CANONICAL PARTICLE TRACKING AND END POLE MATCHING OF HELICAL INSERTION DEVICES

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Abstract

The low emittance 1.7 GeV synchrotron radiation light source BESSY II [1] is presently under construction in Berlin-Adlershof [1]. For the generation of circularly polarized radiation two helical insertion devices (IDs) will be installed [2]. A discussion of beam dynamical effects for such devices is given with special attention to sextupole-like fields of IDs and their compensation by the end poles.

1 INTRODUCTION

The BESSY II storage ring offers 14 out of 16 straight sections for IDs. Two helical undulators of the Sasaki-type [3] will be installed to provide users with up to 1300 eV polarized photon beams. The undulators will be built with NdFeB magnets having a period length of 56 mm (30 periods total) and a minimum half gap of 7.5 mm. Each device will be operated in a double undulator setup mode, where the two halfs of one device are used to generate different polarized photon beams which are separated by about $\pm 200\mu rad$ deflection. A fast chopper is used for polarization switching up to 100 Hz [2]. A photon flux density of $1.7 \cdot 10^{13} photons/s\,mm^2 0.1\% BW 0.1A$ is expected at a degree of polarization of $P_c = 0.72$ for the 5^{th} harmonic and a flux density of $1.8 \cdot 10^{14} photons$ with a polarization of $P_c = 0.99$ for the 1^{st} harmonic. They can be tuned to obtain any degree of polarization, from circular to linear (both horizontal and vertical) for photon energies up to 1300 eV.

Particle tracking methods are applied to verify that these IDs are tolerable for the storage ring. Special attention is given to the effect of the end poles. It is shown that the modeling of the IDs has to be done together with a well matched end pole configuration to obtain stable particle oscillation amplitudes. If these end poles are included in the calculations, only weak interactions of the electron beam with the IDs are predicted.

2 MODELING OF THE TRACKING ROUTINE

The helical undulator consists of 4 long parallel beams of alternating rows of permanent magnets. A variable gap between the upper and the lower row adjusts the on-axis field. The upper left and the lower right row can simultaneously be shifted with respect to the two other rows. This phase shift results in a magnetic field of arbitrary ellipticity. The scalar potential V [4] of the periodic part of the field is obtained by a superposition of the 4 row contributions (x =horizontal, y =vertical and z =longitudinal axis) $V = b_0(V_1 + V_2 + V_3 + V_4)/8$, with

$$V_1 = (e^{+k_y y} cx_-/k_y + e^{+k_z y}/k_z) sz_+$$
$$V_2 = (e^{+k_y y} cx_+/k_y + e^{+k_z y}/k_z) sz_-$$
$$V_3 = (e^{-k_y y} cx_+/k_y + e^{-k_z y}/k_z) sz_+$$
$$V_4 = (e^{-k_y y} cx_-/k_y + e^{-k_z y}/k_z) sz_-,$$

and $\quad cx_\pm = \cos(k_x(x \pm x_0)),\ sz_\pm = \sin(k_z z \pm \psi/2).$

This analytical expression for the scalar potential describes the B_y- and B_x-field with an error of a few percent in the parameter range of interest compared with results of numerical magnet codes. For the present study the parameters choosen are: field strength parameter $b_0 = 1T$, horizontal gap separation $x_0 = 0.02m$, longitudinal period length $\lambda_z = 2\pi/k_z = 0.056m$, horizontal period length $\lambda_x = 2\pi/k_x = 0.0896m$, which yields a vertical $\lambda_y = 2\pi/k_y = 0.0475m$. The parameter ψ measures the shift of the two rows with respect to the other ones. At $\psi = 0$ a magnetic field of 0.6T is obtained on the ID axis.

In [5] an approximated solution of the Hamilton-Jacobi differential equation

$$\partial F/\partial z + H = 0$$

is presented in terms of an analytical Taylor series expansion of the generating function F. A Hamiltonian function of the type

$$H = (p_x - A_x)^2/2 + (p_y - A_y)^2/2 - A_z$$

is used, were A_x, A_y and A_z are the components of the magnetic vector potential divided by the magnetic stiffness $B\rho$ of the electrons. For the present study F is expanded up to the third order with respect to the transverse momenta p_x and p_y and a third variable $b_0/B\rho$ which is proportional to the scaled vector potentials. To obtain the tracking routine the scalar potential V is used as an input for the computer algebraic code REDUCE [6]. With this code parts for an existing FORTRAN mapping module are generated in an automatic way. This mapping routine is extremely fast [7], it can take several periods of the ID in a single step, and it is symplectic because of the generating function approach.

3 MATCHING OF THE END POLES

To obtain helical oscillations of electrons in an ID the potential function has approximately to be like $V \propto a(x,y)\cos k_z z + b(x,y)\sin k_z z$, where the derivatives of $a(x,y)$ and $b(x,y)$ describe the horizontal and

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vertical magnetic field shifted by 90 degrees.

The end poles match the closed orbit from the outside of the ID to a wiggling, periodic orbit inside the ID and back to the outside. Ideally this is done without any residual closed orbit kick or shift of the beam at the exit. To achieve a well matched orbit, two integrals over the fields in both planes B_y and B_x have to vanish, i.e.

$$I_1 = \int_{-L}^{L} B_y dz \equiv 0 \text{ and } I_2 = \int_{-L}^{L}\int_{-L}^{z} B_y dz' dz \equiv 0,$$

and for the other plane as well. This can be achieved by pure dipole-fields, without any higher order field components. However, sextupole-like fields of the ID have to be matched in addition by the end poles [8].

From the potential function terms can be derived which characterize sextupole-like fields:

$$\partial^3 V/\partial x^3,\ \ \partial^3 V/\partial x^2 \partial y,\ \ \partial^3 V/\partial x \partial y^2 \text{ and } \partial^3 V/\partial y^3$$

at $x = y = 0$. These fields have the same order as the two-dimensional sextupole fields. Because of the three-dimensional character of the ID fields there are 4 terms, compared with two expressions (skew and normal) in the two-dimensional case. For the ID one expects an oscillation of these terms along the beam axis with a zero value on average. The amplitude of these oscillations could be large, for example the term $\partial^3 V/\partial y^3$ integrated over half a period could yield as much as 80 T/m for the suggested helical ID. For comparison, the correction sextupoles of the BESSY II optics reach values of 50 T/m. With respect to these fields the ID can be considered as an alternating series of strong sextupole kicks.

There is some similarity between the strong oscillation of the sextupole fields and the oscillation of the dipole fields for the ID. Collins distortion functions [9] are very well suited to demonstrate the effect of these sextupole-like fields. In his note an algorithm is given which can be applied to these fields [8]. Instead of 5 distortion functions for normal two-dimensional sextupoles there are now 7 in case of an ID without skew terms.

If only a single sextupole-like kick inside the ID is considered as a source of distortion neglecting other effects the distortion functions B_i ($i = 1 - 7$) become very simple. All of them can be written as:

$$B_i = \frac{1}{2} s_i(z) \cos(\delta_i + \phi_{i0} - \phi_i)/\sin(\phi_{i0}),$$

where the phase around the ring is $2\phi_{i0}$ and B_i is evaluated outside the ID at ϕ_i. The longitudinal position z and the phases are defined in such a way that $z = \phi_i = \delta_i = 0$ at the same point. All phases are scaled, dependent on the type i of the distortion function. There are 4 different δ_i-scalings:

$$\delta_{1,5} = \tilde{\varphi}_x,\ \ \delta_2 = 3\tilde{\varphi}_x,\ \ \delta_{3,6} = 2\tilde{\varphi}_y + \tilde{\varphi}_x,\ \ \delta_{4,7} = 2\tilde{\varphi}_y - \tilde{\varphi}_x,$$

and similar relations for skew terms (exchange x and y). $\tilde{\varphi}_x$ and $\tilde{\varphi}_y$ are the unscaled horizontal and vertical betatron phases. Similar relations are valid for ϕ_i and ϕ_{i0}. Also the sextupole kicks are scaled. The strength of the kick is proportional to an infinitesimal 'kick-length' Δz and to the sextupole-like field given by the partial derivatives of the potential function multiplied by scaling factors of the local beta functions:

$$s_{1,2}(z) = \frac{1}{3} \Delta z V_{yxx} \beta_x \sqrt{\beta_x}$$

$$s_{3,4}(z) = \frac{1}{6} \Delta z V_{yyy} \beta_x \sqrt{\beta_y}$$

$$s_{5,6,7}(z) = \frac{1}{3} \Delta z V_{yxx} \beta_y \sqrt{\beta_x}$$

For the evaluation of a particular B_i sextupole kicks and phase functions with equal indices have to be combined.

The distortion function B_i can be decomposed into a $\cos(\phi_i - \phi_{i0})$- and a $\sin(\phi_i - \phi_{i0})$-wave. The amplitudes A_c and A_s of these waves for an extended source $s_i(z)$ are proportional to the integrals

$$A_c \propto \int_{ID} s_i(z) \cos \delta_i dz \text{ and } A_s \propto \int_{ID} s_i(z) \sin \delta_i dz.$$

Ideally these amplitudes should be zero to minimize optics distortions. If the phase advance over the ID is small, $\sin \delta_i$ is proportional to z and $s_i(z)$ can be replaced by the unscaled sextupole strength $s(z) = V_{yyy}$, V_{yyx}, V_{yxx} and V_{xxx}, respectively, and setting the beta functions constant. The condition for vanishing A_c and A_s can further be simplified by partial integration:

$$A_c \approx \int_{-L}^{L} s(z) dz \equiv 0,\ A_s \approx \int_{-L}^{L}\int_{-L}^{z} s(z') dz' dz \equiv 0.$$

In this approximation the sextupole fields are matched only due to the ID itself, independent of the machine optics. In this limit all distortion functions can be canceled with a single, properly placed end pole kick. These integrals are of the same type as those of the closed orbit condition, but now applied to the sextupole-like fields. In the potential approximation introduced above the $\cos k_z z-$like term is naturally matched, because the integrals for the closed orbit and for the sextupole matching become zero, whereas at least the $\sin k_z z-$like term requires special end poles.

In case this simplification fails, one has to include the z-dependence of the beta functions as well into the matching. Locations for IDs with fast changing beta functions (low beta insertions) could be critical, or if more complicated symmetries than these simple cos- and sin-expressions are applied.

For example in Fig. 1 the distortion function B_1 is shown. In the $i = 1$ case all scaled phases can be replaced by the corresponding horizontal beta phase value.

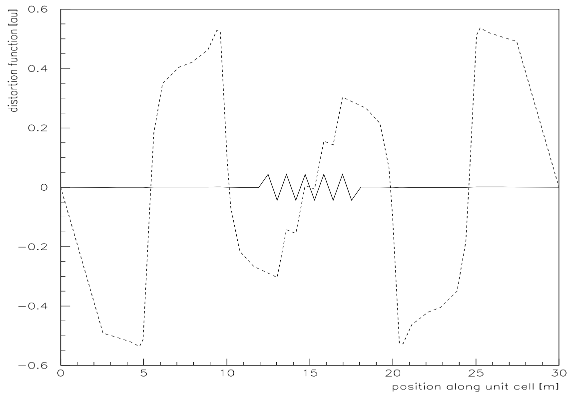

Figure 1: Distortion function B_1 for the matched (line) and unmatched (dotted line) case along one BESSY II cell.

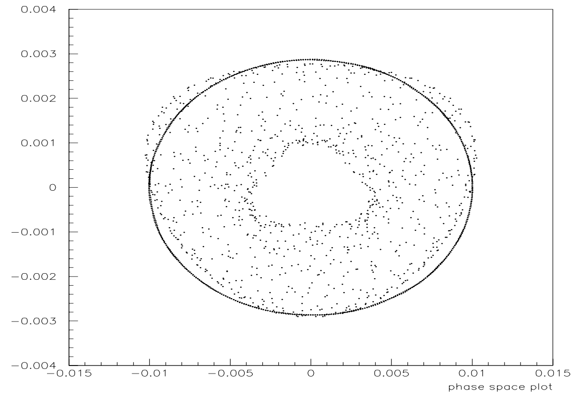

Figure 2: Vertical phase space plot for the matched (line) and unmatched (dots) case.

The distortion generated by the sextupole-like kicks of the ID is plotted for one lattice cell. In this figure the number of ID-periods is reduced to 4, to see clearly the oscillation of the distortion function inside the ID. The distortion is large and spreads out along the cell if no end poles are applied (dotted line). If the parameters are adjusted to the ID parameters, the size of the distortion function is about the same as the one produced by the correction sextupoles of the ring optics. The line in Fig. 1 shows the same configuration but with matched end poles. It is clearly seen that the distortion is nearly perfectly enclosed within the ID, like a closed sextupole bump.

These two situations are compared in Fig. 2 on the basis of tracking simulations for the correctly modeled ID inserted in a linear BESSY II optics. A particle is started in the straight section with 1 cm amplitude in both planes and tracked for 1000 turns. The unmatched case (dots) shows a large smear due to the sextupole fields of the ID. The effect of a good matching is clearly visible, the phase space figure shows a nearly perfect circle. The matching is achieved by an end pole configuration on either side of the ID, each one composed by two poles. These poles are simulated by a step length of the generating function over half a longitudinal period length using the potential function given above, but with reduced field amplitudes. At the entrance side the first half pole has a strength of $+\frac{1}{4}V$ and the second half pole has a strength of $-\frac{3}{4}V$. At the exit side the sequence is reversed, using $+\frac{3}{4}V$ and $-\frac{1}{4}V$. Applying these end poles both closed orbit and distortion functions are well matched. In the matched case the effect of the helical device on the BESSY II optics can be ignored. If the tracking is done with all 14 insertions and one of these replaced by a helical device, no reduction of the dynamic aperture is seen.

4 CONCLUSION

A matched end pole design is required for (not only helical) IDs to damp closed orbit and distortion function oscillations. Both effects can be canceled by properly adjusted end poles. For the full tuning range of the shift parameter ψ of the helical device no effects on the BESSY II dynamic aperture is seen. The required sextupole corrections are in first approximation independent of the ring optics. In case of fast changing beta functions inside the ID the correction has to be done including the ring optics.

5 ACKNOWLEDGEMENTS

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6 REFERENCES

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