

EFFECT OF RF PHASE MODULATION ON THE LONGITUDINAL BEAM DYNAMICS

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Abstract

When the frequency of the RF noise is close to the synchrotron frequency, it will cause longitudinal beam instability. In order to study the effect of the instability, a sinusoidal phase modulation was applied to the RF system. The beam instability was observed with a streak camera. The oscillation amplitude and frequency of the beam versus the frequency of applied phase modulation was measured. The beam splitting into two beamlets in the same RF bucket was also observed when the frequency of phase modulation was close to the synchrotron frequency.

1 INTRODUCTION

Third generation synchrotron light sources are low emittance machines. Any small error source may cause instabilities of the beam stored in the ring. The RF system is a dominating factor of the longitudinal beam instabilities. If the arrival time of the particle at the RF cavity relative to the RF wave is modulated, the effect on the synchrotron motion is equivalent to RF phase modulation. Error sources such as RF noise, RF klystron power supply ripple, driven RF phase shifter, synchrotron-betatron coupling or even wake fields could cause RF phase modulation. To study the instabilities caused by RF phase modulation on the beam, a sinusoidal phase modulation was applied to the RF system and a Hamamatsu C5680 streak camera was used to observe the effects on the synchrotron motion at SRRC. In this report we will discuss the longitudinal beam dynamics with RF phase modulation, the experimental set up, the process of experiment and the results. The theoretical prediction is compared with the experimental results.

2 LONGITUDINAL BEAM DYNAMICS WITH RF PHASE MODULATION

We consider the case when the RF phase is modulated by a sinusoidal function $a \sin(2\pi f_m t)$ with amplitude a and modulation frequency f_m or modulation tune $\nu_m = \frac{2\pi f_m}{\omega_0}$, where ω_0 is the revolution angular frequency. The coupled differential equation for the phase deviation relative to the synchronous phase ϕ , and its conjugate δ defined by $\frac{h\eta - \epsilon}{\nu_s E_0}$ with η the slip factor, h the harmonic number, $\nu_s = \sqrt{\frac{h e V_{RF} |\eta \cos \phi_s|}{2\pi E_0}}$ the natural synchrotron tune, ϵ the energy deviation relative to energy E_0 of synchronous particle, become:

$$\frac{d\phi}{d\theta} = \nu_s \delta \quad (1)$$

$$\frac{d\delta}{d\theta} = \frac{h\eta}{2\pi E_0 \nu_s} [e V_{RF} \sin(\phi_s + \phi + a \sin \nu_m \theta) - U] \quad (2)$$

Where $\theta = \omega_0 t$ is the revolution angle, ϕ_s the synchronous phase, and U the radiation energy loss, which depends on the energy of the particles. The equation of motion of the RF phase modulation becomes:

$$\frac{d^2\phi}{d\theta^2} - \frac{4\nu_s^2}{h\eta} |\tan \phi_s| \frac{d\phi}{d\theta} + \nu_s^2 (\sin \phi + |\tan \phi_s| (\cos \phi - 1)) = a\nu_s^2 \sin \nu_m \theta \cos \phi \quad (3)$$

The damping rate of SRRC is about 100 s^{-1} and is much smaller than the synchrotron angular frequency $\omega_s = 1.8 \times 10^5 \text{ s}^{-1}$. So we will neglect the damping term in the following discussion. The Hamiltonian may now be written as

$$H = \frac{1}{2} \nu_s \delta^2 + \nu_s (1 - \cos \phi) + \nu_s |\tan \phi_s| (\sin \phi - \phi) - a\nu_s \sin \nu_m \theta \sin \phi \quad (4)$$

Employing a canonical transformation to the action-angle variables (ψ, J) , the Hamiltonian becomes:

$$H = (\nu_s - \nu_m)J - \frac{\nu_s}{16} J^2 - \frac{\sqrt{2}J}{2} a\nu_s \cos \psi + \Delta H(\psi, J, \theta) \quad (5)$$

with $J = \frac{1}{2}(\delta^2 + \phi^2)$, $\psi = (\arctan(-\frac{\delta}{\phi}) - \nu_m \theta - \frac{\pi}{2})$, Where ΔH is a superposition of terms oscillating at frequency of even harmonics of ν_m [1]. Near the resonance $\nu_s \approx \nu_m$ the dominating term of the Hamiltonian is given by

$$\langle H \rangle = (\nu_s - \nu_m)J - \frac{\nu_s}{16} J^2 - \frac{\nu_s a \sqrt{2}J}{2} \cos \psi, \quad (6)$$

which is a function of the modulation frequency and amplitude. In terms of the variables $\tilde{\phi} = \sqrt{2}J \cos \psi$, $\tilde{\delta} = \sqrt{2}J \sin \psi$ the Hamiltonian becomes:

$$\langle H \rangle = -\frac{1}{64} \nu_s \tilde{\delta}^4 + \frac{1}{64} (32(\nu_s - \nu_m) - 2\nu_s \tilde{\phi}^2) \tilde{\delta}^2 + \frac{1}{64} (-\nu_s \tilde{\phi}^4 + 32(\nu_s - \nu_m) \tilde{\phi}^2 - 32a\nu_s \tilde{\phi}) \quad (7)$$

The local extrema of the Hamiltonian occurs on the $\tilde{\phi}$ axis where $\frac{\partial H}{\partial \tilde{\phi}} = 0$, $\frac{\partial H}{\partial \tilde{\delta}} = 0$. For $\tilde{\delta} = 0$ the Hamiltonian becomes:

$$H = \frac{1}{64} (-\nu_s \tilde{\phi}^4 + 32(\nu_s - \nu_m) \tilde{\phi}^2 - 32a\nu_s \tilde{\phi}) \quad (8)$$

The local extrema are determined by

$$\tilde{\phi}^3 - 16(1 - \frac{\nu_m}{\nu_s})\tilde{\phi} + 8a = 0 \quad (9)$$

Depending on the value of $\nu_m - \nu_s(1 - \frac{3}{16}(4a)^{2/3})$, there are three different kinds of solutions for the cubic equation[2]. For $\nu_m < \nu_s(1 - \frac{3}{16}(4a)^{2/3})$ there are three unequal real roots, corresponding to two stable fixed points(SFP) and one unstable fixed point(UFP) in the $\tilde{\phi}, \tilde{\delta}$ phase space. For $\nu_m = \nu_c \equiv \nu_s(1 - \frac{3}{16}(4a)^{2/3})$ all the roots are real and at least two roots are equal. In this case a SFP and a UFP are merged together. The modulation tune ν_c is called the bifurcation tune. When $\nu_m > \nu_s(1 - \frac{3}{16}(4a)^{2/3})$ only one real solution exists. In summary, as the modulation frequency is increased with fixed modulation amplitude, the outer SFP and UFP moved inward toward the origin in the $\tilde{\phi}, \tilde{\delta}$ space while the inner SFP moved outward. Figure 1 shows the tori-0(tori passing origin) of Hamiltonian in equation(7) with the value of Hamiltonian equals 0 at modulation amplitude 0.02827 and modulation tune $0.9\nu_s, 0.956\nu_s$ and ν_s which presented the cases for $\nu_m < \nu_c, \nu_m = \nu_c$ and $\nu_m > \nu_c$ respectively. The tori-0 for $\nu_m = 0.944\nu_s$ is also a separatrix and passes USF is also shown in Figure 1. The tracking results of equation (1) and (2) under the same conditions of Figure 1 but neglecting the damping effect are shown in Figure 2. Figure 2 shows the effect of time dependent terms ΔH in equation(7) which is neglected completely in Figure 1.

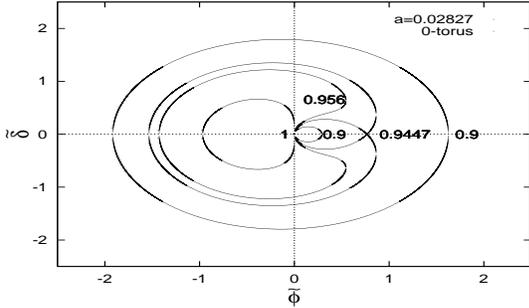


Figure 1: The phase space of RF phase modulation at modulation amplitude 0.02827 and modulation tune 0.9, 0.9447, 0.956 and 1 respectively.

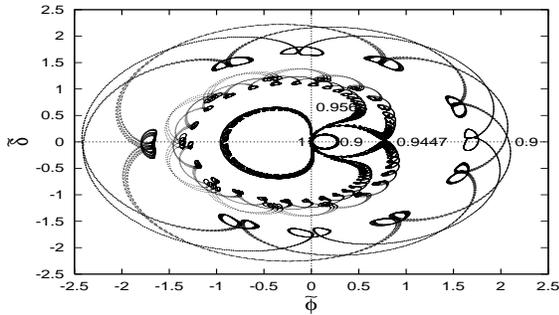


Figure 2: The phase space tracking of RF phase modulation at modulation amplitude 0.02827 and modulation tune 0.9, 0.9447, 0.956 and 1 respectively.

3 EXPERIMENTAL SET UP AND PROCEDURE

The TLS at the SRRC is a 1.3 GeV synchrotron light source. The circumference of the ring is 120m. The revolution time of the ring is 400ns. The frequency of RF cavity is 500 MHz and the harmonic number is 200. The momentum compaction factor of the ring is 6.78×10^{-3} . The natural energy spread is 6.6×10^{-4} . In order to study the beam instability of the ring a Hamamatsu C5680 streak camera was set up at SRRC[3]. In this experiment the synchroscan mode of streak camera combined with the dual time base extender is used to observe the turn by turn electron bunch motion. An amplitude of ± 0.02827 radian phase modulation was applied to the RF system. The source of the phase modulation was generated by a signal generator applied to the phase lock amplifier to generate a phase shift to the RF cavity. In this experiment the single bunch mode beam current was of 1.5 mA and one RF gap voltage was 400kV. The natural synchrotron tune is 8.08×10^{-3} . The RF bucket energy acceptance is 1.029×10^{-2} , which corresponds to 1.696 radian of RF phase angle. The RF bucket acceptance restricts the allowed beam oscillation amplitude. The modulation frequency applied to the beam was roughly scanned from 1 kHz to 40 kHz with 1 kHz step. Then smaller steps with 0.1 kHz per step were swept from 18 kHz to 22 kHz. The behavior of the longitudinal beam motion was recorded with the streak camera.

4 EXPERIMENTAL RESULTS AND DISCUSSIONS

At modulation frequency 18 kHz the beam was observed to start oscillation with a small amplitude. The oscillation amplitude of the beam increased as the modulation frequency was increased. At modulation frequency 19.4 kHz a beam with a large oscillation amplitude started to show up. As the modulation frequency was increased the intensity of beam with large oscillation increased and the intensity of the beam with small oscillation decreased. At modulation frequency 19.8 kHz the beam with small oscillation disappeared. As the modulation frequency was increased further the oscillation amplitude of the large oscillation beam decreased. At modulation frequency 22 kHz there remained very tiny beam motion. The beam motion captured by the streak camera at the modulation frequency from 19.4 kHz to 19.8 kHz was shown in Figure 3. The splitting of the beam into two beamlets were clearly shown in the figure. The oscillation frequency of the beam under the RF phase modulation from modulation frequency 18 kHz to 22 kHz were measured and is displayed in Figure 4. In Figure 4 the tracking results of oscillation frequency at the same RF phase modulation is also shown. The tracking result is an average of hundred particles with different initial values. The oscillation amplitude of beam versus the modulation frequency is shown in Figure 5.

As mentioned in section 2, if the modulation frequency is below bifurcation tune, it is possible for two beamlets to exist in the same RF bucket. Particles with smaller energy

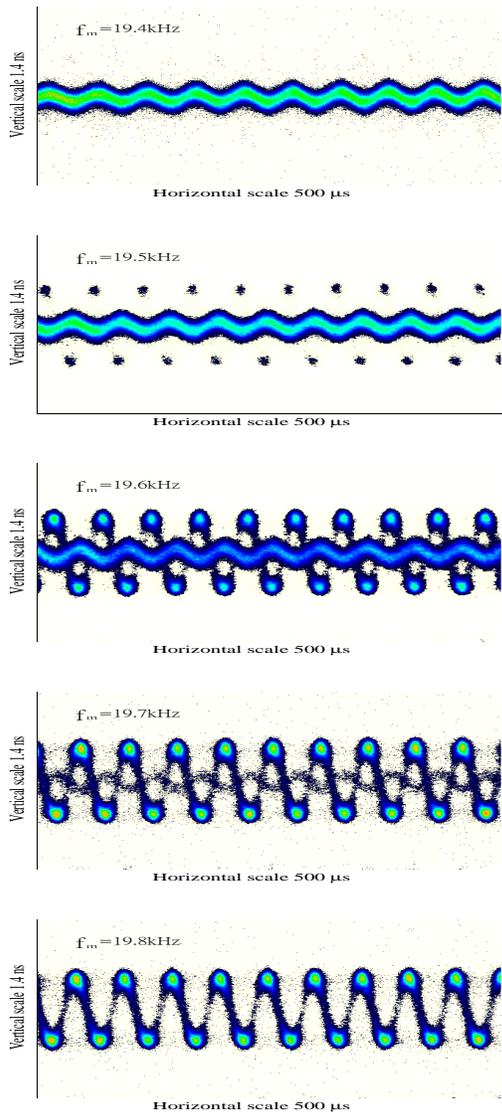


Figure 3: The beam motion observed on the streak camera at modulation amplitude 0.02827 and modulation frequency 19.4 kHz, 19.5 kHz, 19.6 kHz, 19.7 kHz and 19.8 kHz, respectively.

will circulate the inner SFP, while particles with higher energy will circulate the outer SFP. The particles around the inner SFP or outer SFP can interchange through the separatrix. For fixed modulation frequency, the maximum oscillation amplitude of particles around the inner SFP increases as the energy of particles increases, while the maximum oscillation amplitude of particles around the outer SFP decreases as energy increases. For fixed particle energy as shown in Figure 1, the oscillation amplitude of particles around the inner SFP increases as modulation frequency increases. For particles around the outer SFP the oscillation amplitude decreases as modulation frequency increases. At lower modulation frequency the energy and oscillation amplitude of particles around the outer SFP is too large to be accepted in the RF bucket. This explains why at lower

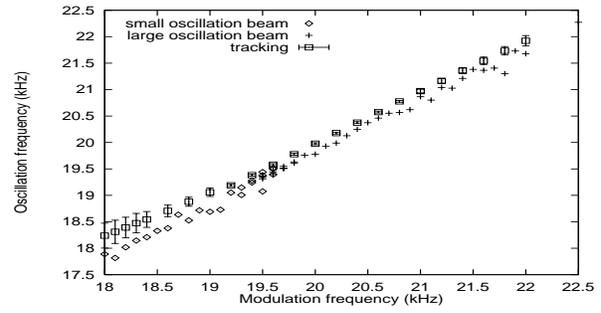


Figure 4: The measured and tracking oscillation frequency at different modulation frequency.

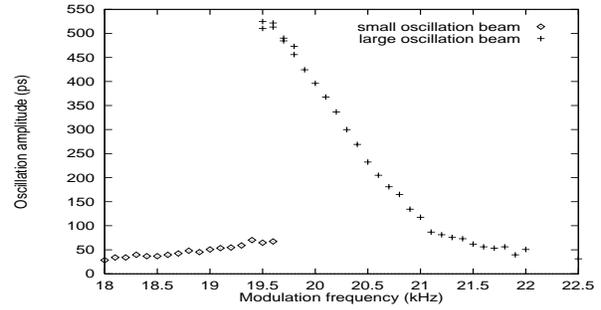


Figure 5: The measured oscillation amplitude at different modulation frequency.

modulation frequency the particles around the outer SFP were not observed. In Figure 3 at modulation frequency 19.5 kHz, the measured large oscillation amplitude was 510 ps which corresponded to 1.6 radian phase oscillation and was below the RF bucket acceptance. From the measurements as shown in Figure 5 the oscillation amplitude of the particles around the outer SFP decreases as modulation frequency increases which is consistent with the discussion in section 2. At modulation frequency 19.8 kHz the small oscillation beam disappeared. However the theoretical bifurcation frequency in this case is 19.3 kHz. The small discrepancy between measured and theoretical values need further investigations.

5 ACKNOWLEDGEMENTS

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6 REFERENCES

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