BEAM DYNAMICS WITH NOISE IN SUPERFERRIC VERY LARGE HADRON COLLIDER ("PIPETRON")

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Abstract

We study transverse and longitudinal beam dynamics in "Pipetron" collider under influence of external noises.

1 INTRODUCTION

Several proposals of the "beyond-LHC" large colliders with 30–100 TeV beam energy and luminosity of 10^{33} – -10^{35} $s^{-1}cm^{-2}$ have been considered in recent years. Two approaches can be distinguished in the trend – namely, smaller circumference ring with high magnetic field dipoles based on high-T_c technology [1], and (presumably) lower cost option of a micro-tunnel low-field superferric magnet machine with large circumference [2]. The later – often referred as "Pipetron" – is a subject of this article. Table 1 shows relevant parameters of the collider [3].

Table 1: Parameters of "Pipetron"

Proton Energy,	E$_p$, TeV	100
Circumference,	C, km	1000
Luminosity,	L, $s^{-1}cm^{-2}$	10^{35}
Intensity,	N_p/bunch	$4.1 \cdot 10^{10}$
No. of Bunches,	N_b	25000
RMS emittance,	$\epsilon_n,\ 10^{-6}\,m$	1
Long. emittance (rms),	A, eV·sec	0.3
Bunch length (rms),	σ_s, cm	10
Rev. frequency,	f_0, Hz	300
Interaction focus	β^*, cm	10
Beam-beam tune shift	ξ_p	0.005

The collider consists of thousands of magnetic elements, and their field imperfections can seriously affect proper machine operation. Depending on the frequency band one can distinguish two mechanisms of beam perturbations in circular accelerator. Slow processes (with respect to revolution period) produce a distortion of the closed orbit of the beam. At higher frequencies (comparable with the revolution frequency), noises cause direct emittance growth.

2 TRANSVERSE EMITTANCE GROWTH

Effect of Transverse Kicks The primary sources which lead to emittance growth in large hadron colliders are quadrupoles (quad) jitter and high-frequency variations of the bending magnetic field in dipoles. Both sources produce angular kicks and excite coherent betatron oscillations. After decoherence time (determined mostly by beam-beam non-linearities, $N_{decoh} = 1200$ turns) filamentation or dilution process due to tune spread within the beam transforms the coherent oscillations into the emittance increase. If the kick amplitude $\Delta\theta$ varies randomly from turn to turn with variance of $\delta\theta^2$, one can estimate the transverse emittance growth as:

$$\frac{d\epsilon_n}{dt} = \frac{1}{2} f_0 \gamma \sum_i^{all\,kicks} \Delta\theta_i^2 \beta_i = \frac{1}{2} f_0 \gamma \delta\theta^2 < \beta > N \quad (1)$$

where $< \beta >$ is the average beta function, γ is relativistic factor, and N is the number of elements which produce uncorrelated kicks. Two major sources of the dipole kicks are fluctuations δB of the bending dipole magnetic field B_0 which give horizontal kick of $\delta\theta = \theta_0(\delta B/B_0)$ ($\theta_0 = 2\pi/N_d$ is bending angle in each dipole, N_d is total number of dipoles); and transverse quadrupole magnets displacements δX (e.g. due to ground motion) which lead to kick of $\delta\theta = \delta X/F$, where F is the quadrupole focusing length.

Non-"white" noise can be described by frequency-dependent power spectral density(PSD) $S_{\delta\theta}(f)$, and causes the emittance growth with rate of [4] $\frac{d\epsilon_n}{dt} = \gamma f_0^2 \sum_i \Big(\beta_i \sum_{n=-\infty}^{\infty} S_{\delta\theta}(f_0|\nu - n|)\Big)$ which consists of the sum of PSDs of angular kicks produced by the i-th source at frequencies of $f_0|\nu - n|$, n is integer, the lowest of them is fractional part of the tune times revolution frequency $f_1 = \Delta\nu f_0$.

Beam lifetime in the Pipetron is about $\tau_c = 5$ hours (determined mostly by longitudinal intrabeam scattering [5] $\tau_{\parallel}^{IBS} \approx 6$ hrs, while synchrotron radiation transverse damping time is about 42 hours). Let us constrain that external noise should lead to less than 10% emittance increase while the beam circulates in the accelerator, then we get tolerable the noise-induced emittance growth rate of $\frac{d\epsilon_n}{dt} \leq 0.1\frac{\epsilon_n}{\tau_c} = 5.6 \cdot 10^{-12}\, m/s$. Taking into consideration 500-m long FODO cell (i.e. $L = 250$m) focusing structure with $\mu = 90^o$ phase advance per cell [3] one can estimate the tune $\nu \simeq 500$, total number of focusing quadrupoles as $N_q = 4000$ and about the same number of dipoles N_d. Now, the acceptable transverse emittance growth rate requires:

a) the PSD of single quadrupole transverse vibration is limited by the value of $\sum_n S_{\delta X}(f_0|\nu - n|) \approx S_{\delta X}(f_0 \Delta\nu) \leq 2 \cdot 10^{-11}\, \frac{\mu m^2}{Hz} = 20\, \frac{pm^2}{Hz}$, where $\Delta\nu$ is fractional part of ν;
b) or the rms amplitude of turn-to-turn jitter of each quadrupole (white noise in frequency band f_0) $\delta X_{rms} \leq 7.6 \cdot 10^{-11}$m;
c) and a tolerable level of bending magnetic field fluctuations to its mean value B_0 in the dipole: $\Big(\delta B/B_0\Big)_{rms} \leq 3.4 \cdot 10^{-10}$.

Measured Ground Motion Let us make a comparison of the above calculated constraints with experimental data on ground motion. Fig.1 presents PSDs of ground velocity

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$S_x(f)(2\pi f)^2$ in units of $(\mu m/s)^2/Hz$ for the USGS "New Low Noise Model" – a minimum of the PSD observed by geophysicists worldwide – and data from accelerator facilities of HERA, KEK, CERN, SLAC, and FNAL (see references in [5]). These spectra indicate that: 1) accelerators are essentially "noisy" places; 2) ground vibrations above 1 Hz are strongly determined by cultural noises – they manifest themselves as numerous peaks in Fig.1; 3) even among accelerator sites the difference is very large, that gives a hint for the Pipetron builders.

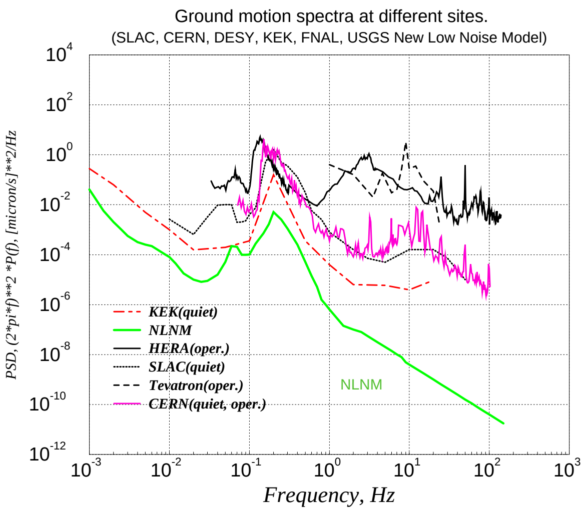

Figure 1: Measured ground velocity spectra.

Below 1 Hz the ground motion amplitude is about 0.3-1 μm due to remarkable phenomena of "7-second hum". This hum is waves produced by oceans – see a broad peak around 0.14 Hz in Fig.1 – with wavelength of about $\lambda \simeq 30$ km. It produces negligible effect on Pipetron, because λ is much bigger than typical betatron wavelength $2\pi\beta \simeq 2$ km. Investigations of spatial characteristics of the fast ground motion have shown that above 1-4 Hz the correlation significantly drops at dozens of meters of distance between points.

Table 2 compares requirements for the Pipetron with three particular tunes $\Delta\nu = 0.18, 0.31$ and 0.45 and experimental data.

Table 2: PSD of Ground Motion (in $(pm)^2/Hz$)

$\Delta\nu$	0.18	0.31	0.45
$f_1 = \Delta\nu f_0$	54 Hz	93 Hz	135 Hz
Pipetron tolerance	**20**	**20**	**20**
SLAC (quiet)	100	-	-
DESY (tunnel)	10^5	7000	1700
CERN (tunnel)	300	20	-

One can see that none of the accelerator data shows vibrations which are less than the Pipetron requirements, although PSDs at higher frequencies (say $f_1 = 135$ Hz) are much less than at lower frequency of 54 Hz, and, therefore, larger $\Delta\nu$ – closer to half integer resonance – are preferable from this point of view. At $\Delta\nu = 0.18$ one needs the vibration power reduction factor of $R = 5 - 5000$. We have not enough experimental data on dipole field fluctuations at 50-150 Hz which may drastically increase the emittance growth.

Feedback System A transverse feedback frequency allows one to suppress the emittance growth caused by excitation of the betatron oscillations simply by damping the coherent beam motion faster then they decohere. The system monitors the dipole offset X of the beam centroid and tries to correct it by dipole kicks θ which are proportional to the offset, applied a quarter of the betatron oscillation downstream. We operate with dimensionless amplification factor g of the system (gain) which is equal to $g = \frac{\theta\sqrt{\beta_1\beta_2}}{X}$, where β_1 and β_2 are the beta-functions at the positions of the pick up and the kicker electrodes respectively. In the limit of $g \ll 1$ the decrement due to the feedback is equal to $\frac{1}{2}f_0 g$, i.e. the amplitude of the betatron oscillations being reduced $1/e$ times after $2/g$ revolution periods. Theory of the feedback (see e.g. [4]) gives the transverse emittance evolution formula:

$$\frac{d\epsilon_n}{dt} = \left(\frac{4\pi\delta\nu_{rms}}{g}\right)^2 \left[\left(\frac{d\epsilon_n}{dt}\right)_0 + \frac{\gamma f_0 g^2}{2\beta_1} X^2_{noise}\right], \quad (2)$$

$g \gg 4\pi\delta\nu_{rms}$, where emittance growth rate without feedback $(d\epsilon_n/dt)_0$ is given by (1), X_{noise} is the rms noise of the system (presented as equivalent input noise at the pick-up position), and $\delta\nu_{rms}$ is the rms tune spread within a beam.

Major source of the tune spread (and, consequently, decoherence) is nonlinear beam-beam force which results in the rms tune spread of $\delta\nu_{BB} \approx 0.167\xi = 8.4 \cdot 10^{-4}$.

Analytical consideration of the feedback system resulted in maximum useful gain factor $g_{max} \simeq 0.3$ – there is no reduction of the emittance growth rate with further increase of g because of higher-(than dipole)-order kicks effect, the system noise contribution grows, while the coherent tune shift due to feedback becomes too large, and affects multibunch beam stability in presence of resistive wall impedance.

Therefore, maximum reduction factor $R_{max} = (g_{max}/4\pi\Delta\nu_{BB})^2$ is about 800 for the Pipetron design parameter of $\xi = 0.005$, while the minimum practical gain which still can lead to the damping is about $4\pi\delta\nu_{BB} \approx 0.01$.

As it is seen from (2), feedback noise also leads to emittance growth and its relative contribution grows as $\propto g^2$. Taking the beta function at the pick-up $\beta_1 = 500$m we get limit on the rms noise amplitude:

$$X_{noise} \ll \left[\frac{2\beta_1(d\epsilon_n/dt)_0}{f_0(4\pi\delta\nu_{BB})^2\gamma}\right]^{1/2} \approx 1.4\,\mu m. \quad (3)$$

Power of the output amplifier of the system depends on maximum noise amplitude of the proton beam oscillations and is estimated to be about 50 kW for a bunch-by-bunch system[5].

RF Phase Noise Turn-to-turn jitter of the RF phase $\Delta\phi$ results in fast momentum variation $(\Delta p/p) = (eV_0/E_p)\Delta\phi$ which leads to an instant change of the horizontal orbit of $\Delta X = D_x(\Delta p/p)$, where D_x is the dispersion function at the RF cavities. Measured $\Delta\phi$ is found to be two orders of magnitude less than estimated tolerances [5] that take the jitter out of list of Pipetron problems.

3 LONGITUDINAL EMITTANCE GROWTH

The RF phase errors at frequencies of the order of synchrotron one $f_s = \nu_s f_0$ and higher lead to the longitudinal emittance growth of:

$$\frac{dA}{dt} = \frac{eV_0}{f_{RF}}\frac{d\phi^2}{dt} = \frac{eV_0}{f_{RF}}2\pi f_0^2 \nu_s^2 S_\phi(f_0\nu_s), \qquad (4)$$

where $\omega_s = 2\pi\nu_s f_0 > 0$, S_ϕ is the PSD of the phase noise

The synchrotron frequency $f_0\nu_s$ varies from 3.1 Hz at the beginning of the ramp to 0.33 Hz at the end of the ramp at 100 TeV, and then it is about 0.076 Hz during the collision time with $V_0 = 20$ MeV RF.

If one requires less than 10% emittance increase during half an hour of ramp time τ_R, than the tolerance on the phase jitter PSD in $f_{RF} = 450$ MHz RF system is $S_\phi(\omega_s) = \frac{0.1Af_{RF}}{\tau_R(eV_0)\pi\omega_s^2} \approx \frac{6.4\cdot10^{-6}}{\omega_s^2}$. Measurements with the SSC RF system HP8662 synthesizer [6] show that in frequency band of 1-100 Hz the PSD of phase noise can be approximated by $S_\phi(\omega_s) = \frac{1.3\cdot10^{-5}}{\omega^{2.65}}$, that is only twice the tolerance at frequencies about 1 Hz. Equivalent rms phase jitter tolerance is $\delta\phi \simeq \sqrt{\omega_s S_\phi(\omega_s)} \approx 0.3$ mrad at $f_s = 3$ Hz.

The same 10% tolerance for 5 hours of the collision operation with $eV_0 = 20$ MeV gives $S_\phi(\omega_s) \approx \frac{1.2\cdot10^{-5}}{\omega_s^2}$ that is very close to the measured PSD.

We can conclude that with minor improvement of the RF phase stability with respect to the SSC synthesizer, no longitudinal feedback will probably be required.

Another possible source of the RF phase errors, the change of the circumference due to non-zero dispersion function D_x at the position of dipole kick [7], is found to give negligible contribution to the emittance growth [5].

4 CLOSED ORBIT DISTORTIONS

Alignment Tolerances The rms closed orbit distortion dX_{COD} is proportional to the rms error dX of quads alignment, and if these errors are not correlated, then in the FODO lattice we can get:

$$dX^2_{COD} = \frac{\beta dX^2}{4sin^2(\pi\nu)}\sum_i \frac{\beta_i}{F_i^2} = \frac{\beta N_q tg(\mu/2)dX^2}{Lsin^2(\pi\nu)}. \qquad (5)$$

Let us take the "safety criteria", i.e. ratio of maximum allowable COD to the rms one, equal to 5, then for maximum COD of dX^{max}_{COD}=1 cm (this is about half aperture of the vacuum chamber) at the focusing lenses where $\beta_F = 765$ m ($L = 250$ m, $\mu = 90^o$) we get requirement on the rms alignment error of $dX \approx 15\ \mu$m (here we take $\Delta\nu = 0.31$). This value sets a challenging task, its solution needs the most sophisticated alignment techniques and two questions arise in this connection: 1) temporal stability of the magnets positions; and 2) applicability of the beam-based alignment.

Slow Ground Motion Numerous data on uncorrelated slow ground motion support an idea of "space-time ground diffusion". An empirical rule that describes the diffusion – so called "the ATL law" [8] – states the rms of relative displacement dX (in any direction) of two points located at a distance L grows with time interval $T < dX^2 >= ATL$, where A is site dependent coefficient of the order of $10^{-5\pm1}\ \mu m^2/(s\cdot m)$.

The ground diffusion should cause corresponding closed orbit diffusion (COD) in accelerators [1] with rms value over the ring approximately equal to $\langle dX^2_{COD}\rangle \simeq 2\sqrt{ATC}$. It clearly shows that the diffusive orbit drift is not very sensitive to the focusing lattice type (only the circumference C plays role), in particular, there is almost no difference between the combined- and separated-function lattices responses on the ATL-like diffusion.

If one applies the ATL law with $A \approx 4\cdot10^{-5}\ \mu m^2/(s\cdot m)$ to the Pipetron (see [5]) then rms COD at $\beta_{max} = 850$ m is equal to $dX_{COD} \approx 800[\mu m]\sqrt{T[hrs]}$. Requirement of "safe" rms COD of 2 mm yields in T=6.3 hours of mean time between necessary realignments to an initial "smooth" orbit. It does not seem to be an easy task to do it mechanically, even with use of robots, especially taking into account 15 μm precision of the procedure. "Beam-based alignment" technique looks as an appropriate method but requires numerous (of the order o the number of quads) correctors with 4.3 Tm maximum strength.

5 CONCLUSIONS.

Preceding consideration shows that natural and man-made vibrations at Pipetron can lead to dangerous transverse emittance growth rate (high-frequency part of spectrum) and closed orbit distortions (at low frequencies). The transverse feedback system can drastically reduce the emittance increase. Sophisticated alignment methods are necessary to keep Pipetron beam on a "golden orbit".

It seems reasonable to carry out "on-site" ground motion studies and magnet vibrations measurements, as well as get data on long-term tunnel movements, the RF phase and amplitude stability, and dipole field jitter.

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