FEMTOSECOND ELECTRON BUNCHES FROM COLLIDING LASER PULSES IN PLASMAS

E. Esarey, R.F. Hubbard, W.P. Leemans,* A. Ting, and P. Sprangle
Beam Physics Branch, Plasma Physics Division, Naval Research Laboratory, Washington DC 20375

Abstract

An injector and accelerator is analyzed that uses three collinear laser pulses in a plasma: a pump pulse, which generates a large wakefield ($\geq$ 20 GV/m), and two counterpropagating injection pulses. When the injection pulses collide, a slow phase velocity beat wave is generated that injects electrons into the fast wakefield for acceleration. Particle tracking simulations in 1-D with injection pulse intensities near 10^{17} W/cm^2 indicate the production of high energy electrons with bunch durations as short as 3 fs, energy spreads as small as 0.3%, and densities as high as 10^{18} cm^{-3}.

1 INTRODUCTION

Plasma-based accelerators [1] may provide a compact source of high energy electrons due to their ability to sustain ultrahigh electric fields E_z on the order of $E_0 = cm\omega_p/e \simeq n_0^{1/2}[\text{cm}^{-3}]$ V/cm, where $\omega_p = (4\pi n_0 e^2/m)^{1/2}$ is the plasma frequency and n_0 is the plasma density. Accelerating fields of 10-100 GV/m have been generated over distances of a few mm [2-4] in both the standard [5] and self-modulated [6,7] regimes of the laser wakefield accelerator (LWFA). The characteristic scale-length of the accelerating plasma wave is the plasma wavelength $\lambda_p = 2\pi c/\omega_p$, which is typically ≤ 100 μm. Although several recent experiments [3,4] have demonstrated the self-trapping and acceleration of plasma electrons in the self-modulated LWFA, the production of electron beams with relatively low momentum spread and good pulse-to-pulse energy stability will require injection of ultrashort electron bunches into the wakefield with femtosecond timing accuracy. These requirements are beyond the current state-of-the-art performance of photo-cathode radio-frequency electron guns.

Recently an all-optical method for injecting electrons in a standard LWFA has been proposed [8]. This method (referred to as LILAC) utilizes two laser pulses which propagate either perpendicular or parallel to one another. The first pulse (the pump pulse) generates the wakefield, and the second pulse (the injection pulse) intersects the wakefield some distance behind the pump pulse. The ponderomotive force $F_p \sim \nabla a^2$ of the injection pulse can accelerate a fraction of the plasma electrons such that they become trapped in the wakefield, where $a^2 \simeq 7\times 10^{-19}\lambda^2[\mu\text{m}]I[\text{W/cm}^2]$, $\lambda = 2\pi c/\omega$ is the laser wavelength, and I the intensity. Simulations, which were performed for ultrashort pulses at high densities ($\lambda_p/\lambda = 10$ and $E_z/E_0 = 0.7$), indicated the production of a 10 fs, 21 MeV electron bunch with a 6% energy spread. However, high intensities ($I > 10^{18}$ W/cm^2) are required in both the pump and injection pulses ($a \simeq 2$). An all optical electron injector would be a significant step in reducing the size and cost of a LWFA.

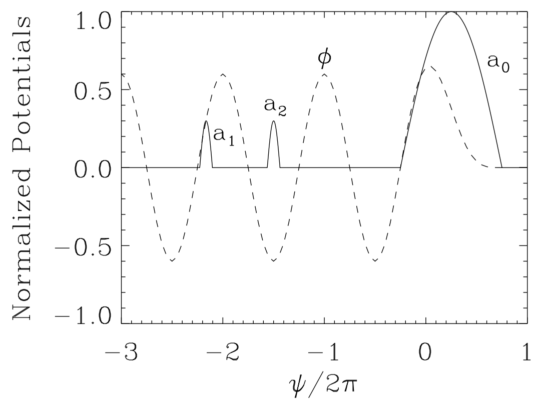

Figure 1: Profiles of the pump laser pulse a_0, the wakefield ϕ, and the forward a_1 injection pulse, all of which are stationary in the $\psi = k_p(z - v_{p0}t)$ frame, and the backward injection pulse a_2, which moves to the left at $\simeq 2c$

In the following, a colliding pulse optical injection scheme for a LWFA is proposed and analyzed that uses three short laser pulses: an intense pump pulse (denoted by subscript 0), a forward going injection pulse (subscript 1), and a backward going injection pulse (subscript 2), as shown in Fig. 1. The frequency, wavenumber, and normalized intensity are denoted by ω_i, k_i, and a_i ($i = 0, 1, 2$). Furthermore, $\omega_1 = \omega_0$, $\omega_2 = \omega_0 - \Delta\omega$ ($\Delta\omega \geq 0$), and $\omega_0 \gg \Delta\omega \gg \omega_p$ are assumed such that $k_1 = k_0$, and $k_2 \simeq -k_0$. The pump pulse generates a fast ($v_{p0} \simeq c$) wakefield. When the injection pulses collide (some distance behind the pump) they generate a slow ponderomotive beat wave with a phase velocity $v_{pb} \simeq \Delta\omega/2k_0$. During the time in which the two injection pulses overlap, a two-stage acceleration process can occur, i.e., the slow beat wave injects plasma electrons into the fast wakefield for acceleration to high energies. It will be shown that injection and acceleration can occur at low densities ($\lambda_p/\lambda \sim 100$), thus allowing for high single-stage energy gains, with normalized injection pulse intensities of $a_1 \sim a_2 \sim 0.2$ ($\sim 10^2$ less intensity than required by the LILAC scheme). Furthermore, the colliding pulse concept offers detailed control of the injection process: the injection phase can be controlled via the position of the forward injection pulse, the beat phase velocity via $\Delta\omega$, the injection energy via the

* Lawrence Berkeley National Laboratory, Berkeley CA 94720

pulse amplitudes, and the injection time (number of trapped electrons) via the backward pulse duration.

2 ANALYSIS

The colliding pulse injection mechanism will be analyzed in 1-D with the plasma wave and laser fields represented by the normalized scalar $\phi = e\Phi/mc^2$ and vector $a = eA_\perp/mc^2$ potentials, respectively. The axial component of the normalized electron momentum $u_z = p_z/mc = \gamma\beta_z$ obeys $du_z/dct = \partial\phi/\partial z - (2\gamma)^{-1}\partial a^2/\partial z$, where $\gamma = \gamma_z\gamma_\perp$, $\gamma_\perp = (1+a^2)^{1/2}$, and $\gamma_z = (1-\beta_z^2)^{-1/2}$. This can be written in terms of the phase of the electron with respect to the wakefield $\psi = k_p(z - v_{p0}t)$, i.e.,

$$\frac{d^2\psi}{d\tau^2} = \frac{(1-\beta_z^2)}{\gamma}\frac{\partial\phi}{\partial\hat{z}} - \frac{1}{\gamma^2}\left(\frac{\partial}{\partial\hat{z}} + \beta_z\frac{\partial}{\partial\tau}\right)\frac{a^2}{2}, \qquad (1)$$

where $k_p = \omega_p/c$, $v_{p0} = c\beta_{p0}$ is the wakefield phase velocity, $\hat{z} = k_p z$, $\tau = \omega_p t$, and $\beta_z = d\psi/d\tau + \beta_{p0}$.

The effects of three waves will be considered: a plasma wakefield $\phi = \hat{\phi}(\psi)\cos\psi$, and a forward and a backward injection laser pulse, both of the form $a_i = \hat{a}_i(z - v_{gi}t)\,(\sin\theta_i e_x + \cos\theta_i e_y)$. Here, $\theta_i = k_i z - \omega_i t$ and the amplitudes $\hat{a}_i$ and $\hat{\phi}$ are assumed to be slowly varying compared to the phases θ_i and ψ. Also, k_i and ω_i satisfy $k_i = \sigma_i\omega_i(1-\omega_p^2/\omega_i^2)^{1/2}$, where $\sigma_1 = 1$ and $\sigma_2 = -1$, which implies a group velocity $v_{gi} = c\beta_{gi} = c^2k_i/\omega_i$ ($v_{p0} = v_{g0} = v_{g1}$). Furthermore, $a^2 = \hat{a}_1^2 + \hat{a}_2^2 + 2\hat{a}_1\hat{a}_2\cos\psi_b$, where $\psi_b = \theta_1 - \theta_2 = \Delta k(z - v_{pb}t)$ is the beat phase, $v_{pb} = c\beta_{pb} = \Delta\omega/\Delta k$, and $\Delta k = k_1 - k_2 \simeq 2k_0$.

In the absence of the injection pulses, electron motion in the wakefield is described by the Hamiltonian [9] $H_w = \gamma - \beta_{p0}(\gamma^2-1)^{1/2} - \phi$, where $\phi = \phi_0\cos\psi$. The boundary between trapped and untrapped orbits is given by the separatrix $H_w(\gamma,\psi) = H_w(\gamma_{p0},\pi)$, where $\gamma_{p0} = (1-\beta_{p0}^2)^{-1/2}$. The minimum momentum of an electron on the separatrix is given by $u_{min} \simeq (1/\Delta\phi - \Delta\phi)/2$, where $\Delta\phi = \phi_0(1+\cos\psi)$, assuming $\gamma_{p0}\Delta\phi \gg 1$ and $\beta_{p0} \simeq 1$. In particular at $\psi = 0$, $u_{min} = 0$ for $\phi_0 = 1/2$, which means that an electron that is at rest at the phase $\psi = 0$ will be trapped. The background plasma electrons, however, are untrapped and are undergoing a fluid oscillation with a momentum $u_f \simeq -\phi$ ($\phi^2 \ll 1$). Hence, at $\psi = 0$, the plasma electrons are moving backward with $u_f \simeq -\phi_0$, which is far from the trapping threshold.

The beat wave leads to formation of phase space buckets (separatrices) of width $2\pi/\Delta k \simeq \lambda_0/2$, which are much shorter than those of the wakefield (λ_p), thus allowing for a separation of time scales. In particular, it can be shown that both the transit time $2\pi/\Delta\omega$ of an untrapped electron through a beat wave bucket and the synchrotron (bounce) time $\pi/(\hat{a}_1\hat{a}_2)^{1/2}\omega_0$ of a deeply trapped electron in a beat wave bucket are much shorter than a plasma wave period $2\pi/\omega_p$. Hence, on the time scale in which an electron interacts with a single beat wave bucket, the wakefield can be approximated as static.

In the combined fields, the electron motion can be analyzed in the local vicinity of a single period of the beat wave by assuming that the wakefield electric field $E_z = -k_p^{-1}E_0\partial\phi/\partial z \simeq E_{z0}$ is constant. The Hamiltonian associated with Eq. (1) is given by

$$H_b \simeq \gamma - \beta_{pb}\left[\gamma^2 - \gamma_\perp^2(\psi_b)\right]^{1/2} + \epsilon\psi_b, \qquad (2)$$

where $\epsilon = E_{z0}k_p/E_0\Delta k$ is constant and $\gamma_\perp^2 = 1 + \hat{a}_1^2 + \hat{a}_2^2 + 2\hat{a}_1\hat{a}_2\cos\psi_b$. When $\epsilon = 0$, the phase space orbits are symmetric with respect to ψ_b. In terms of the normalized axial momentum, the maximum and minimum points on the separatrix are given by $u_{bm} \simeq \beta_{pb}\gamma_{bp}(1+4\hat{a}_1^2)^{1/2} \pm 2\hat{a}_1\gamma_{pb}$, where $\gamma_{pb} = (1-\beta_{pb}^2)^{-1/2}$ and $\hat{a}_1 = \hat{a}_2$ is assumed. When $\epsilon \neq 0$, the separatrix distorts into fished-shape islands. In particular, when $\epsilon < 0$ ($\epsilon > 0$), the "fish tail" of the separatrix opens to the right (left).

A scenario by which the beat wave leads to trapping in the plasma wave is the following. In the phase region $-\pi/2 < \psi < 0$, the plasma electrons are flowing backward, $u_f = -\phi_0\cos\psi < 0$, and the electric field is accelerating, $E_z/E_0 = \phi_0\sin\psi < 0$. Here $\epsilon < 0$ and the beat wave buckets open to the right. Consider an electron that is initially flowing backward and resides below the beat wave separatrix. Since the separatrix opens to the right, there exists open orbits which can take an electron from below to above the beat wave separatrix. Such an electron can acquire a sufficiently large positive velocity to allow trapping and acceleration in the plasma wave. These open phase space orbits, which provide the necessary path for electron acceleration, can exist when the beat wave resides within $-\pi/2 < \psi < 0$.

An estimate for the threshold for injection into the wakefield can be obtained by considering the effects of the wakefield and the beat wave individually and by requiring (i) the maximum energy of the beat wave separatrix exceed the minimum energy of the wakefield separatrix, $u_{bmax} \geq (\Delta\phi^{-1} - \Delta\phi)/2$, and (ii) the minimum momentum of the beat wave separatrix be less than the plasma electron fluid momentum, $u_{bmin} \leq -\phi$. These two conditions imply that the beat wave separatrix overlaps both the wakefield separatrix and the plasma fluid oscillation, thus providing a phase-space path for plasma electrons to become trapped in the wakefield. For a given wakefield amplitude ϕ_0, conditions (i) and (ii) imply the optimal phase location $3\phi_0\cos\psi \simeq 3^{1/2} - 2\phi_0 - 2\beta_{pb}$ and threshold amplitude $6\hat{a}_1 > 3^{1/2} - 2\phi_0 + \beta_{pb}$ of the injection pulse, where $\phi_0^2\cos^2\psi \ll 1$, $\hat{a}_1^2 \ll 1$, and $\beta_{pb}^2 \ll 1$ were assumed. For example, $\phi_0 = 0.6$ and $\beta_{pb} = 0.05$ imply $\psi = -1.3 - 2\pi j$ and $\hat{a}_1 > 0.11$.

3 SIMULATION

To further evaluate the colliding laser injection method, the motion of test particles in the combined wake and laser fields was simulated by numerically solving Eq. (1). At $\tau = 0$, the forward (backward) pulse profile $\hat{a}_1$ ($\hat{a}_2$) is a half-period of a sine wave with maximum amplitude

a_{1m} (a_{2m}), centered at $\psi = \psi_1 < 0$ ($\psi_2 > 0$), with length L_1 (L_2). Test particles are loaded uniformly from $\psi = 0$ to $\psi = \psi_{max}$ with $d\psi/d\tau = -\beta_{p0}$ (initially at rest) and pushed from $\tau = 0$ to $\tau = \tau_{max}$. Also, $\hat{\phi} = \phi_0 \left[1 - \exp(-\psi^2/\pi^2)\right]$ for $\psi \leq 0$.

To validate the analytical predictions for the trapping thresholds, a "near threshold" case was simulated with $\omega_1/\omega_p = 100$, $\omega_2/\omega_p = 90$, and $\phi_0 = 0.6$, which for $\lambda_1 = 2\pi c/\omega_1 = 1$ μm implies $n_0 \simeq 10^{17}$ cm^{-3} and $E_z = 0.6E_0 \simeq 19$ GV/m. Also, $a_{1m} = a_{2m} = 0.3$ (1.2×10^{17} W/cm^2), $L_1 = L_2 = \lambda_p/8$ (42 fs), $\psi_1 = -13.6$ and $\psi_2 = 21.4$ (chosen so the beat wave and test particles overlap). After a propagation distance of $\tau_{max} = 300$ (0.48 cm), trapped electrons were observed with a bunch length $L_b = 6.3$ μm (21 fs) with 60% of the electrons are contained within 66 MeV $\pm 8\%$. The fraction f_{tr} of those particles which encounter the beat wave that become trapped was $f_{tr} \simeq 30\%$. A numerical optimization of the parameters was also performed to determined the trapping threshold. The optimal phase for injection (which minimizes the value of a_{1m} required for trapping) was found to be $\psi_1 = -13.8$, in good agreement with theory. Furthermore, trapping was observed for $a_{1m} > 0.17$, somewhat higher than the analytical prediction (0.11). Additional simulations indicate that trapping occurs when the center of the $L_1 = \lambda_p/8$, $a_{1m} = 0.3$ pulse is located within $-14.2 \leq \psi_1 \leq -13.5$. This implies that the forward pulse must be synchronized to the wake with an accuracy < 37 fs, which is not a serious constraint and can be relaxed somewhat by using a longer forward pulse.

More dramatic results can be obtained by moving the position of the injection pulse slightly forward and increasing both duration and amplitude of the injection pulses (in comparison to the previous "near threshold" example). As an example, a simulation was performed with $\psi_1 = -12.6$, $a_{1m} = a_{2m} = 0.5$, $\phi_0 = 0.7$, $L_1 = L_2/4 = \lambda_p/4$, $\omega_1/\omega_p = 100$, and $\omega_2/\omega_p = 85$ ($\lambda_1 = 0.85$ μm, $\lambda_2 = 1$ μm, and $\lambda_p = 85$ μm). After a distance of $\tau_{max} = 100$ (0.14 cm), the results are quite dramatic: a bunch duration of 2.9 fs was obtained due to natural compression provided by the axial electric field, with a mean energy of 27 MeV and a standard deviation in energy of 0.32%. The trapping fraction is $f_{tr} \simeq 19\%$ and the bunch density is $n_b = 1.8 \times 10^{18}$ cm^{-3}. Furthermore, in this run, the trapped electron are injected into and remain within a phase region of the wakefield that is both accelerating and focusing.

The bunch density is $n_b \simeq f_{tr} n_0 L_z/L_b$, where $L_z \simeq (L_1 + L_2)/2$ is the length of plasma that encounters the overlapping pulses. Assuming that the 1-D results hold for a pump laser of radius r_0 implies a total number of trapped electrons $N_b \simeq f_{tr} n_0 L_z \pi r_0^2$, e.g., $N_b \simeq 7.7 \times 10^9$ for Fig. 4 with $r_0 = 40$ μm. Note that N_b can be increased by increasing n_0, r_0, a_{1m} (via f_{tr}) and, in particular, L_z by increasing the duration of the backward pulse L_2. The ratio of N_b to the theoretical beam loading limit N_0 [10] is $N_b/N_0 = f_{tr} k_p L_z E_0/E_z$, which can easily approach unity. For N_b near N_0, however, space-charge effects become important and a self-consistent simulation is required.

In summary, a method has been proposed and analyzed for injecting plasma electrons into a large wakefield using two colliding laser pulses. Simulations of test electrons in prescribed 1-D fields indicate the production of relativistic (≥ 25 MeV) electrons with bunch durations as short as 3 fs, energy spreads as small as 0.3%, and densities as high as 10^{18} cm^{-3}.

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