EXPERIMENTAL NON LINEAR BEAM DYNAMICS STUDIES AT SPEAR[1]

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Abstract

The frequency map analysis of a Hamiltonian system [1],[2],[3] recently introduced to accelerators physics in combination with turn-by-turn phase space measurements opens new experimental opportunities for studying non linear dynamic in storage rings. In this paper we report on the experimental program at SPEAR having the goal of measuring the frequency map of the machine. In this paper we discuss the accuracy of the instantaneous tune extraction from experimental data and demonstrate the possibility of the frequency map measurement.

1 TURN-BY-TURN PHASE SPACE MONITOR

We have previously reported on experimental beam dynamics studies at SPEAR and hardware used for this purpose [4], [5]. We have upgraded the electronics following the BPM base-band processor in order to achieve higher resolution by adding 6 channels of custom built high speed low noise track and holds serving as a front end for 14-bit Pentek™ ADCs in the VME mainframe. The resolution achieved in the single pass is 125 μm for current per bunch 3mA. We have modified the fast kicker triggering circuitry to initiate the data acquisition cycle at fixed phase with respect to AC to avoid the influence of the 60 Hz ripple in the magnetic field [6]. The control of the data acquisition has also been automated using LabView™ running on Pentium™ workstation for effective use of machine time dedicated to accelerators physics.

2 INSTANTANEOUS TUNE EXTRACTION

We apply Numerical Analysis of Fundamental Frequency (NAFF) to a subset of numerically or experimentally obtained turn-by-turn trajectories starting at turn *m* containing N turns

$$f_n, \text{ where } n = m \ldots m + N - 1, \qquad (1)$$

f_n represents turn-by-turn data for one of the canonical variables. We search for the fundamental frequencies $\nu_{m,N}^{(k)}$ that maximize the absolute value of the correlator:

$$I(\nu_{m,N}^{(k)}) = \sum_{n=m}^{m+N-1} f_n \exp\left(-i2\pi \cdot \nu_{m,N}^{(k)} n\right) \chi_{m-n} \qquad (2)$$

$$\chi_n = \sin(\pi \cdot n / N)$$

These frequencies extracted from the numerical tracking data asymptotically converge for $N \to \infty$ to the tunes (winding numbers) associated with invariant surfaces in the phase space.

If the method is applied to experimental tracking data, the interpretation of these frequencies needs to be modified for two reasons:

1. due to synchrotron radiation in electron machine the particle does not stay on the invariant tori.
2. BPMs measure the trajectory in the phase space of the center of mass of a distribution which differs from a single particle trajectory due to decoherence[6].

Given a subset of the center of mass trajectory data (1) NAFF picks out the *instantaneous* (associated with particular starting turn *m*) tunes $\nu_{m,N}^{(k)}$, that make an approximation of (1) in the form

$$f_n = \sum_k a_{m,N}^{(k)} \cos\left(2\pi \cdot \nu_{m,N}^{(k)} n + \psi_{m,N}^{(k)}\right) \chi_{m-n} \qquad (3)$$

The instantaneous amplitudes $a_{m,N}^{(k)}$ can be computed afterwards by chi-square fitting.

3 ACCURACY OF THE INSTANTANEOUS TUNE EXTRACTION

We find it necessary to investigate the accuracy of the instantaneous tune extraction in order to be able to interpret the results correctly. Although the functions $\exp(-i2\pi \cdot \nu \cdot n)$ can serve as basis vectors, the correlator (2) is not a legitimate projection operator for any finite N. Therefore the method will introduce systematic errors due to the finite number of turns used. Systematic errors also increase by several orders of magnitude as the tune approaches to an integer or half integer.

We numerically tested the accuracy of the NAFF by applying it to a test sequence

$$f_n = A\cos\left(2\pi \cdot n\nu + \psi\right) + R_n(\sigma)$$

where $R_n(\sigma)$ Gaussian white noise.

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The error on Fig.1. is the r.m.s. error for tunes and phases randomly seeded in the range 0.05 … 0.45 and 0 - π/2 respectively.

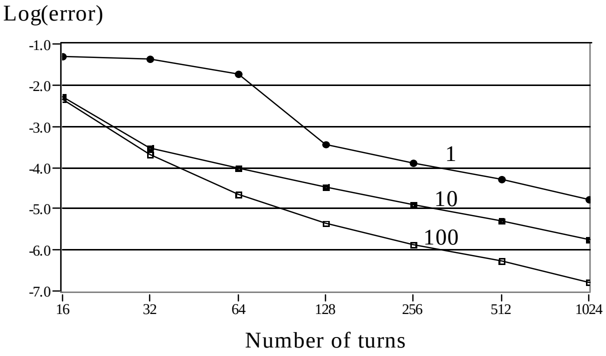

Fig.1 r.m.s. error for a range of experimentally useful numbers of turns and signal to noise ratios.

4 PROPOSED AND CARRIED OUT EXPERIMENTS, RESULTS AND DISCUSSION

4.1 Frequency map measurements.

Frequency map analysis [1][2][3] provides a useful visualization of the dynamics of a Hamiltonian system. The non linear resonance lines act as attractors or repellers in the tune space. Since the width of the resonant structures associated with high order resonances is within the achievable resolution of the method, we have started experiments to extract the frequency map experimentally.

The frequency map (footprint) on Fig.2. is obtained by numerical tracking with initial conditions chosen on a uniform grid in J_x, J_y action space. For each initial condition the particle is tracked for 1024 turns and the horizontal and vertical tunes are computed using NAFF. The linear working point of SPEAR (7.166;5.26) is in the upper right corner.

The four octupoles installed in SPEAR introduce positive horizontal tune shift with horizontal amplitude[5], so a transverse kick places the instantaneous tunes in a new point in the tune space different from the linear working point.

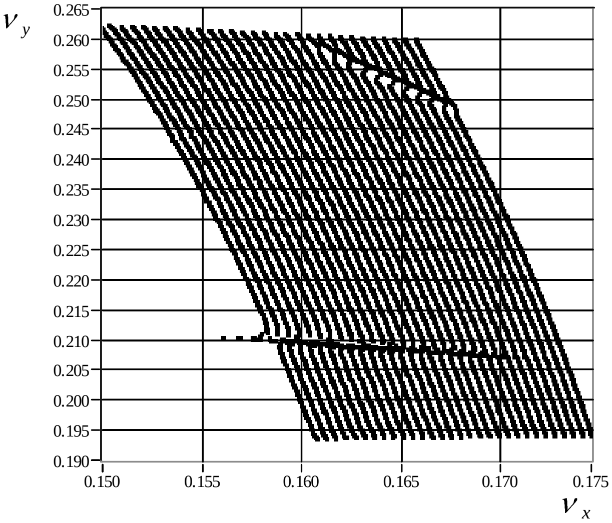

Fig.2 Frequency map of the SPEAR model.

To detect the presence of a resonant line we tune the machine such that linear working point is slightly above it. We then follow the evolution of the tunes after applying a horizontal kick. The experiment was conducted with 5mA in a single bunch stored in the machine.

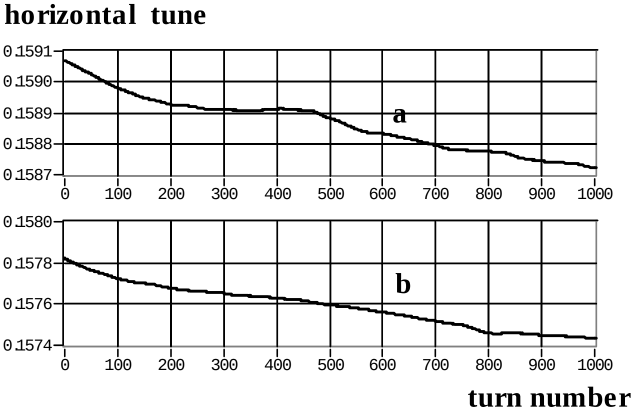

Fig.3 Experimental evidence of the $3\nu_x + 2\nu_y = 32$ resonance crossing. Horizontal tune vs. turn number for 2 different nearby linear working points set approximately to (a) (7.158 ;5.261) and (b) (7.157 ; 5.261)

In case (a), after the kick the instantaneous tune point starts on the other side of the resonance line due to the positive horizontal tune shift with amplitude and then crosses the line as the amplitude decreases. The flat part of the upper graph is a manifestation of the resonance crossing. In case (b), the linear tune is chosen below the line so it is not crossed.

4.2 Modulation of instantaneous tune and amplitude with synchrotron frequency.

It is can be seen from Fig.1. that even for N=32, the accuracy of the NAFF method is between 1×10^{-3} and 1×10^{-4}. In SPEAR, the synchrotron period is ~50 turns. This suggests that we should be able to resolve the modulation of the transverse tunes at the synchrotron frequency due to coherent synchrotron oscillations and centroid amplitude modulations due to the decoherence-recoherence effect, and due to non-zero energy spread within the bunch [7]. All these effects were indeed observed using instantaneous tune measurements.

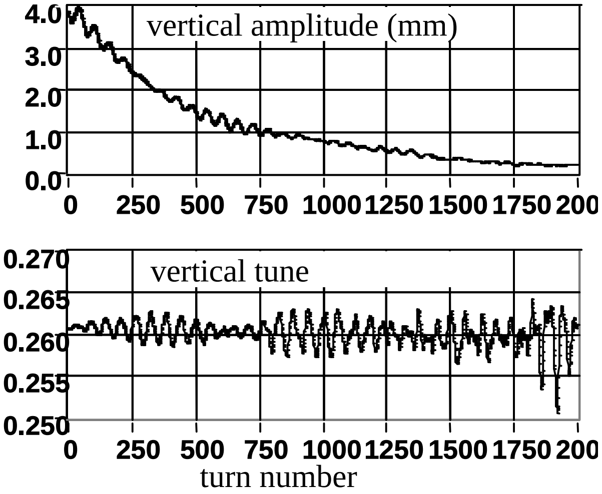

Fig.4 Evolution of the instantaneous vertical tune $\nu_{m,32}^{(y)}$ and amplitude $a_{m,32}^{(y)}$ as a function of turn number.

5 CONCLUSIONS

The instantaneous tune extraction technique can be applied to experimental tracking data with reasonable accuracy. Frequency map can be experimentally determined using the existing turn-by-turn phase space measurement techniques and NAFF instantaneous tune extraction.

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