THE COUPLING-CAVITY SCHEME FOR THREE-DIMENSIONAL LASER COOLING

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Abstract

We study three-dimensional laser cooling of fast circulating beams, employing the tracking code SAD. As an example, the lattice parameters of the storage ring ASTRID in Denmark are taken to figure out the efficiency of transverse cooling by synchrobetatron coupling. In this paper, the stress is put upon the so-called coupling-cavity scheme of laser cooling[1]. A possible design of the coupling cavity is also presented.

1 INTRODUCTION

The high potential of laser cooling in a storage ring has been demonstrated during the last several years through extensive experimental efforts by the TSR group of MPI and by the ASTRID group of Aarhus University[2]. The longitudinal temperature of $\sim$1 mK has already been achieved for low-energy heavy ion beams. However, concerning the transverse degrees of freedom, laser cooling of fast circulating beams is not particularly efficient although natural Coulomb coupling reduces the transverse temperatures to the level achievable with the electron cooling technique[3].

In order to obtain the transverse cooling rates comparable to the longitudinal rate, we studied, in the previous publications, the effect of dynamical coupling due either to a coupling cavity[1] or to dispersion at an ordinary rf cavity[4]. It was then shown that the transverse cooling rates could considerably be enhanced under the linear resonance conditions; namely,

$$\nu_x - \nu_L = \text{integer and } \nu_x - \nu_y = \text{integer}, \tag{1}$$

where ν_x, ν_y, and ν_L are, respectively, the horizontal, vertical, and longitudinal tune of a storage ring.

In the present paper, we compare the cooling properties of the two coupling schemes, giving some simulation results obtained with the tracking code SAD[5].

2 DISPERSIVE COUPLING

In the dispersive coupling scheme, we need nothing new to develop transverse-longitudinal coupling; namely, it is only necessary to put an ordinary rf cavity at a location with a finite dispersion while a skew quadrupole magnet must be introduced in order to couple the two transverse degrees of freedom.

Fig.1 shows a simulation result obtained with SAD, where the lattice parameters of the ASTRID ring have been taken into account. A skew quadrupole magnet is installed, for horizontal-vertical coupling, at the opposite side of the laser cooling section, and the field gradient optimized to realize the largest transverse cooling rates is $(L/B\rho)\cdot(\partial B/\partial y) = 0.06\,\mathrm{m}^{-1}$ where L is the axial length of the magnet. The stored beam assumed here is 100 keV $^{24}\mathrm{Mg}^{+}$ which has been employed in recent laser-cooling expriments in ASTRID. To approximately meet the resonance requirement in Eq.(1), the horizontal and vertical tunes have been adjusted to be 3.11 and 3.09 respectively. The longitudinal tune should then be set at around 0.1. This high value of longitudinal tune can easily be achieved with a low rf voltage because of the very low beam energy. In fact, adopting the harmonic number of 260, the rf gap voltage of only 34 volts is needed to have $\nu_L = 0.1$. To adiabatically capture the initial continuous Mg-beam into the rf bucket, the rf voltage is linearly increased from 0 to 34 volts within the first 0.022 seconds in this simulation. Then, the upper half of the longitudinal phase space of the beam is scanned with a cooling laser in the next 0.45 seconds. It is evident, from Fig.1, that all three degrees of freedom have efficiently been cooled.

Needless to say, beam density becomes higher as the cooling process advances. Consequently, the tunes should be depressed by strong Coulomb repulsive forces, leading to the breakdown of the resonance conditions. It might thus be important to know how strongly this coupling scheme

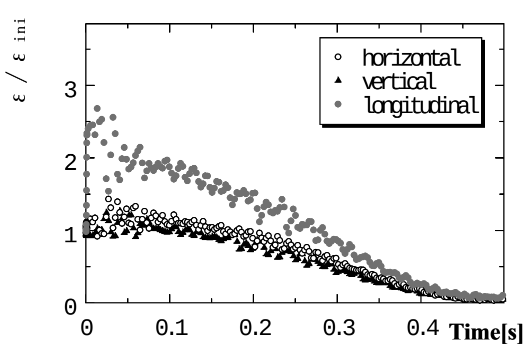

Figure 1: Time evolution of root-mean-squared(rms) emittances of a laser-cooled $^{24}\mathrm{Mg}^{+}$-beam. ε_{ini} denotes the initial value of the rms emittance. An ordinary rf cavity has been set 10 meters away from the cooling section where the size of dispersion is 2.15 meters.

depends on the resonance conditions. Fig.2 shows the ratio of the rms emittances after cooling to the initial values. We see that the transverse cooling efficiency gets worse rapidly as the longitudinal tune is deviated from the resonant value.

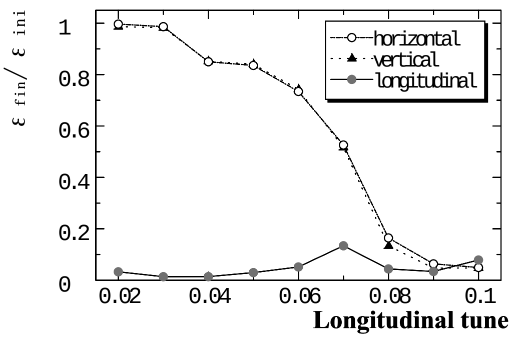

Figure 2: The rms-emittance ratio $\varepsilon_{fin}/\varepsilon_{ini}$ vs. longitudinal tune in the dispersive coupling scheme. ε_{fin} stands for rms emittance after laser scan is completed.

3 THE COUPLING CAVITY

3.1 A possible conceptual design

Let us now explore the cooling properties of the coupling-cavity scheme. The vector potential of an ideal coupling cavity can be given by

$$\boldsymbol{A} = (0, 0, Vx \sin \omega t) \tag{2}$$

where ω is the rf frequency, V is the constant proportional to the gap voltage, and x is the horizontal coordinate measured from the closed orbit. Apparently, this potential yields a longitudinal electric field linearly dependent on the transverse coordinate, which enables us to have a transverse-longitudinal coupling. Note here that, in the coupling-cavity scheme, we additionally need an ordinary rf cavity to provide a finite longitudinal tune such that the resonance condition in Eq.(1) is satisfied. The coupling-cavity frequency ω should then be taken roughly the same as the operation frequency of the ordinary cavity.

The easiest way to approximately obtain the potential in Eq.(2) is to excite TM_{210} mode in a simple rectangular pill-box cavity. This is, however, impractical in our case since ω is usually not high enough; namely, ω should be of the order of a few tens of MHz or less. In this frequency region, the cavity dimension becomes huge as far as a simple pill-box is used as a resonator. Therefore, we propose here a lumped circuit as illustrated in Fig.3. The approximate electric field distribution is shown in Fig.4. In order to verify that the coupling cavity in Fig.3 can roughly generate the vector potential proportional to the horizontal coordinate x, we evaluated the integrated effect of the longitudinal electric field in the cavity. The result is given in Fig.5 which actually demonstrates a good linearity with respect to x.

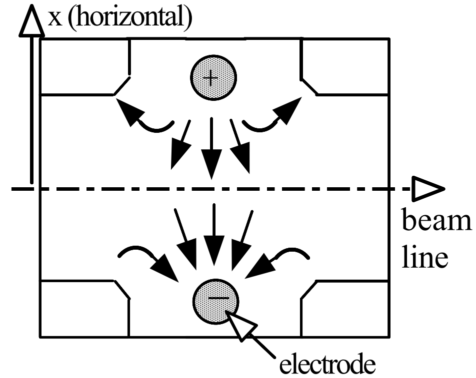

Figure 3: A possible geometry of coupling cavity.

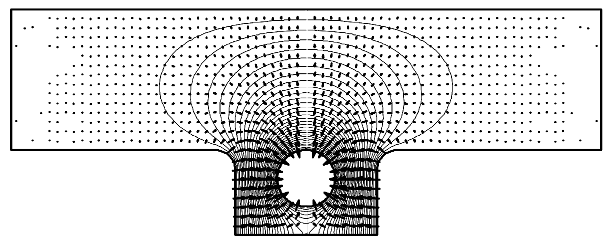

Figure 4: The approximate electric field and equipotential line of the coupling cavity. Only the lower half of Fig.3 has been shown.

3.2 Simulation results

To simulate the motion of laser-cooled beams under the influence of a coupling cavity, we have added a new subroutine to SAD. In the subroutine, the effect of a coupling cavity is analyzed, based on the electric-field data as shown in Fig.5. The field distribution is fitted with power series. Since the field linearity is sufficiently good, we have neglected the terms of higher than third order.

We have performed simulations again with the ASTRID lattice, modifying some fundamental parameters. The rf frequency of the coupling cavity has been fixed at about 5.8 MHz corresponding to the harmonic number of 260. This frequency is the same as that of the ordinary rf cavity. The peak voltage applied to the two electrodes of the coupling cavity is 487.1 volts. It has been confirmed, as anticipated, that the coupling cavity scheme works excellently under

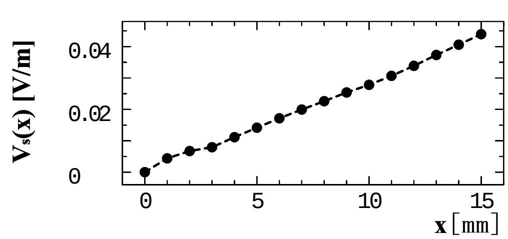

Figure 5: The dependence of the effective longitudinal electric field strength on the transverse coordinate. The absolute value of the electrode voltage has been set at 1 volt.

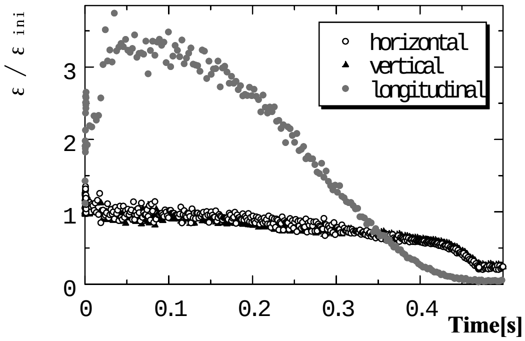

Figure 6: Time evolution of rms emittances of a laser-cooled $^{24}Mg^{+}$-beam under the influence of a coupling cavity. The longitudinal tune has been set at 0.04 while the two transverse tunes are roughly equal to 3.1. Note that an ordinary rf cavity is sitting at a dispersive position. We thus have synchrobetatron coupling not only by a coupling cavity but also by dispersion at the ordinary cavity.

the resonance conditions satisfied. Furthermore, we have found that this scheme is less sensitive to the longitudinal-transverse resonance condition, compared to the dispersive coupling scheme. As an example, we show, in Fig.6, a simulation result in which off-resonant value of the longitudinal tune has been assumed; namely, the longitudinal tune is 0.04 while the transverse tunes are identical to those employed in Fig.1. We see that both transverse rms emittances have been reduced to about 20% of the initial values after the laser scan is completed. The sensitivity to the resonance condition is given in Fig.7 which exhibits much better cooling properties than the dispersive coupling cases.

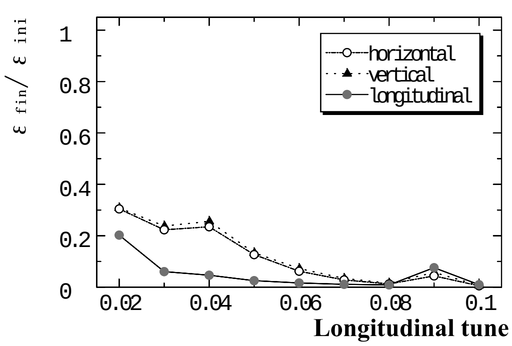

Figure 7: The rms-emittance ratio $\varepsilon_{fin}/\varepsilon_{ini}$ vs. longitudinal tune in the coupling-cavity scheme.

4 CONCLUSION

Two different schemes, i.e. dispersive coupling scheme and coupling-cavity scheme, have been considered to accomplish three-dimensional laser cooling of stored and circulating ion beams in a storage ring. We have confirmed that the transverse cooling rates can considerably be enhanced through dynamical coupling under the linear resonance conditions.

A possible design of a coupling cavity has been presented. Although the coupling-cavity scheme seems a bit more complicated than the dispersive coupling scheme, it might deserve further study because it is much less sensitive to the resonance conditions.

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