FRACTIONAL FILLING INDUCED LANDAU DAMPING OF LONGITUDINAL INSTABILITIES AT THE ESRF

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Abstract

Longitudinal coupled bunch instabilities are a major obstacle for the increase of beam current in modern electron storage rings. At the ESRF, threshold limits for multi-bunch operation have been considerably increased by using fractional fillings, from about 60 mA for a homogeneous filling to well beyond the nominal intensity of 200 mA for a filling of one third of the circumference. The gap in the bunch train induces a modulation of the cavity voltage and a subsequent spread in synchrotron frequencies. This results in additional Landau damping. An appropriate set of coupled equations, which completely models the problem, has been derived. With slight simplifications one obtains analytical formulae which still accurately describe the observed effect. The theoretical results have been soundly confirmed by experiments carried out at the ESRF.

1 INTRODUCTION

Longitudinal coupled bunch instabilities (LCBIs) arise from the resonant coupling of multi-bunch modes (MBMs) with higher order modes (HOMs) in RF cavities. Avoiding the resonance by tuning away the HOMs is one remedy, reduction of the coupling by damping the HOM or staying at low beam currents is another. In the present paper we show how, additionally, Landau damping [1] can be used in high energy electron storage rings to maintain strong beam current levels. It is routinely applied at the ESRF in combination with a dedicated temperature regulation system [2].

In section 2 we use a set of coupled equations to treat the combined effect of Landau damping coming from different synchrotron frequencies of the individual bunches and the strong natural synchrotron damping in a high energy storage ring. We also present methods to accurately calculate current thresholds. Section 3 gives results on the determination of the frequency spread induced by the beam loading due to a fractional filling. We employ this to compute the threshold current. In section 4 we present experimental results that validate our theory.

Unless stated otherwise we use standard ESRF parameters [2], [3]. Notably the revolution time is $T_0 = 2.8\ \mu$s, the revolution frequency $\omega_0/(2\pi) = 355$ kHz, the energy $E_0 = 6$ GeV, the loss per turn $U_0 = 4.75$ MeV, the momentum compaction factor $\alpha = 1.9 \cdot 10^{-4}$, the harmonic number $h = 992$, the natural damping constant $\delta_{\rm n} = 277$ Hz the mean synchrotron frequency $f_{\rm s} = \omega_{\rm s}/(2\pi) = 1.97$ kHz and the peak cavity voltage $\widehat{V} = 8$ MV. HOMs with shunt impedances up to about 4 MΩ have to be considered at the ESRF, mainly at 500 and 910 MHz.

2 LCBI MODEL WITH A SPREAD IN SYNCHROTRON FREQUENCIES

2.1 Interaction Equations

For short bunches, rigid bunch MBMs are responsible for LCBIs. We model the beam in the storage ring as N rigid bunches, obeying synchrotron equations with frequencies $\omega_{{\rm s}k}$ spread over a range $\Delta\omega_{\rm s}$:

$$\ddot{\tau}_k + 2\delta_{\rm n}\dot{\tau}_k + \omega_{{\rm s}k}^2\tau_k = 0 \quad , \quad k = 1,\ldots,N \qquad (1)$$

τ_k is the temporal displacement of bunch k w.r.t. a synchronous particle at phase $\phi_{{\rm s}k}$. It is well known that

$$\delta_{\rm n} \approx \frac{U_0}{T_0 E_0} \quad , \quad \omega_{{\rm s}k}^2 = \frac{\alpha}{T_0 E_0/e}\left.\frac{{\rm d}V}{{\rm d}\tau}\right|_{\phi_{{\rm s}k}} \quad . \qquad (2)$$

Distinct $\omega_{{\rm s}k}$ arise from a modulation of ${\rm d}V/{\rm d}\tau$ and $\phi_{{\rm s}k}$.

An MBM giving rise to a synchrotron sideband in the beam spectrum at $(n + mh)\omega_0 + \omega$ can be described by

$$\tau_k(t) = \widehat{\tau}_k {\rm e}^{{\rm j}(\omega t + 2\pi nk/h)} \quad , \quad k = 1,\ldots,N \qquad (3)$$

where n is the MBM number, $\widehat{\tau}_k$ the complex amplitude and ω the common complex frequency of the bunch oscillation. Due to its high Q, an HOM is excited only by spectral lines near $\omega_{\rm HOM}$. Developing the phase modulated beam signal up to first order yields an expression for the HOM-voltage to be added to the energy budget of each bunch, linear in each $\widehat{\tau}_k$. Using this and the ansatz (3) in (1), we acquire a system of coupled equations for the $\widehat{\tau}_k$ and ω:

$$(-\omega^2 + {\rm j}2\omega\delta_{\rm n} + \omega_{{\rm s}k}^2)\widehat{\tau}_k = {\rm j}H\sum_{i=1}^{N} I_i\widehat{\tau}_i \qquad (4)$$

with $k = 1,\ldots,N$. Here I_i is the DC current in bunch i, the total current is $I_{\rm b} = \sum_{i=1}^{N} I_i$. H is given by

$$H = \omega_{\rm HOM} Z_{\rm HOM}\frac{\alpha}{T_0 E_0/e} \qquad (5)$$

with $Z_{\rm HOM}$ the impedance of the HOM (linac-Ohms).

2.2 Dispersion Relation

Eq. (4) leads to the dispersion relation

$$1 = \sum_{1=k}^{N}\frac{{\rm j}HI_k}{\omega_{{\rm s}k}^2 - \omega^2 + 2\delta_{\rm n}{\rm j}\omega} \quad . \qquad \text{(DR)}$$

Its N solutions ω_i are the eigenfrequencies of the system (4). For small currents the ω_i are near $\omega_{{\rm s}i} + {\rm j}\delta_{\rm n}$, increasing I_b will move them. The stability limit is reached at the threshold current $I_{\rm th}$, where the first of the ω_i becomes purely real. Fig. 1 shows how the rhs of (DR) maps the positive real axis ($0 < \omega < \infty$). As in feedback theory, the system is instable if the critical point 1 is encircled.

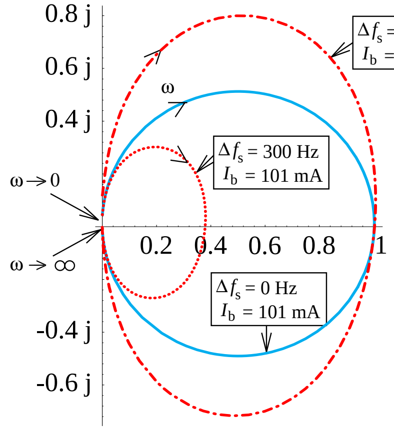

Figure 1: Stability traces, rhs of (DR) $R_{\mathrm{HOM}} = 2\ \mathrm{M}\Omega$ 16 bunches

The instability limit 1 is reached at higher currents, if $\Delta f_{\mathrm{s}} > 0$.

2.3 *Eigenvalue Approach*

The eigenvalues (μ_i) of the matrix obtained from eq. (4)

$$\begin{pmatrix} \mathrm{j}HI_1 - \omega_{\mathrm{s}1}^2 & \mathrm{j}HI_2 & \cdots & \mathrm{j}HI_N \\ \mathrm{j}HI_1 & \mathrm{j}HI_2 - \omega_{\mathrm{s}2}^2 & \cdots & \mathrm{j}HI_N \\ \vdots & \vdots & \ddots & \vdots \\ \mathrm{j}HI_1 & \mathrm{j}HI_2 & \cdots & \mathrm{j}HI_N - \omega_{\mathrm{s}N}^2 \end{pmatrix}$$

are related to the solutions ω_i by $\mu_i = \omega_i^2 - 2\delta_{\mathrm{n}}\mathrm{j}\omega_i$ and it is easier to compute them than to evaluate (DR) directly. An iteration in I_{b} yields I_{th}, as shown in Fig. 2. This illustrates best what Landau damping means here: because of the frequency spread energy is continuously transferred from the HOM excited MBM to the other MBMs. A narrow band feedback on the $n = 0$ MBM could be applied to damp the LCBI, as is addressed in [4].

Using natural damping we are reversing this idea: *all* MBMs now damp the *single* MBM that actively participates in the LCBI. In fact, the MBMs that do not couple back to the HOM dissipate their energy freely due to δ_{n}. Note that this scheme attacks *any* LCBI.

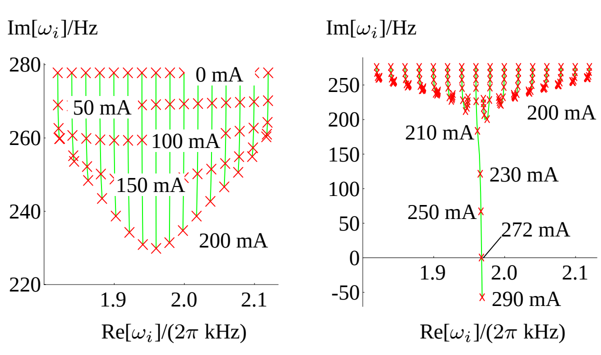

Figure 2: Evolution of the ω_i with I_{b}. $R_{\mathrm{HOM}} = 2\ \mathrm{M}\Omega$, 16 bunches, $\Delta f_{\mathrm{s}} = 300$ kHz.

2.4 *Simplifications for the Calculation of I_{th}*

In the worst case the crest of the HOM impedance will be exactly on the synchrotron sideband, then H is real. Knowing that at the threshold the dominating solution ω of (DR) is real, we have two equations for I_{th} and ω:

$$I_{\mathrm{th}} = \frac{N}{2\delta_{\mathrm{n}}H\omega}\left(\sum_{k=1}^{N}\frac{I_k/I_{\mathrm{th}}}{(\omega_{\mathrm{s}k}^2-\omega^2)^2+4\delta_{\mathrm{n}}^2\omega^2}\right)^{-1} \quad (6)$$

$$0 = \sum_{k=1}^{N}\frac{I_k(\omega_{\mathrm{s}k}^2-\omega^2)}{(\omega_{\mathrm{s}k}^2-\omega^2)^2+4\delta_{\mathrm{n}}^2\omega^2\delta_{\mathrm{n}}} \quad . \quad (7)$$

They are appropriate numerically for small N.

For equally populated bunches and sufficient natural damping, ω will be given approximately by the root mean square $\omega_{\mathrm{s}} = \sqrt{1/N\sum_{k=1}^{N}\omega_{\mathrm{s}k}^2}$. We write the total frequency spread as $\Delta\omega_{\mathrm{s}}$ and assume an even distribution in frequencies. Then in the case of many bunches we can replace the sum in eq. (6) by an integral and find a simple *analytical formula*:

$$I_{\mathrm{th}}(\Delta\omega_{\mathrm{s}}) = \frac{\omega_{\mathrm{s}}}{H}\frac{\Delta\omega_{\mathrm{s}}}{\arctan(\Delta\omega_{\mathrm{s}}\tau_{\mathrm{nat}}/2)} \quad . \quad (8)$$

Eq. (8) is a very good approximation for solutions obtained by the eigenvalue method or eqs. (6) and (7), as long as $\Delta\omega_{\mathrm{s}}/N \ll \omega_{\mathrm{s}}$. Note that

$$I_{\mathrm{th}}(\Delta\omega_{\mathrm{s}})\begin{cases} \rightarrow \quad 2\delta_{\mathrm{n}}\omega_{\mathrm{s}}/H & \text{for} \quad \Delta\omega_{\mathrm{s}} \rightarrow 0 \\ \approx \quad \dfrac{2\omega_{\mathrm{s}}}{\pi H}\left(\Delta\omega_{\mathrm{s}} + 4\dfrac{\delta_{\mathrm{n}}}{\pi}\right) & \text{for} \quad \Delta\omega_{\mathrm{s}} \gg 2\delta_{\mathrm{n}} \end{cases}$$

For $\Delta\omega_{\mathrm{s}} \rightarrow 0$ the well known threshold formula is recovered, for large spreads the contributions of natural damping and Landau damping add up.

2.5 *Threshold Current Calculations*

As can be perceived in fig. 3, Landau damping in conjunction with natural damping is more effective at higher energies and for lower values of R_{HOM}. For the ESRF it allows an increase of the beam current by more than a factor 3. Low energy machines, however, may have difficulties in countering strong HOMs just by using this effect, as it does not change orders of magnitude.

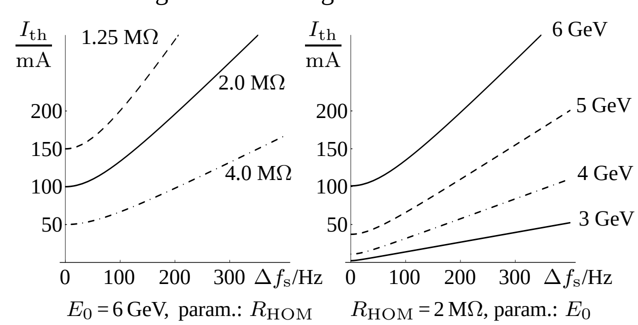

Figure 3: $I_{\mathrm{th}}(\Delta\omega_{\mathrm{s}})$ from eq. (8), $f_{\mathrm{HOM}} = 500$ MHz.

3 EFFECT OF FRACTIONAL FILLINGS

When filling only a fraction of the storage ring circumference, beam loading strongly modulates the cavity voltage. This gives the bunches different zero motion positions (see fig. 4) and different synchrotron frequencies (cf. eq. (2)).

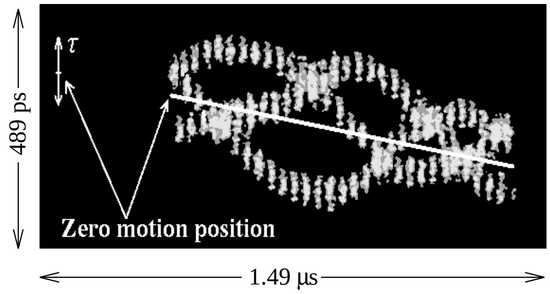

Figure 4: Instability in a 1/3 filling.
Streak camera image, 154 mA, HOM at 911 MHz

The resulting increase of the instability threshold can be calculated with the results of the preceding section if we have quantitative knowledge of the distribution of the ω_{sk}.

Since the beam loading itself is influenced by the position of each bunch, we have contrived an iterative process which converges towards the zero motion positions of all bunches. This fix-point problem has been treated numerically: fig. 5 shows how the overall spread changes with I_b and the filling ratio p.

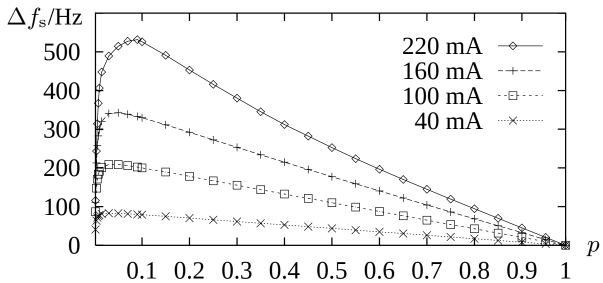

Figure 5: $\Delta\omega_s(p)$, calculated numerically.

Knowing $\omega_{sk}(I_b, p)$, we can determine the threshold current by solving the following equation for I_b:

$$I_b = I_{th}\left[(\omega_{sk}(I_b, p))_{k=1,\dots,N}\right] \tag{9}$$

where I_{th} from eqs. (6) or (8) is used. Fig. 6 shows $I_{th}(p)$ in analogy to fig. 3. Since raising the beam current increases the spread of the ω_{sk}, self-stabilization is observed for smaller fractions and nonviolent HOMs: Landau damping overcomes the normal LCBI growth rate for any current (e.g. at 6 GeV for $p \leq 0.4$, if $R_{HOM} \leq 2$ MΩ).

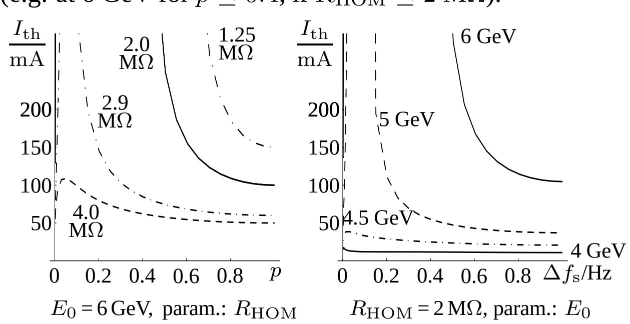

Figure 6: $I_{th}(p)$ from eq. (9), $f_{HOM} = 500$ MHz.

4 EXPERIMENTAL RESULTS

The theoretical results presented so far have been verified by experiments at the ESRF. By means of the temperature control system of the RF-cavities we deliberately tuned HOMs onto a synchrotron sideband. Varying the beam current and observing the presence or absence of longitudinal oscillations (cf. Fig. 4) the threshold current was determined. We present some results for an LCBI due to an HOM at 500 MHz and MBM number 417.

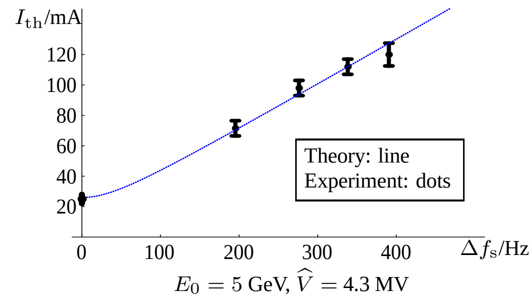

Figure 7: $I_{th}(\Delta f_s)$, theory and experiment.

A direct validation of the results in section 2 was possible at 5 GeV by operating one of the two RF units at $(h+1)\omega_0$, cf. fig. 7 and [2]. The spread in synchrotron frequencies due to the modulation $\Delta\widehat{V}$ is obtained from $\Delta\omega_s/\omega_s = (1+\tan^2\phi_s)\Delta\widehat{V}/(2\widehat{V})$.

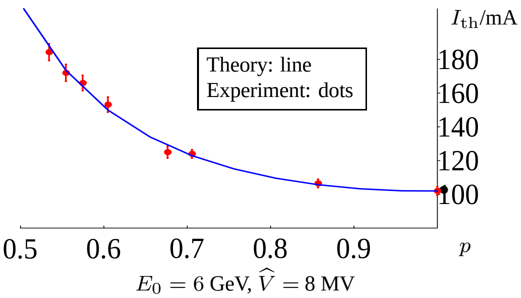

Figure 8: $I_{th}(p)$, theory and experiment.

Fig. 8 shows a striking confirmation of the results from section 3. However, we experienced some deviations for fractions $p \lesssim 0.4$ on strong HOMs around 910 MHz, with measured thresholds below theoretical predictions.

5 CONCLUSIONS AND OUTLOOK

A theory of Landau damping of LCBIs incorporating natural damping was elaborated and the spread in synchrotron frequencies from beam loading due to fractional fillings was computed. This was verified by experiments. Fractional fillings are a simple and efficient way to fight LCBIs in high energy storage rings, our results permit the deliberate choice of the appropriate filling ratio at the ESRF [3]. However, to allow high intensity operation in homogeneous fillings at 6 GeV, direct modulation is envisaged in the near future [2].

6 REFERENCES

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