CANONICAL PARTICLE TRACKING AND END POLE MATCHING OF HELICAL INSERTION DEVICES

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Abstract
The low emittance 1.7 GeV synchrotron radiation light source BESSY II \(^1\) is presently under construction in Berlin-Adlershof \([1]\). For the generation of circularly polarized radiation two helical insertion devices (IDs) will be installed \([2]\). A discussion of beam dynamical effects for such devices is given with special attention to sextupole-like fields of IDs and their compensation by the end poles.

1 INTRODUCTION
The BESSY II storage ring offers 14 out of 16 straight sections for IDs. Two helical undulators of the Sasaki-type \([3]\) will be installed to provide users with up to 1300 eV polarized photon beams. The undulators will be built with NdFeB magnets having a period length of 56 mm (30 periods total) and a minimum half gap of 7.5 mm. Each device will be operated in a double undulator setup mode, where the two halves of one device are used to generate different polarization amplitudes. If these end poles are included in the matched end pole configuration to obtain stable particle oscillations \([4]\), Particle tracking methods are applied to verify that these IDs are tolerable for the storage ring. Special attention is given to the effect of the end poles. It is shown that the modeling of the IDs has to be done together with a well matched end pole configuration to obtain stable particle oscillation amplitudes. If these end poles are included in the calculations, only weak interactions of the electron beam with the IDs are predicted.

2 MODELING OF THE TRACKING ROUTINE
The helical undulator consists of 4 long parallel beams of alternating rows of permanent magnets. A variable gap between the upper and the lower row adjusts the on-axis field. The upper left and the lower right pole can simultaneously be shifted with respect to the other two rows. This phase shift results in a magnetic field of arbitrary ellipticity. The scalar potential \(V\) \([4]\) of the periodic part of the field is obtained by a superposition of the 4 row contributions \((x = \text{horizontal}, y = \text{vertical} \text{ and } z = \text{longitudinal axis}) V = b_0(V_1 + V_2 + V_3 + V_4)/8\), with

\[
V_1 = (e^{+k_x y}c_x + k_y + e^{-k_y y}k_z)s_z +
V_2 = (e^{+k_x y}c_x + k_y + e^{-k_y y}k_z)s_z -
V_3 = (e^{-k_x y}c_x + k_y - e^{-k_y y}k_z)s_z +
V_4 = (e^{-k_x y}c_x + k_y + e^{-k_y y}k_z)s_z -
\]

and \(c_x = \cos(k_x(x \pm x_0)), s_z = \sin(k_z z \pm \psi)/2\).

This analytical expression for the scalar potential describes the \(B_y\) and \(B_x\)-field with an error of a few percent in the parameter range of interest compared with results of numerical magnet codes. For the present study the parameters chosen are: field strength parameter \(b_0 = 1T\), horizontal gap separation \(x_0 = 0.02m\), horizontal period length \(\lambda_z = 2\pi/k_z = 0.056m\), horizontal period \(\lambda_x = 2\pi/k_x = 0.0896m\), which yields a vertical \(\lambda_y = 2\pi/k_y = 0.0475m\). The parameter \(\psi\) measures the shift of the two rows with respect to the other ones. At \(\psi = 0\) a magnetic field of \(0.6T\) is obtained on the ID axis.

In \([5]\) an approximated solution of the Hamilton-Jacobi differential equation

\[
\frac{\partial F}{\partial z} + H = 0
\]

is presented in terms of an analytical Taylor series expansion of the generating function \(F\). A Hamiltonian function of the type

\[
H = (p_x - A_x)^2/2 + (p_y - A_y)^2/2 - A_z
\]

is used, were \(A_x\), \(A_y\) and \(A_z\) are the components of the magnetic vector potential divided by the magnetic stiffness \(B\rho\) of the electrons. For the present study \(F\) is expanded up to the third order with respect to the transverse momenta \(p_x\) and \(p_y\) and a third variable \(b_0/B\rho\) which is proportional to the scaled vector potentials. To obtain the tracking routine the scalar potential \(V\) is used as an input for the computer algebraic code REDUCE \([6]\). With this code parts for an existing FORTRAN mapping module are generated in an automatic way. This mapping routine is extremely fast \([7]\), it can take several periods of the ID in a single step, and it is symplectic because of the generating function approach.

3 MATCHING OF THE END POLES
To obtain helical oscillations of electrons in an ID the potential function has approximately to be like \(V \propto a(x, y) \cos k_z z + b(x, y) \sin k_z z\), where the derivatives of \(a(x, y)\) and \(b(x, y)\) describe the horizontal and

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vertical magnetic field shifted by 90 degrees.

The end poles match the closed orbit from the outside of the ID to a wiggling, periodic orbit inside the ID and back to the outside. Ideally this is done without any residual closed orbit kick or shift of the beam at the exit. To achieve a well matched orbit, two integrals over the fields in both planes \( B_y \) and \( B_x \) have to vanish, i.e.

\[
I_1 = \int_{-L}^{L} B_y \, dz = 0 \quad \text{and} \quad I_2 = \int_{-L}^{L} \int_{-L}^{L} B_y \, dz' \, dz = 0,
\]

and for the other plane as well. This can be achieved by pure dipole-fields, without any higher order field components. However, sextupole-like fields of the ID have to be matched in addition by the end poles [8].

From the potential function terms can be derived which characterize sextupole-like fields:

\[
\frac{\partial^3 V}{\partial x^3}, \quad \frac{\partial^3 V}{\partial x^2 \partial y}, \quad \frac{\partial^3 V}{\partial x \partial y^2} \quad \text{and} \quad \frac{\partial^3 V}{\partial y^3}
\]

at \( x = y = 0 \). These fields have the same order as the two-dimensional sextupole fields. Because of the three-dimensional character of the ID fields there are 4 terms, compared with two expressions (skew and normal) in the two-dimensional case. For the ID one expects an oscillation of these terms along the beam axis with a zero value on average. The amplitude of these oscillations could be large, for example the term \( \frac{\partial^3 V}{\partial y^3} \) integrated over half a period could yield as much as 80 components. However, sextupole-like fields of the ID have the same order as those of the closed orbit condition, but now applied to the sextupole-like field given by the partial derivatives of the potential function multiplied by scaling factors of the local beta functions:

\[
s_{1,2}(z) = \frac{1}{3} \Delta z \frac{V_{yxx} \beta_x}{\sqrt{\beta_x}}
\]

\[
s_{3,4}(z) = \frac{1}{6} \Delta z \frac{V_{gyy} \beta_y}{\sqrt{\beta_y}}
\]

\[
s_{5,6,7}(z) = \frac{1}{3} \Delta z \frac{V_{yxx} \beta_y}{\sqrt{\beta_x}}
\]

For the evaluation of a particular \( B_i \) sextupole kicks and phase functions with equal indices have to be combined.

The distortion function \( B_i \) can be decomposed into a \( \cos(\phi_i - \phi_{i0}) \)- and a \( \sin(\phi_i - \phi_{i0}) \)-wave. The amplitudes \( A_c \) and \( A_s \) of these waves for an extended source \( s_i(z) \) are proportional to the integrals

\[
A_c \propto \int_{ID} s_i(z) \cos \delta_idz \quad \text{and} \quad A_s \propto \int_{ID} s_i(z) \sin \delta_idz.
\]

Ideally these amplitudes should be zero to minimize optics distortions. If the phase advance over the ID is small, \( \sin \delta_i \) is proportional to \( z \) and \( s_i(z) \) can be replaced by the unscaled sextupole strength \( s(z) = V_{ypp}, V_{yyx}, V_{yxx} \) and \( V_{xxz} \), respectively, and setting the beta functions constant. The condition for vanishing \( A_c \) and \( A_s \) can further be simplified by partial integration:

\[
A_c \approx \int_{-L}^{L} s(z) \, dz = 0, \quad A_s \approx \int_{-L}^{L} \int_{-L}^{L} s(z') \, dz' \, dz = 0.
\]

In this approximation the sextupole fields are matched only due to the ID itself, independent of the machine optics. In this limit all distortion functions can be canceled with a single, properly placed end pole kick. These integrals are of the same type as those of the closed orbit condition, but now applied to the sextupole-like fields. In the potential approximation introduced above the \( \cos k_z z \)-like term is naturally matched, because the integrals for the closed orbit and for the sextupole matching become zero, whereas at least the \( \sin k_z z \)-like term requires special end poles.

In case this simplification fails, one has to include the \( z \)-dependence of the beta functions as well into the matching. Locations for IDs with fast changing beta functions (low beta insertions) could be critical, or if more complicated symmetries than these simple cos- and sin-expressions are applied.

For example in Fig. 1 the distortion function \( B_1 \) is shown. In the \( i = 1 \) case all scaled phases can be replaced by the corresponding horizontal beta phase value.
The distortion generated by the sextupole-like kicks of the ID is plotted for one lattice cell. In this figure the number of ID-periods is reduced to 4, to see clearly the oscillation of the distortion function inside the ID. The distortion is large and spreads out along the cell if no end poles are applied (dotted line). If the parameters are adjusted to the ID parameters, the size of the distortion function is about the same as the one produced by the correction sextupoles of the ring optics. The line in Fig. 1 shows the same configuration but with matched end poles. It is clearly seen that the distortion is nearly perfectly enclosed within the ID, like a closed sextupole bump.

These two situations are compared in Fig. 2 on the basis of tracking simulations for the correctly modeled ID inserted in a linear BESSY II optics. A particle is started in the straight section with 1 cm amplitude in both planes and tracked for 1000 turns. The unmatched case (dots) shows a large smear due to the sextupole fields of the ID. The effect of a good matching is clearly visible, the phase space figure shows a nearly perfect circle. The matching is achieved by an end pole configuration on either side of the ID, each one composed by two poles. These poles are simulated by a step length of the generating function over half a longitudinal period length using the potential function given above, but with reduced field amplitudes. At the entrance side the first half pole has a strength of $+\frac{1}{4} V$ and the second half pole has a strength of $-\frac{1}{4} V$. At the exit side the sequence is reversed, using $+\frac{1}{4} V$ and $-\frac{1}{4} V$. Applying these end poles both closed orbit and distortion functions are well matched. In the matched case the effect of the helical device on the BESSY II optics can be ignored. If the tracking is done with all 14 insertions and one of these replaced by a helical device, no reduction of the dynamic aperture is seen.

### 4 CONCLUSION

A matched end pole design is required for (not only helical) IDs to damp closed orbit and distortion function oscillations. Both effects can be canceled by properly adjusted end poles. For the full tuning range of the shift parameter $\psi$ of the helical device no effects on the BESSY II dynamic aperture is seen. The required sextupole corrections are in first approximation independent of the ring optics. In case of fast changing beta functions inside the ID the correction has to be done including the ring optics.

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### 6 REFERENCES

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