COLLECTIVE CENTROID OSCILLATIONS AS AN EMITTANCE PRESERVATION DIAGNOSTIC IN LINEAR COLLIDER LINACS

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Abstract

Transverse bunch centroid oscillations, induced at operating beam currents at which transverse wakefields are substantial, and observed at Beam Position Monitors, are sensitive to the actual magnetic focusing, energy gain, and rf phase profiles in a linac, and are insensitive to misalignments and jitter sources. In the 'pulse-stealing' set-up implemented at the SLC, they thus allow the frequent monitoring of the stability of the in-place emittance growth inhibiting or mitigating measures—primarily the energy scaled magnetic lattice and the rf phases necessary for BNS damping—independent of the actual emittance growth as driven by misalignments and jitter. We have developed a physically based analysis technique to meaningfully reduce this data. Oscillation beta-beating is a primary indicator of beam energy errors; shifts in the 'invariant' amplitude reflect differential internal motion along the longitudinally extended bunch and thus are a sensitive indicator of the real rf phases in the machine; shifts in betatron phase advance contain corroborative information sensitive to both effects. Examples from initial SLC applications illustrate the method.

Differential internal motion due to intra-bunch energy or amplitude spread, or bunch spatial extension which makes collective or multiparticle interactions possible, causes striking differences between the behavior of the centroid of a bunched beam, which is measured by a beam position monitor, and a single particle. Bunch inhomogeneities that respond differentially to the beamline environment engender decoherence, and possibly recoherence and echo phenomena; current dependent collective effects can strongly excite novel patterns of centroid motion.

We first discuss general aspects of the difference between centroid and single particle mechanics in the context of developing a general parameterization scheme for the former, and then move to some more specific features of beam dynamics with transverse wakefields.

1 CENTROID KINEMATICS

To develop a useful description of centroid motion that provides more or less machine-error-specific indicators, we mimic to a large degree the familiar parameterization of single particle motion. The parameters acquire new meanings in the centroid context that fully incorporate its distinctive features, but recover their single particle significance in the limiting case in which the bunch as a whole moves rigidly. At every location in the beamline, every trajectory in the two dimensional phase space is mapped into a point on some unit circle, where it is located by a phase angle which can accordingly be taken to 'advance' from beamline location to location. Choosing the mapping to be linear makes families of trajectories comprising whole circles generically images of ellipses in the phase space observables. Both the ellipse and the phase angle contain nonredundant information about the motion, and are inextricably linked in that the phase angle and hence the phase advance is undefined without an associated ellipse. For linear motion choosing an ellipse at one beamline location determines an ellipse—its image under the transport map—at all other locations. It is usually especially useful to choose an initial family of geometrically similar ellipses whose shape has a conceptually or mnemonically useful property, like the same periodicity as the beamline itself, and/or, the distinction that it describes the bunch 'beam envelope'. Here the ellipse in phase space is a family of centroid trajectories, which could *e.g.*, correspond to the ensemble of trajectories generated by a specially distributed jitter source at the beginning of the beamline; it is emphatically not directly related to the family of collectively interacting single particle trajectories that constitute the beam envelope. The fact that linear transport maps ellipses into ellipses is all that we use; we do not assume the invariance of the ellipse area, even though it was this aspect of the single particle case that historically drew attention to ellipses in phase space. Phase advance is necessary to then fully describe the transport of the particular trajectories on the ellipse.

To be more explicit, the phase space centroid, or beam-average position $\langle x(s)\rangle$ and angle $\langle x'(s)\rangle$) coordinates at machine location s, is parameterized by the non-restrictive *ansatz*

$$\begin{bmatrix}\langle x(s)\rangle\\ \langle x'(s)\rangle\end{bmatrix} = \sqrt{2a(s)}\,A(s)\begin{bmatrix}\cos\big(\psi(s)\big)\\ -\sin\big(\psi(s)\big)\end{bmatrix} \qquad (1)$$

that linearly maps all trajectories into unit circles, on which they are located by an advancing phase $\psi(s)$. $A(s)$ is a 2×2 matrix; taking it to have unit determinant defines the numerical 'amplitude' factor $\sqrt{a(s)}$. Since circles are invariant under rotation the re-definition $A \rightarrow A\mathcal{O}$, where $\mathcal{O}$ is a rotation matrix, just re-defines the phase function; thus A has only two meaningful parameters. A set of trajectories at an 'initial' beamline location constructed to fill a unit circle and mapped in x, x' space into a fiducial ellipse that is geometrically characterized by the symmetric matrix $a_0 A_0 A_0^{\mathsf{T}}$, will be transported at each downstream location into a specific ellipse whose geometric form is given by aAA^{T}. By linearity the shape depends only on the initial shape parameters $A_0 A_0^{\mathsf{T}}$ and the area scales like a_0, *i.e.*,

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only the ratio $a(s)/a_0$ is characteristic of the transport.

Since the circle that the ellipse is mapped to is invariant under rotations, to associate specific trajectories with phase angles $\psi(s)$, and hence be able to speak of a 'phase advance' requires a further but final convention. The historical choice made by Courant and Snyder for single particles assigns a $90°$ phase everywhere in the beamline to a trajectory on the x'-axis, or equivalently imposes $A_{12}(s) \equiv 0$. It has the unique property that zeros in x are $180°$ apart for any choice of the initial ellipse geometry.

The centroid transfer matrix R now satisfies $\sqrt{2a_0}RA_0 = \sqrt{2a}A\mathcal{O}(\Delta\psi)$, giving the representation

$$R = a_g\sqrt{\frac{E_0}{E}}A\mathcal{O}A_0^{-1} \tag{2}$$

where the *amplitude growth/damping factor* $a_g = \sqrt{aE/a_0E_0}$ parameterizes the *specific* amplitude shift (exclusive of 'adiabatic' damping due to acceleration).

In the single particle limit, where the bunch either is a single particle or behaves like one (*i.e.*, is rigid) $a_g \to 1$ and $A \to \frac{1}{\sqrt{\beta}}\begin{pmatrix}\beta & 0\\ -\alpha & 1\end{pmatrix}$, where β and α are the familiar functions describing pseudo-harmonic oscillations in the quadrupole magnetic focusing field for the initial condition A_0. In the single particle case A is *local*—it depends only on the beamline location at which the trajectory is observed and not on the oscillation's prior history, and the phase advance is *additive*, *i.e.*, the amount by which it increases as one moves along the beamline is independent of the initial location. Directly equivalent to these properties is the *factorization* of the R matrix: $R(a \to b) = R(c \to b)R(a \to c)$ for any intermediate point c. For generic centroid oscillations factorization/locality is not true, and caution should be applied to avoid being misled by the product decomposition in (2). Centroid transport depends on the detailed internal initial state of the bunch, something which is not completely specified by giving its centroid phase space coordinates alone; in fact without a specification of the suppressed internal variables, any centroid transfer map is seriously ill-defined. In the linear case it is convenient in practice to take all transfer matrices to correspond to rigid excitations (kicks or instantaneous displacements) of a homogeneous bunch (one whose internal transverse coordinates all line up with the centroid). Superpositions of appropriately distributed excitations then can describe any coherent β-tron oscillation. The set of centroid transfer maps in the space of centroid variables thus constitutes a complete physical description, at the price of tolerating hysteresis, or non-factorization/non-locality, at a basic level.

An instructive and practical application is to consider 'steering-out' a coherent oscillation by applying appropriate kicks in some neighborhood in the beamline. A rigidly excited oscillation (as with dipole magnets) can be superposed to precisely cancel an incoming oscillation at some point; however, since the internal bunch distribution in the incoming oscillation would generically differ from the homogeneous distribution associated with the excitation and accordingly transport differently, the cancellation will break down as the superposition propagates. An oscillation will grow 'spontaneously', a behavior impossible to achieve if the R matrix were to factorize.

It is frequently very useful to view the A matrix in terms of its deviation from a fiducial reference A_r, which in practice is usually chosen to be the single particle periodic lattice function already mentioned. The *oscillation* 'Bmag' $B = \frac{1}{2}\mathrm{tr}[A_r^{-1}A(A_r^{-1}A)^\mathsf{T}]$ is invariant downstream of an isolated discrepancy with respect to the reference, although the effective β and α functions continuously 'beat'; $\sqrt{B^2-1}$ is in fact the beat amplitude. B is equivalently the average of the squares of the semi-major and semi-minor axes of the ellipse corresponding to the transport of an initial unit circle of trajectories in the normal coordinates defined by A_r. The additional variable completing the description is the β-beat phase or orientation angle of the anomalous ellipse incorporated as Ψ in the parameterization $A_r^{-1}A(A_r^{-1}A)^\mathsf{T} = \begin{pmatrix}B+\sqrt{B^2-1}\cos\Psi & -\sqrt{B^2-1}\sin\Psi\\ -\sqrt{B^2-1}\sin\Psi & B-\sqrt{B^2-1}\cos\Psi\end{pmatrix}$. Collective and decoherence intrabunch effects produce a negligible deviation in $B-1$ in a lattice that is periodic and 'smooth' on the scale of the β-tron wavelength (*cf.* Section 2). Therefore when applied to data, *e.g.*, B serves as a useful indicator isolating effective magnetic strength errors. To the extent to which this is the case the beat phase in an error free region will advance according to $\Psi \to \Psi + 2\psi$, where ψ is the single particle phase shift, *i.e.*, is *exclusive* of any coherent phase shift. It is important not to ignore the β-beat phase since it is possible for a quadrupole strength error to be manifested as a rapid Ψ shift through $-2\times$ the prior Ψ at the error, while $B, > 1$ already due to an upstream error, accidentally does not change. Note again that the oscillation or coherent 'Bmag' is inequivalent to and has nothing direct to say about the beam envelope Bmag that expresses the possible elevation of the matched equivalent emittance.

An alternative representation is sometimes used, will be used in the next section, and is instructive to consider. Two orthogonal oscillations are treated as having individual *pseudo*-phase-advances $\Delta\psi_{1,2}$ and 'amplitudes' $\zeta_{1,2}$. The centroid 'normalized' transfer matrix

$$\sqrt{\frac{E}{E_0}}A_r^{-1}RA_{r\,0} = \begin{pmatrix}\zeta_2\cos(\Delta\psi_2) & \zeta_1\sin(\Delta\psi_1)\\ -\zeta_2\sin(\Delta\psi_2) & \zeta_1\cos(\Delta\psi_1)\end{pmatrix} \tag{3}$$

$$= \mathcal{O}(\Delta\psi_+)\begin{pmatrix}\zeta_2\cos(\Delta\psi_-) & \zeta_1\sin(\Delta\psi_-)\\ \zeta_2\sin(\Delta\psi_-) & \zeta_1\cos(\Delta\psi_-)\end{pmatrix} \tag{4}$$

where $\Delta\psi_\pm = \frac{1}{2}(\Delta\psi_1 \pm \Delta\psi_2)$ are the average and (half) difference pseudo-phase-advances. The term *pseudo*-phase advance is used to emphasize that even in the single particle case where the R matrix factorizes, these 'phase advance'-like parameters are not additive, in the sense described above. The phase independent amplitude growth $a_g = \sqrt{\zeta_1\zeta_2|\cos(2\Delta\psi_-)|}$. It is reduced by a *pseudo*-phase advance difference for fixed $\zeta_{1,2}$. That $a_g \to 1$ in the single particle limit implies a constraint among the parameters in that case. $B = \frac{1}{2}\left[(\zeta_1/\zeta_2) + (\zeta_2/\zeta_1)\right]/|\cos(2\Delta\psi_-)|$. Thus $B > 1$ reflects disparate 'orthogonal amplitudes', and/or disparate pseudo-phase advances—this is perhaps the most useful way to conceptualize oscillation, as opposed to envelope, β-beating. Either condition $\zeta_2 \not\approx \zeta_1$ or $\Delta\psi_2 \not\approx \Delta\psi_1$ accordingly indicates a non-'smooth' lattice and significant collective effects (*cf.* above and Section 2), or the presence of effective magnetic field errors.

2 COLLECTIVE DYNAMICS

The beam dynamics of a longitudinally extended bunch in which transverse dipole wakefields are the dominant intra-bunch forces, reduces to the problem of the motion of a 'string' beam in which the variables are the centroid coordinates $x(s;\tau)$ and $x'(s;\tau)$, at longitudinal position τ. The restriction to a pure dipole wakefield makes an exact 'hydrodynamic' description possible; *i.e.*, the dynamics closes in terms of 'slice' first moments. Finite transverse emittance and energy spread within-a-slice effects can be recovered after this problem is solved.

As already discussed, the problem of interest here may be further reduced to a study of the transfer matrices for rigid excitations in terms of the whole bunch centroid, or the observable N particle slice average $\langle x(s)\rangle = \int dN(\tau)\, x(s;\tau)/N = \int d\tau\, (dN/d\tau)\, x(s;\tau)/N$, and similarly for $\langle x'(s)\rangle$. Each slice freely oscillates due to the initial dipole magnet impulse, to which is superposed free oscillations originating in deflections occurring along the entire length of accelerator through which the beam has moved, which are in turn proportional to the charge weighted accumulation of the instantaneous transverse offsets of the slices preceding it in the bunch, and the wakefield function of their longitudinal distance:

$$x(s;\tau) = R_{12}(s,s_0;\tau)\,\Delta x' + \int_{s_0}^{s} ds_1 R_{12}(s,s_1;\tau) \int dN(\tau_1)\, \frac{eW_\perp(\tau-\tau_1)}{E(s_1)}\, x(s_1;\tau_1) \quad (5)$$

and similarly with x'. $R(s,s_0;\tau)$ is the single particle transverse transfer matrix for longitudinal position τ, on which it depends significantly through the nominal energy spread arising from the accelerating rf waveform and the longitudinal wakefield. Since the slice centroid we are trying to find is related to the preceding slice centroids in the bunch, (5) is a linear integral equation to be solved. It is convenient to combine x and x' into a new complex variable by setting $x - i(\beta x' + \alpha x) = -i\sqrt{E_0/E}\,\beta\,\Delta x' \exp\big(i\Delta\psi(s,\tau)\big)\,\xi(s,\tau)$. It satisfies

$$\begin{aligned}\xi(s,\tau) = 1 - \frac{i}{2}\int_{s_0}^{s} ds_1\, \frac{\beta}{E} \int dN(\tau_1)\, eW_\perp(\tau-\tau_1) \\ \times \{\exp\left[-i\left(\psi(s_1,\tau) - \psi(s_1,\tau_1)\right)\right]\, \xi(s_1,\tau_1) \\ + \exp\left[-i\left(\psi(s_1,\tau) + \psi(s_1,\tau_1)\right)\right]\, \xi^*(s_1,\tau_1)\} \quad (6)\end{aligned}$$

$\psi(s,\tau)$ is the single particle phase, and the τ dependence of β has been suppressed on the grounds that it should be negligible in a viable, at least approximately periodic, lattice. The centroid $\xi(s) \equiv \exp\big(-i\psi(s,0)\big)\,\big\langle \exp\big(i\psi(s,\tau)\big)\xi(s,\tau)\big\rangle$ where $\tau = 0$ (the 'center' of the bunch, say) is chosen to define a reference single particle phase advance. The centroid transfer matrix in the 'two-phase' form (3) is then fixed since $\zeta_1 \exp[i(\Delta\psi_1 - \Delta\psi)] = \xi(s)$ for $\psi(s_0,\tau) = -90°$, and $\zeta_2 \exp[i(\Delta\psi_2 - \Delta\psi)] = \xi(s)$ for $\psi(s_0,\tau) = 0$.

The overall factor in the kernel in (6), $\int ds\,(\beta/E)NeW_\perp$, is $\sim$ the (Courant-Snyder) α-shift of the bunch tail due to the defocusing effect of the wakefield, and scales its excitatory strength.

The first term in the kernel has the factor $\exp\left[-i\left(\psi(s_1,\tau) - \psi(s_1,\tau_1)\right)\right]$, which oscillates with the 'tail-head' phase advance *difference*, and is independent of the absolute β-tron phase. It is the agent for BNS 'damping'.

The second term factor oscillates like the *sum* of 'tail' and 'head' phase advances and flips its sign for rigid excitations with a $90°$ initial phase difference. It is strongly suppressed relative to the first term if the lattice is periodic or 'smooth' over several cycles at twice the nominal β-tron phase advance, and may be neglected in the leading eikonal approximation. Thus the dominant wakefield effect is a phase independent phase shift $= \arg \xi(s)$, and amplitude growth/damping shift $a_g = |\xi(s)|$. To the extent this approximation is accurate $B = 1$.

Although interesting *gedankenphysik*, the asymptotic 'long' machine results that have also been obtained [1] are not quantitatively germane to any known extant or seriously contemplated machine. The perturbation series, in which distributed (non-macroparticle) $n+1$ particle-like contributions, each associated with an order n in the α-shift are superposed, is straightforward. Under SLC conditions ($N \approx 4\cdot 10^{10}$, $\tau_{\rm rms} \approx 1\,$mm, $W'_\perp \cong 4.1\,$GeV/($10^{10}\,$mm-m^2)), $n_{\max} = 5$ suffices for $\sim 0.1\%$ accuracy in the centroid R-matrix.

A companion paper [2] provides details of the SLC "diagnostic pulse" implementation and examples of high bunch current centroid data. Another [3] is a case study in which the coherent oscillation physics discussed here contributed crucial diagnostic insight.

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3 REFERENCES

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