ON THE ANALYTIC REPRESENTATION OF PERIODIC MAGNETOSTATIC FIELDS

A.V. Smirnov
Russian Research Centre `Kurchatov Institute`, Moscow 123182, Russia

Abstract

A generalized form is considered in a source-free region for the magnetic scalar potential in focusing planar undulators and insertion devices. A number of particular cases is outlined: periodic quadrupole and sextupole focusing, combined-function nonlinear wigglers, twisted undulator, alternating longitudinal field focusing system, an adjustable phase insertion device. A special attention is paid to quadrupole type intrinsic fields including strong periodic focusing. The generalised formalism can be used in particle tracing codes or 3D codes for free electron lasers simulation.

1 INTRODUCTION

Free electron lasers (FELs), synchrotron radiation sources, microwave sources use a great variety of periodic magnetic systems. The development of the shortest wavelength high power FELs [1] requires a long undulator with strong focusing to achieve high gain. Therefore the fields providing intrinsic focusing should be considered in more details. Besides, some novel structure of insertion devices (ID) such as twisted undulator [2] need in specific description of undulator fields in paraxial region.

2 FIELDS IN CARTESIAN FRAME FOR PERIODIC SYSTEMS

There is a well known formula for the magnetic scalar potential in the planar wiggler [3] with a periodic intrinsic focusing of sextupole type. However this formula does not include the real case of intrinsic focusing with a periodic transverse field gradient in planar wigglers (undulators).

Let us consider a magnetostatic system with space period $\lambda_w = 2\pi / k_w$ and adiabatic tapering along the longitudinal axis OZ. The magnetic field $\boldsymbol{B}$ ($\boldsymbol{B} = -\nabla\psi$) can be expanded as a Fourier series in a free-space (interaction) region:

$$\psi = \sum \psi_n(x,y) e^{ink_w z}.$$

For rectangular gap configuration the analytic solutions of Helmholtz equation $\Delta_\perp \psi_n - n^2 k_w^2 \psi_n = 0$ can be derived in the following form:

$$\psi = \mathrm{Re}\sum_n e^{i(n\int_0^z k_w dz + \varphi_n)} \begin{pmatrix} A_n^{(1)} \cosh \mu_{nx}^{(1)} x \dfrac{\sinh \mu_{ny}^{(1)} y}{\mu_{ny}^{(1)}} + \\ A_n^{(2)} \dfrac{\sinh \mu_{nx}^{(2)} x}{\mu_{nx}^{(2)}} \cosh \mu_{ny}^{(2)} y + \\ A_n^{(3)} \cosh \mu_{nx}^{(3)} x \cosh \mu_{ny}^{(3)} y + \\ A_n^{(4)} \dfrac{\sinh \mu_{nx}^{(4)} x}{\mu_{nx}^{(4)}} \cdot \dfrac{\sinh \mu_{ny}^{(4)} y}{\mu_{ny}^{(4)}} \end{pmatrix},$$

$$\mu_{nx}^{(m)2} + \mu_{ny}^{(m)2} = n^2 k_w^2, \; m = 1,2,3,4, \qquad (1)$$

where n is the field harmonic number; φ_n is the initial space shift for the n-th harmonic, $A_n^{(m)}$, $\mu_{nx}^{(m)}$, $\mu_{ny}^{(m)}$, k_w are slowly varying functions of the longitudinal coordinate z: $\frac{d}{dz}\ln\left\{A_n^{(m)}, \mu_{nx,y}^{(m)}, k_w\right\} << k_w$; $A_n^{(m)}$ coefficients (m≠4) define on-axis field Fourier-amplitudes or field gradient Fourier-amplitudes (m=4), and $\mu_{nx}^{(m)}, \mu_{ny}^{(m)}$ eigenvalues describe the field shape in transverse plane for each Fourier-harmonic.

2.1 Particular cases

Herewith five particular cases **i**)÷ ∨) are considered.

i) $A_n^{(1)} \neq 0$, $A_n^{(m)} = 0, m \neq 1$

We deal with Scharlemann`s expression (see ref. [3]) for the magnetic scalar potential in the planar undulator with a horizontal wiggle plane. The first term in (1) being expanded in powers of x,y gives the series of periodic fields with an even symmetry with respect to YOZ plane: dipole, sextupole, decapole, etc.
We can consider two subclasses.

a) A conventional wiggler (undulator) with sextupole type (`natural`) focusing (see ref. [3]) at $\mu_{nx}^{(m)2} > 0$, $\mu_{ny}^{(m)2} > 0$.

b) If $\mu_{ny}^{(m)2}$(z) periodically changes its sign along the undulator ($\mu_{ny}^{(m)2} < 0$, $\mu_{nx}^{(m)2} > 0$ and vice versa) we can derive an alternating sextupole strong focusing that was investigated in matrix formalism in [4]. K-V equations [5] describe beam envelope behaviour as Mathieu-Hill equations and allow to treat such focusing in a source-free region for undulators also. Particle tracing codes can solve the beam transport problem in such undulators taking into account nonlinear effects and the measured magnetic field data (see [6,7] for example).

ii) $A_n^{(2)} \neq 0, \quad A_n^{(m)} = 0, m \neq 2.$

The second term in (1) describes the undulator fields with the sextupole type periodic focusing and the vertical (YOZ) wiggle plane. So this case can be obtained from the previous (**i**) case by means of replacement: x ↔ y.

iii) $A_n^{(3)} = 0 = A_n^{(4)}$, $\mu_{nx}^{(2)} \Rightarrow 0, \mu_{ny}^{(1)} \Rightarrow 0$

at $A_n^{(1)} = -iA_n^{(2)} = const \neq 0.$

This case approximates the fields of a well known helical undulator in Cartesian frame. This form is more suitable for a permanent magnet undulator having only two dipoles over $\lambda_w / 2$ length and angular shift π/2 between them in the transverse plane.

iv) $A_n^{(3)} \neq 0$.

a) $A_n^{(m)} = 0, m \neq 3.$

We see that the longitudinal magnetic field is not zero on the OZ axis. The transverse magnetic field components are equal to zero on the OZ axis. The transverse field components vanish at $n = 0$ and $\vec{B}_\perp \sim \vec{r}_\perp$ near the axis. This case describes alternating longitudinal field (solenoidal) focusing system. Such systems are very compact and effective for electron beam focusing in travelling wave tubes (TWT) and backward wave oscillator (BWO) tubes.

b) $A_n^{(1)} = 2B_r G_n \cos\left(\frac{nk_w z_o}{2}\right), \quad A_n^{(2)} = 0, A_n^{(4)} = 0,$

$A_n^{(3)} = -i2B_r G_n \sin\left(\frac{nk_w z_o}{2}\right).$

This case appropriates to the adjustable phase insertion device (APID, see ref. [8]). Here z_o denotes the space shift along the OZ axis between two parallel magnet rows with a constant gap and other notations B_r, G_n are the same as in the ref. [8]. It can be shown that the focusing properties of APID are independent on the undulator strength (i.e. z_o value for APID) in contrary to the adjustable gap insertion devices (AGID) for ultra relativistic electron beams.

2.1.1 Field component having odd symmetry in transverse plane

v) $A_n^{(4)} \neq 0$, $A_n^{(3)} = 0.$

Here $A_n^{(4)} = Q_n(z)$ is Fourier-amplitude of the field gradient for the n-th field harmonic.

The last term in (1) describes the intrinsic periodic focusing fields with odd symmetry with respect to the longitudinal OZ axis. Unlike the previous cases (**i**, **ii**) corresponding fields can be regarded as quadrupole, octupole, dodecapole, etc. series of fields. Besides, an interchange of signs of $\mu_{nx}^{(4)2}$ and $\mu_{ny}^{(4)2}$ corresponds no longer to a conventional strong focusing. The alternating gradient focusing is caused by $A_n^{(4)} = Q_n(z)$ variation along the undulator. It can be provided by means of both external quadrupoles and special undulator schemes with intrinsic quadrupole focusing.

The combination of the first and the last term in (1) can describe the so-called quadrupole-sextupole wiggler (see ref. [6]) in which, as the name suggests, there are superposed quadrupole and sextupole fields. Such multipole combination produces nonlinear damping effect with negligible energy loss for fluctuating synchrotron radiation in storage rings. Another combined-function nonlinear wigglers (for example, dipole-octupole magnets) produce a large additional energy loss [6].

Let us consider the contribution of ψ_q to a total scalar potential ψ from the last term in (1):

$$\psi_q = \mathrm{Re} \sum_n e^{i(n\int_0^z k_w dz + \varphi_n)} A_n^{(4)} \frac{\sinh \mu_{nx}^{(4)} x}{\mu_{nx}^{(4)}} \cdot \frac{\sinh \mu_{ny}^{(4)} y}{\mu_{ny}^{(4)}} . \quad (2)$$

We can outline three following subclasses.

a) $A_n^{(4)} \neq 0$, $A_n^{(4)}$ =0 for n≠0 , $\varphi_o = 0 : \psi_q = Q_o xy$.

This is a conventional quadrupole focusing with constant gradient. For intrinsic quadrupole focusing Q_o is the gradient averaged over undulator period.

b) $A_0^{(4)} \neq 0$, $A_n^{(4)}$ =0 for n≠1.

This is the case of an alternating gradient with quadrupole lenses placed between the poles (φ_1 =0, see also ref. [4]) or with canted poles having the same tilt ($\varphi_1 = \pi / 2$).

c) $A_0^{(4)} \neq 0$, $A_0^{(4)} \neq 0$, $A_n^{(4)}$ =0 for n≠0 and n≠2.

This case takes place when the local gradient $Q(z) = -\partial B_y / \partial x$ is a periodic function with period $\lambda_w / 2$. As it follows from direct measurements [9-12] the undulators with canted poles [11,12], side magnets [9] and C-shape poles [10] have a such property. It can be assumed that for some other planar undulator schemes (for example, with trapezoidal blocks or offset pole pieces) we have Q_2 ≠0 as well.

Expression (2) describes intrinsic quadrupole focusing and contains Q_o , Q_2 , μ_{2x} , φ_2 parameters in addition to the conventional undulator parameters (period, amplitude, etc.). These additional parameters can be defined by means of direct measurements or 3D magnetostatic fields calculation. To take into account Q_2-effect on beam size, phase synchronism detuning (see [9,10]) we can not apply K-V equations and should use non-averaging and non-linearized particle tracing calculations (e.g. [7]). We expect this effect can appear as a coupling between the vertical and horizontal motions due to the nonlinear cross terms in the undulator focusing strengths expanded as a Taylor series. It would be similar to the effect noticed for the equal focusing undulators with the sextupole type focusing [9,13].

3 TWISTER UNDULATOR FIELDS

The geometry [2,14] looks like helical transformation with period λ_t applied to an original linear undulator structure having period λ_w and provides a combination of fast and slow oscillations of the electron beam. In a source-free region the fields can be represented in terms of $\psi(r,\varphi, z)=Re\Sigma R(r)\Phi(\varphi)Z(z)$. Since the longitudinal structure combines two periods λ_t and λ_w, we use nh_t+mh_w as a separation constant. It gives the followng solution:

$$\psi(r,\varphi,z) = \mathrm{Re} \sum_{l,m,n} \begin{pmatrix} A_l C_{n,m} I_l(r(nk_t + mh_w)) \times \\ \exp(i(l\varphi - (nk_t + mh_w)z)) \end{pmatrix}$$

To define the constants A_l, $C_{m,n}$ the following transverse fields we imply at $r=0$:

$B_r=(B_w \cos k_w z+B_t)\sin(\varphi+k_t z)$

$B_\varphi=(B_w \cos k_w z+B_t)\cos(\varphi+k_t z)$,

where B_w and B_t are the amplitudes of the untwisted and helical undulator fields respectively. This form follows from the expression for on-axis undulator field derived from that for planar undulator under twisting transformation [14]:

$B_x=(B_w \cos k_w z+B_t)\sin(k_t z)$

$B_y=(B_w \cos k_w z+B_t)\cos(k_t z)$.

Under these assumptions one can derive:

$$\psi(r,\varphi,z) = \mathrm{Re} \sum_{m=-1}^{m=+1} \begin{pmatrix} C_m I_1(r(mh_w + k_t)) \times \\ \exp(i(\varphi + (mh_w + k_t)z)) \end{pmatrix},$$

where $C_0 = \dfrac{2iB_t}{k_t}, \quad C_{\pm1} = i\dfrac{2B_w}{k_t \pm h_w}$

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