AN ORBIT AND DISPERSION CORRECTION SCHEME FOR PEP-II *

M.H.R. Donald, Y. Cai, H. Shoaee and G.White
Stanford Linear Accelerator Center, Stanford University, Stanford, CA 94309 USA

Abstract

A scheme of simultaneously correcting the orbits and dispersion has been implemented in the simulation code [1] and on-line control system [2] for PEP-II [3]. The scheme is based on the eigenvector decomposition method. An important ingredient of the scheme is to choose the optimum eigenvectors that minimize the orbit, dispersion and corrector strength. Simulations indicate this to be a very effective way to control the vertical residual dispersion.

1 INTRODUCTION

To achieve optimum luminosity in a storage ring it is vital to control the residual vertical dispersion. In the original PEP storage ring, a scheme [4] to control the residual dispersion function was implemented using the ring orbit as the controlling element. The "best" orbit not necessarily giving the lowest vertical dispersion. A similar scheme has been implemented in both the on-line control code [2] and in the simulation code LEGO [1]. The method involves finding the response matrices (sensitivity of orbit/dispersion at each Beam-Position-Monitor (BPM) to each orbit corrector) and solving in a least squares sense for minimum orbit, dispersion function or both. The optimum solution is usually a subset of the full least squares solution.

2 RESPONSE MATRICES

The orbit response matrices for a beamline (R_{12}) and a closed ring (C_{12}) are well known.

In a beamline The response to an orbit corrector at position s_p observed as a change in orbit at position s_o is given by for $\mu_{s_o} > \mu_{s_p}$

$$y_o = \sqrt{\beta_p \beta_o}\Theta_p \sin(\mu_{s_o} - \mu_{s_p}) \tag{1}$$

for $\mu_{s_o} < \mu_{s_p}$

$$y_o = o \tag{2}$$

In a closed ring the response to an orbit corrector at position s_p, observed as a change in orbit at position s_o is given by

$$y_o = \frac{\sqrt{\beta_p \beta_o}}{2\sin\pi\nu}\Theta_p \cos(\pi\nu + \mu_1 - \mu_2) \tag{3}$$

where μ_1 and μ_2 denote the smaller and larger of the phases at the corrector/monitor positions respectively.

The response matrices for the dispersion function can be found by differentiating these expressions with respect to $\delta = dp/p$ the relative momentum.

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In a beamline we observe how equation 1 changes as a function of momentum. β_p, β_o, μ_p and μ_o all depend on momentum, as does Θ_p the applied kick from the corrector.

$$\begin{aligned}\frac{dy_o}{d\delta} &= \frac{1}{\Theta_p}\frac{d\Theta_p}{d\delta}y_o \\ &+ \frac{1}{2}\left[\frac{1}{\beta_o}\frac{d\beta_o}{d\delta} + \frac{1}{\beta_p}\frac{d\beta_p}{d\delta}\right]y_o \\ &+ \left[\frac{d\mu_1}{d\delta} - \frac{d\mu_2}{d\delta}\right]\frac{y_o}{\tan(\mu_2 - \mu_1)}\end{aligned} \tag{4}$$

We find the derivative of the first term to be

$$\frac{d\Theta_p}{d\delta} = -\Theta_p$$

In the LEGO code the derivatives $\frac{1}{\beta_p}\frac{d\beta}{d\delta}$ and $\frac{d\mu}{d\delta}$ are found by a finite difference method. The Twiss parameters are evaluated at momenta slightly below and slightly above the nominal momentum and the finite differences in β and ν are recorded.

In a closed ring we observe how equation 3 changes as a function of momentum. As before β_p, β_o, μ_p and μ_o and Θ_p all depend on momentum, but in this case the dependence of the machine tune ν on momentum must also be taken into account.

$$\begin{aligned}\frac{dy_o}{d\delta} &= \frac{1}{\Theta_p}\frac{d\Theta_p}{d\delta}y_o \\ &- \frac{\pi}{2}\frac{1}{sin^2\pi\nu}\sqrt{\beta_p\beta_o}\cos(\mu_1 - \mu_2)\Theta_p\frac{d\nu}{\delta} \\ &+ \frac{1}{2}\left[\frac{1}{\beta_o}\frac{d\beta_o}{d\delta} + \frac{1}{\beta_p}\frac{d\beta_p}{d\delta}\right]y_o \\ &- \tan(\pi\nu + \mu_1 - \mu_2)\left[\frac{d\mu_1}{d\delta} - \frac{d\mu_2}{d\delta}\right]y_o\end{aligned} \tag{5}$$

Alternatively the D_{12} matrix for the ring can be found from the expression [6]

$$\frac{dy_m}{d\delta} = \frac{\sqrt{\beta_m\beta_c}}{2\sin\pi\nu}\Theta_c \left\{ \begin{array}{c} \sum_{qs} \left[\begin{array}{c} \frac{(-K1+K2\eta)}{2\sin\pi\nu}\beta_{qs} \\ \cos(\pi\nu - |\mu_m - \mu_{qs}|) \\ \cos(\pi\nu - |\mu_c - \mu_{qs}|) \end{array} \right] \\ -\cos(\pi\nu - |\mu_m - \mu_c|) \end{array} \right\} \tag{6}$$

where the change in orbit due to a corrector change is tracked through every quadrupole and sextupole and the momentum dependence of the kicks is summed around the ring. Note that, although this equation gives the vertical dispersion function, the dispersion function used for the kick (η) is the horizontal dispersion function. The subscripts m, c and qs refer to monitor, corrector and quadrupole or sextupole respectively. A similar expression can be found for the response in a beamline.

3 THE MICADO METHOD

Knowing the response of the orbit/dispersion to a change in strength of each orbit corrector we can solve for the change in corrector strengths required to correct the measured orbit/dispersion. If y is the orbit generated by a corrector set x and the response matrix is M

$$y = Mx \tag{7}$$

and the solution is

$$x = M^{-1}y \tag{8}$$

for a square matrix.

$$x = (M^t M)^{-1}y \tag{9}$$

is the the least squares solution of a non-square system of equations.

The MICADO [5] package uses The Householder Transform method to solve the least squares problem, but at the reduction of each column of the matrix the intermediate result is stored. the pivot is chosen on the basis of which column matches the measured orbit best.

The net result is that the first solution obtained picks out a single corrector that best corrects the orbit/dispersion. The next column chosen finds the corrector that (in conjunction with the first corrector) best corrects the orbit/dispersion, and so on.

4 THE EIGENSOLUTION METHOD

The equation

$$y = Ax \tag{10}$$

is solved in the least squares sense by pre-multiplying by M^t

$$A^t Ax = A^t y \tag{11}$$

and then decomposing the matrix $A^t A$ into its eigenvalues (a diagonal matrix D) and eigenvectors (a square matrix U).

$$\begin{aligned} & y &=& Ax \\ & A^t y &=& A^t Ax \\ but \quad & A^t A &=& UDU^t \\ therefore \quad & A^t y &=& UDU^t x \\ & x &=& UD^{-1}U^t A^t y \end{aligned}$$

The terms $U^t A^t y$ result in a column vector, the values of which indicate how well each of the orthogonal eigenvectors best fit the data. Ordering the eigenvalues and eigenvectors on these numbers allows applying a subset of the eigensolution determined on how well the data fits. The size of the eigenvalues is also important however, the larger the eigenvalue, the more sensitive the correction is, a small change in strength giving a large change in the correction. In a ring, for example, the largest eigenvectors correspond to eigenvectors with patterns close to the betatron tune of the machine. The eigenvectors (and eigenvalues) are ordered on the *product* of the best fit and the eigenvalue.

After the eigenvectors and eigenvalues are ordered, the correction (-x) is found by multiplying out $D^{-1}U^t A^t y$ then summing the corrector sets $U_{ij}(D^{-1}U^t A^t y)_j$ for each corrector i

5 MERITS OF THE METHODS

Both methods overcome one of the problems of the full solution. In the presence of noise (BPM errors, machine errors and an otherwise non-perfect machine model) and a possibly ill-conditioned matrix, the strength of the correctors can suddenly jump to very large values when trying to correct for non-physical BPM readings. Cutting the solution short (fewer correctors or corrector sets) gives acceptable solutions with weaker correctors.

The MICADO method tends to pick out the most effective individual correctors to give an acceptable orbit/dispersion and is used often in correcting the orbit with very few correctors. The eigensolution method is particularly useful for correction of the dispersion function, where patterns in the orbit are used for the correction. Either method may be used to correct orbit, dispersion or a weighted combination of the two. In PEP-I a weighting factor, where 2 cm of dispersion was worth 1 mm of orbit, proved to be satisfactory.

6 SIMULATIONS USING LEGO

Simulations have been performed using the code LEGO to check out the on-line code and also to simulate start-up of the High Energy Ring of PEP-II with alignment, field setting and multipole errors. A typical simulation would:

- correct first turn orbits until a stable machine results
- correct closed orbits both horizontal and vertical
- correct vertical dispersion
- correct orbit and dispersion together

Table 1 shows the result of just such a simulation on PEP-II with expected tolerances on alignment, field setting and multipole errors on all elements. In this case the rms of the quadrupole roll errors was increased from the expected 0.5 mr to the value 2 mr.

Figure 1 shows how the rms of the corrector strengths grows steadily while the rms of the residual dispersion function falls rapidly at first then more slowly. This figure corresponds to the first dispersion correction of table 1. The rms of the measured dispersion function (reduced from 40.25 mm to 5.67 mm) was close to the predicted rms value of 3.87 mm.

Figure 2 shows how the correction goes unstable when all the corrector sets are chosen (full correction). For the 145 correctors and 151 BPMs in the High Energy Ring it is best to take no more than 50 sets, then measure the dispersion function (and orbit) and repeat the correction. The case shown is the same as that shown in table 1. and figure 1 except that the correction was not stopped at 50 sets. The

Table 1: Results of a simulated correction

Action	Horizontal orbit (mm)	Vertical orbit (mm)	Vertical dispersion (mm)
First turn orbits	21.559	14.220	
Correct H orbit	4.380	11.662	
Correct V orbit	3.628	0.990	
Closed orbits	4.384	0.575	34.053
Correct H orbit	0.552	1.122	40.248
Correct V orbit	0.248	0.100	41.265
Correct V orbit again		0.0285	40.438
Correct V dispersion		1.794	5.676
Correct dispersion again		1.808	3.927
Correct both orbit and dispersion		0.792	11.188

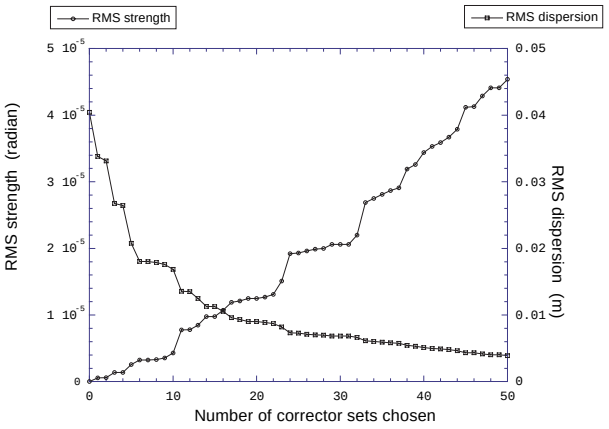

Figure 1: A correction of the vertical dispersion function showing how the strength of the orbit correctors increases while the rms of the dispersion function decreases. Only the first 50 corrector sets are recorded

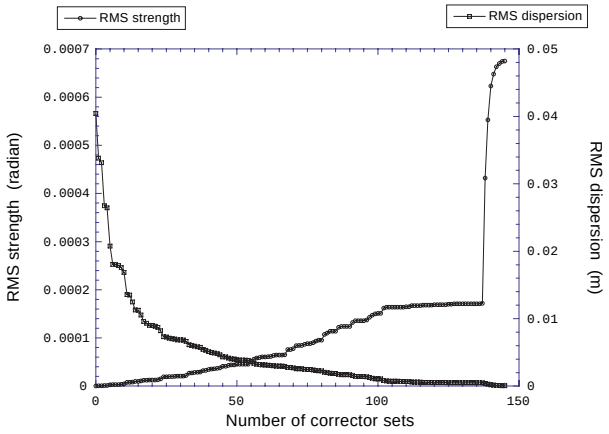

Figure 2: The same correction as figure 1 but in this case the predictions for all 145 corrector sets are shown

results for the case of full correction were very much worse than those for the limited correction. It was predicted that the rms of the dispersion would be 0.04 mm. On measurement it was found to be 15 mm. much worse than previously, the orbit was also much worse at 6.24 mm.

7 SUMMARY

The scheme for control of unwanted residual dispersion function has been implemented, both on-line and in the simulation code LEGO. The on-line code has been checked against the simulation code. Simulations indicate that the scheme will work well when the two storage rings of the PEP-II B-Factory are commissioned.

8 REFERENCES

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