕ_M are the total gap voltage and synchronous phase of the main RF system; V_L and ϕ_L are the total gap voltage and synchronous phase of the harmonic RF system. The accelerating voltage $V_T(0)$ is equal to the voltage for compensating the energy loss of the synchronous electron. Notice that the time displacement τ is defined to be positive if the electron lags behind the synchronous electron. In this paper, the subscript L, M represent the quantities related to the system of the harmonic Landau cavity and main RF system, respectively.

V_M in (1) is combined with the induced voltage V_b and the generated voltage V_g [4]. The relations between them can be expressed as

$$V_M \cos(\phi_M) = -V_b \cos(\psi) + V_g \cos(\phi_g) \tag{2}$$

where ϕ_g is the phase of generated voltage with reference to the synchronous electron, ψ is the tuning angle of the main RF cavity. Without beam current in cavity, ψ is equal to the tuning angle offset ψ_0, which is defined as

$$\tan(\psi_0) = 2Q_l \frac{\omega_r - \omega_{RF}}{\omega_r} \tag{3}$$

where Q_l is the loaded quality factor, ω_r is the resonance frequency of the cavity. By this definition, the positive tuning angle means that the resonance frequency ω_r is higher than the frequency of the electromagnetic field of the mode, ω_{RF}. Similarly, if the Landau cavity is operated in active mode, V_L can be expressed as

$$V_L \cos(\phi_L) = -V_{bL} \cos(\psi_L) + V_{gL} \cos(\phi_{gL}) \tag{4}$$

If the Landau cavity is operated in passive mode, ψ_L is not dependent on the beam current, and V_L is derived only from the induced voltage. Equation (4) needs to be modified as

$$V_L \cos(\phi_L) = -V_{bL} \cos(\psi_L) \tag{5}$$

where $\phi_L = \psi_L$. In (2), ψ is varied with the beam loading effect. The variation ψ_b is dependent on the operating condition. For our main RF system the feedback control always keeps the amplitude of V_M and the relative phase between V_M and V_{gr} constant. V_{gr} is the generated voltage at resonance frequency. On this condition, ψ_b can be obtained by the equation below [5].

$$\psi_b = -\tan^{-1}\left(\frac{V_{br}\sin(\phi_M - \psi_0)}{V_M/\cos(\psi_0) - \tan(\psi_0) V_{br}\sin(\phi_M - \psi_0)}\right) \tag{6}$$

where V_{br} is the induced voltage at the resonance frequency. With beam current in the cavity, ψ becomes

$$\psi = \psi_0 + \psi_b \tag{7}$$

The phase of V_g with reference to the synchronous electron becomes

$$\phi_g = \phi_M + \psi_b \tag{8}$$

If we consider the main RF system with the couple coefficient β and ψ in (7) which are optimized for minimizing the reflected RF power from the cavity, the compensated condition [5]. And the bunch length is short enough to be ignored, in comparison with the cavity size. With these conditions, we can obtain V_{br} and V_b as follows:

$$V_{br} = 2I_aR_s \tag{9}$$

$$V_b = V_{br}\cos(\psi) \tag{10}$$

where R_s is the shunt impedance of the cavity, I_a is the average beam current. ϕ_g is decreasing with the increase of the beam current. As ϕ_g becomes zero, the phase is at the stability limit [4]. In passive Landau cavity, the interaction between the beam current and the RF field is stronger as ψ_L is getting close to zero. As shown in Fig. 1, the maximal available beam current is reduced from 512 mA to 153 mA as ψ_L is tuned from $90°$ to $0°$. From the energy conservation law, we can obtain the RF power for the cavity to keep V_M constant

$$P_M = \frac{V_M^2}{2.0\cdot R_s} + V_M\cdot\cos(\phi_M)\cdot I_a \tag{11}$$

The discussion above for the main RF system are still valid with the Landau cavity operated in active mode.

For the Landau cavity operated in passive mode, V_{brL} is given by

$$V_{brL} = \frac{2I_aR_{sL}}{1+\beta_L} \tag{12}$$

and the power loading of the Landau cavity is given by

$$P_L = \frac{[V_{brL}\cdot\cos(\psi_L)]^2}{2.0\cdot R_{sL}} \tag{13}$$

P_L in equation (13) is dependent on the beam current and ψ_L. For the case in Fig. 2, P_L is obtained at the phase instability limit. The maximal P_L, at $\psi_L = 0$, is beyond 50 kW.

3 BUNCH LENGTH

If we neglect the radiation damping term, the time deviation in synchrotron oscillation can be described by the equation

$$\frac{d^2\tau}{dt^2} = \frac{\alpha}{E_0T_0}\cdot\{e\cdot V_T(\tau) - U_0\} \tag{14}$$

where U_0 is the radiation loss of the electron with nominal energy in one revolution, E_0 is the nominal energy, T_0 is the revolution time, α is the momentum compaction. We define α as

$$\frac{d\tau}{dt} = \alpha\frac{\epsilon}{E_0} \tag{15}$$

where ϵ is the energy deviation from the nominal energy. Eq. (14) is similar to the equation of motion under the

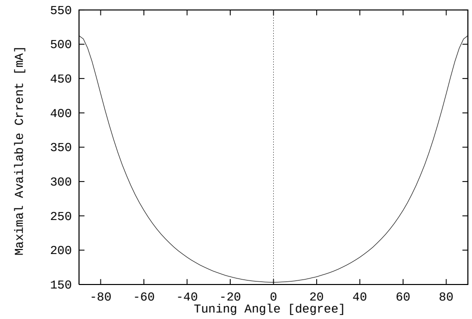

Figure 1: The beam current at phase instability limit versus the tuning angle of the passive Landau cavity. The machine parameters are listed as follows: nominal energy is 1.5 GeV, harmonic number of Landau cavity is 3.0, number of main cavities is 3, energy spread is 6.6×10^{-4}, accelerating frequency is 500 MHz, $R_s = 3.0M\Omega$, $R_{sL} = 1.2M\Omega$, $\alpha = 6.78\times10^{-3}$, $V_T = 157kV$, $V_M = 1200kV$.

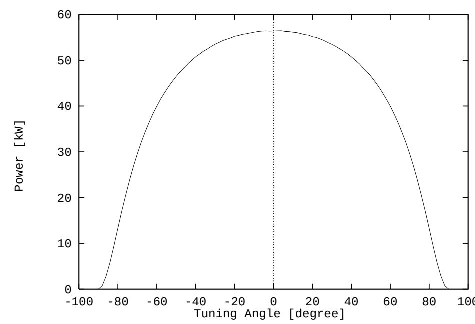

Figure 2: The power loading of the Landau cavity with maximal available beam current (at the phase stability limit). The machine parameters are the same as in Fig. 1.

conservative force $(d^2\tau/dt^2)$. The " potential energy " can be obtained by integrating the "force" [Eq. (14)].

$$\phi(\tau) = -\frac{\alpha}{E_0T_0}\int_0^\tau\{e\cdot V_T(t) - U_0\}dt \tag{16}$$

For the electron with peak energy deviation $\hat{\epsilon}$, the maximal potential energy, which is equal to the maximal kinetic energy $[1/2(d\tau/dt)^2]$, can be obtained from Eq. (15)

$$\hat{\phi} = \frac{1}{2}(\alpha\frac{\hat{\epsilon}}{E_0})^2 \tag{17}$$

The maximal time displacement is at the point where the potential energy is maximal.

$$\phi(\tau_p) = \phi(\tau_n) = \hat{\phi} \tag{18}$$

where τ_p (τ_n) is the maximal time displacement behind (ahead) the synchronous phase. The bunch length of the electrons with peak energy deviation $\hat{\epsilon}$ is then given by

$$\sigma_l(\hat{\epsilon}) = c\cdot(\tau_p - \tau_n) \tag{19}$$

where c is the velocity of the photon. For an electron beam with Gaussian spread in energy deviation, we may express the normalized electron density as

$$n(\hat{\epsilon}) = \sqrt{\frac{2}{\pi}} \cdot \epsilon_{rms} \cdot exp(-\hat{\epsilon}^2/2\epsilon_{rmx}^2) \qquad (20)$$

where ϵ_{rms} is the root-mean-square deviation of energy. The root-mean-square bunch length is given by

$$\sigma_{rms} = \sqrt{\int n(\hat{\epsilon})\sigma_l(\hat{\epsilon})^2 d\hat{\epsilon}} \qquad (21)$$

For the machine with a Landau cavity operated in passive mode, ψ_L is the parameter that we can use to manipulate the bunch length.

As shown in Fig. 3, the bunch is lengthened with the larger beam current if ψ_L is positive. In this case, the bunch is lengthened more than 70% with the beam current at 200 mA. But the power loading is beyond 45 kW (see Fig. 2).

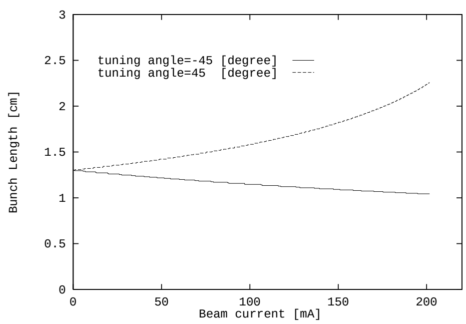

Figure 3: The bunch length versus the beam current for two different tuning angles. The parameters are the same as in Fig. 1.

4 SYNCHROTRON FREQUENCY

From energy conservation law, the " velocity" ($d\tau/dt$) is given by

$$\frac{d\tau}{dt} = \sqrt{2}[\hat{\phi} - \phi(\tau)]^{1/2} \qquad (22)$$

From Eq.(22), we can obtain the time period of synchrotron oscillation.

$$t_\nu = 2.0 \int_{\tau_n}^{\tau_p} \frac{1.0}{\sqrt{2}[\hat{\phi} - \phi(\tau)]^{1/2}} d\tau \qquad (23)$$

The synchrotron frequency is obtained from the inverse of t_ν

$$f_\nu = \frac{1}{t_\nu} \qquad (24)$$

Fig. 4 shows that the shift of synchrotron frequency, introduced by the addition of a Landau cavity, is dependent on the peak energy deviation. It results in a synchrotron tune spread for a bunched beam, which will enhance the Landau damping against longitudinal coupled bunch instability.

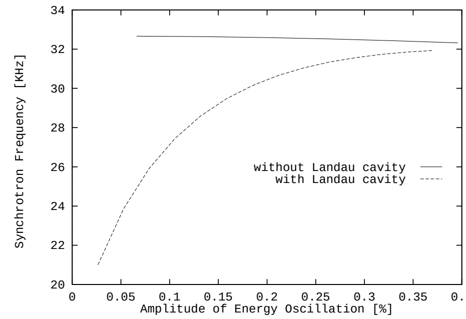

Figure 4: The synchrotron frequency versus the peak energy deviation. $I_a = 180mA$, $\psi_L = 45°$, and the other parameters are the same as in Fig. 1.

5 DISCUSSION

The Landau cavity proposes an approach to lengthen the bunch, and enhances the Landau damping against the longitudinal coupled bunch instability. But it also causes unwanted effects, such as the reduction of the available beam current. V_b in (10) is obtained assuming that the RF system is operated on the compensated condition. In reality, the compensated condition can be met only at a certain beam current. In our machine, the couple coefficient of the main RF cavity is 1.2. It means that without beam current, the ratio between the reflection power and the power loss in the cavity is about 0.84 $\%$. With this mismatch, the error for V_b in (10) is about 1 $\%$ when beam current is very small. If the couple coefficient is getting larger, a further study is necessary. Moreover in passive mode, if R_{sL} is large enough and the beam current is above some threshold value, the "potential energy " may be deformed to two valleys. In such case, Eq.(19) is invalid for the bunch length calculation.

6 REFERENCES

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