SORTING STRATEGIES FOR THE LHC DIPOLES

R. Bartolini, W. Scandale
CERN, CH-1211 Geneva 23, Switzerland
M. Giovannozzi, E. Todesco
INFN, Sezione di Bologna, V. Irnerio 46, I-40126 Bologna, Italy

1 INTRODUCTION

Unavoidable field-shape imperfections are present in the superconducting magnets of modern hadron colliders, due to mechanical tolerances, persistent currents, design imperfections, coil deformations and iron saturation. The residual multipolar errors, typically of the order of a few units in 10^{-4} at the usual reference radius of 1 cm, have a detrimental effect on the stability of the particle orbits. A sound design of the machine layout, with well focused orbit functions and a suited set of multipolar correctors, may help in improving the beam stability. However, the dynamic aperture (hereafter DA), may strongly depend on the random part of the multipolar imperfections and on the specific distribution of the errors along the machine azimuth. In such a case, sorting strategies can be applied to provide the mutual compensation of the residual errors. This procedure is quite demanding, therefore it is crucial to evaluate in detail the expected beneficial effects on beam dynamics. In fact, all the magnets have to be carefully measured as they are manufactured and qualified in terms of the random imperfections. In addition, a sizeable number of magnets must be available for sorting before the final installation.

Two techniques are proposed for sorting the LHC dipoles. The first one uses the perturbative analysis of the nonlinear betatron motion to find dynamical quantities which allow to evaluate rapidly the DA. These quantities are called Quality Factors (hereafter QF) [1, 2]: the search of a satisfactory rearrangement is performed by generating random permutations of the magnetic elements, selecting the permutation which optimises the QF and hence the DA. The effectiveness of this method depends on the degree of correlation of the QF with the numerical estimate of the dynamic aperture. The second approach is based on the local cancellation of the random errors by pairing the magnets with similar errors in magnitude and sign and placing the pairs in strategic positions along the azimuth of the accelerator. We refer to these methods as deterministic algorithms [3, 4].

In Section 2 we present in detail the sorting strategies. In Section 3 we apply them to a realistic LHC model. Conclusions are drawn in Section 4.

2 SORTING STRATEGIES

2.1 Analytical indicators of the dynamic aperture

A good arrangement of the magnets along the azimuth of the accelerator can be found by comparing a sufficiently large number of random permutations [5]. Each permutation produces a different lattice with a different DA and the search is stopped when a satisfactory value of the DA is found. The direct calculation of the DA is extremely time consuming and allows to explore only a very limited number of permutations. It is therefore necessary to find dynamical quantities which can be computed much faster while being well correlated with the DA, in order to explore more configurations. The perturbative theory of the betatron motion provides a powerful tool to parametrise the effects of non linearities on beam dynamics and to build analytical indicators of the DA. The QF considered in this paper are either the tuneshift or the strength of a certain resonance.

The first step of the sorting procedure is the analysis of the correlation of various QF with the DA. A QF is retained if it allows to better disentangle the good realizations (i.e. the ones with a large DA) from the bad ones. The second step consists in generating a set of random permutations of the errors and for each permutation to compute only the QF. The permutation which gives the minimum value of the QF is selected. The third and last step of this technique consists in the verification of the effectiveness of the sorting strategy. The analysis of the correlation of the QF's with the DA is made typically considering only the 4D betatronic motion, and the short-term DA. We therefore have to check, a posteriori, that the sorted sequence is more stable for long-term tracking, including synchrotron motion and various tune ripple sources. It is also interesting to verify whether the improvement is robust with respect to significant changes of the working point of the machine. This last step will be treated in a separate paper in preparation.

The DA is calculated by tracking particles for 10^3 turns, with different initial conditions on a polar grid in the plane (x, y) assuming zero initial momenta. The coordinates of the last stable particle on each radius are retained and the value of the DA is given by the average of the radius over the different angles. The QF's used are defined in Ref. [2] and they were calculated with the FORTRAN code PLATO [6].

2.2 Deterministic algorithms

Local compensation schemes allow to define deterministic algorithms to rearrange the magnetic elements along the azimuth of the accelerator. They are based on the pairing of magnets with similar random errors which mutually compensate by an appropriate location in the lattice.

<u>Pairing at zero phase advance.</u> Taking into account that two adjacent dipoles have close values of the optical functions and almost the same betatron phase, one can obtain a local compensation scheme by placing in adjacent posi-

0-7803-4376-X/98/$10.00

tions two errors equal in strength but with opposite signs. In the LHC cell the average phase advance between two dipoles is approximately 15 degrees and the optical functions vary by less than 50%. This method will be denoted by Sort-1 in what follows.

Pairing at 180 or 360 degrees. Two equal and opposite errors are quite well compensated at 360 degrees, while equal errors with the same sign cancel at 180 degrees, assuming that the motion is quasi-linear between the two locations. In the LHC, each cell contains 6 dipoles and the phase advance is about 90 degrees. Positions separated by 24 (12) magnets correspond to a phase advance of 360 (180) degrees. In these positions the optical functions are the same. Therefore one can easily obtain quasi-local compensation of the errors. This method will be denoted as Sort-2.

2.3 Mixed techniques

It is possible to define sorting procedures based on more than one of the previous strategies. The pairing of two adjacent magnets can be improved by a compensation at 360 degrees. In this case 4 dipoles are paired. This method will be denoted as Sort-3. Furthermore, the techniques based on the QF's can be used in combination with deterministic algorithms. In this case the compensated pairs of magnets are considered as a new unit and the permutations are generated not on the single magnets but on the pairs. In fact we found very effective to combine the QF's analysis on permutations of blocks of 4 dipoles previously paired with the Sort-3 method. This method will be denoted as Sort-3-QF, where QF is the dynamical quantity which has been minimised.

3 THE LHC MODEL

The LHC model used in our simulation is the old LHC version 2, with the injection optics. It is made of 8 octants, each of them carrying 16 dipoles in the dispersion suppressor region and 144 dipoles in the arcs. Each arc is composed of 24 FODO cells each carrying 6 dipoles. The overall number of dipoles is 1280. The set of 376 chromatic sextupoles is considered in the simulations. The distribution of the random errors is assumed to be Gaussian truncated at 3 σ and the σ of the multipolar coefficients used in the simulation are given in Table 1. We assume that the magnets will be installed as the production goes on. Only a limited number of dipoles will be stored and available for sorting. We applied the sorting strategies on groups of 144 dipoles. Two extreme cases were analysed in detail: dipoles with only random b_3, and dipoles with the full set of random errors given in Table 1.

3.1 Random b_3

We applied the deterministic algorithms to a set of 100 realizations of the random errors. Furthermore we analysed the correlation of the QF's related to the tuneshifts and the strength of several resonances. Several of those were found to be well correlated as shown in Fig. 1(a) and (b). The correlations can vary when the dipoles are paired, i.e. with the scheme Sort-3 as shown in Fig. 1 (c) and (d). Some QF's lose their correlation with the dynamic aperture after the pairing of the dipoles while others are still well correlated. This implies that the process of pairing acts only on particular resonant terms as, for instance, the strength of the resonance $(2,-2)$. The QF's which are still correlated can be used to improve the DA: the norm of the resonance $(1,2)$ has been used to sort a set of 100 random realizations of the errors. The characteristics of the distribution of the DA and the results of the sorting strategies are reported in Table 2. The average value over 100 realizations is denoted by $<\rho>$ and the R.M.S. by σ_ρ. The improvement of the DA due to the sorting of the cases with an initially small value of the DA are denoted as 'Worst Cases' in Table 2. The effect of sorting on the DA can be put in evidence by plotting the relative gain in DA as a function of the DA of the unsorted realizations of the errors, as shown in Fig. 3.

Table 1: Random errors in the LHC dipoles at injection, in unit 10^{-4} referred to a radius of 1 cm.

Order	Normal	Skew
1	-	-
2	0.372	1.227
3	0.882	0.186
4	0.055	0.186
5	0.083	0.041
6	0.014	0.022
7	0.012	0.011
8	0.005	0.005
9	0.003	0.004
10	0.002	0.002
11	0.001	0.001

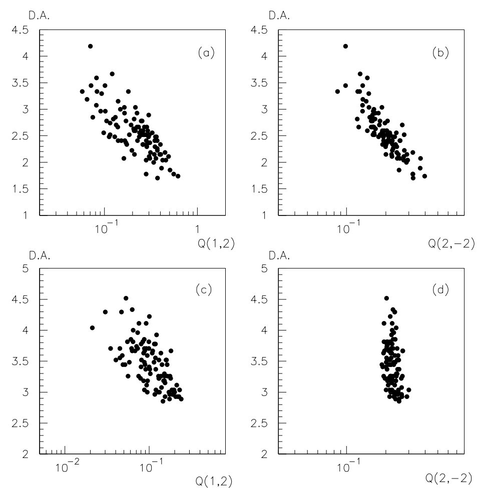

Figure 1: Correlation of DA and QF for 100 random realizations of b_3: (a) norm of (1,2), (b) norm of (2,-2). After the pairing of the dipoles according to the scheme Sort-3, the same correlation plots are shown in (c) and (d) respectively.

Table 2: Characteristics of the DA distribution over 100 random realizations of b_3, with different sorting procedures.

	$<\rho>$	σ_ρ	Gain	Worst Cases
unsorted	2.57	0.44	-	
sort-1	3.24	0.39	26%	74%
sort-2	3.02	0.37	18%	53%
sort-3	3.45	0.38	35%	82%
sort-3-$Q(1,2)$	3.78	0.42	48%	91%

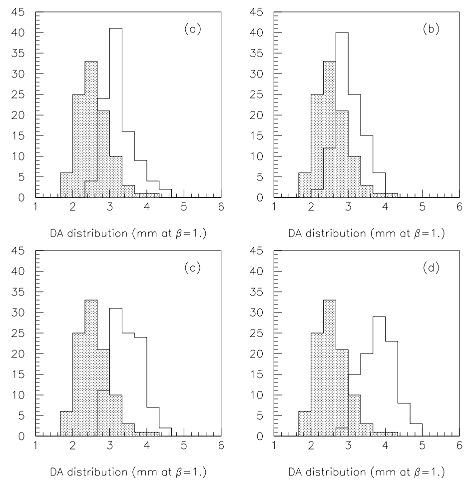

Figure 2: DA distribution over 100 random realizations of b_3 paired and sorted with $Q(1,2)$; the DA is given in normalized mm at β_x = 1 m.

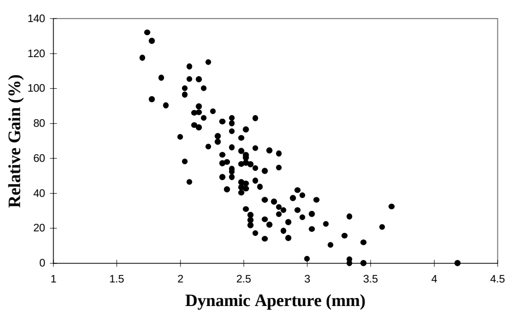

Figure 3: Relative gain as a function of the DA of the unsorted sequence of random errors for 100 realizations. Random b_3.

3.2 All random errors

We applied the deterministic algorithm described in Sec. 2.2 to the case where all the random errors up to the 11-th order are considered in the dipoles. The correlation of the QF's was also investigated generating permutation of the block of magnets previously paired. Owing to the presence of multipoles of order as high as 11, the one turn map has to be calculated at the same order. Resonances up to 10-th order were investigated and the norm of the resonance $(7,-1)$ was chosen to select the best permutation

Table 3: Characteristics of the DA distribution over 100 random realizations of all the random errors, with different sorting procedures.

	$<\rho>$	σ_ρ	Gain	Worst Cases
unsorted	1.57	0.09	-	-
sort-1	1.63	0.07	4%	13%
sort-2	1.64	0.05	4%	15%
sort-3	1.65	0.07	5%	15%
sort-3-$Q(7,-1)$	1.68	0.05	7%	17%

of the blocks obtained from the Sort-3 method. The effects on the DA were investigated on a set of 100 random realizations of the errors and the results are reported in Table 3 and in Fig. 4. The improvement of the dynamic aperture is still non negligible especially for the worst cases.

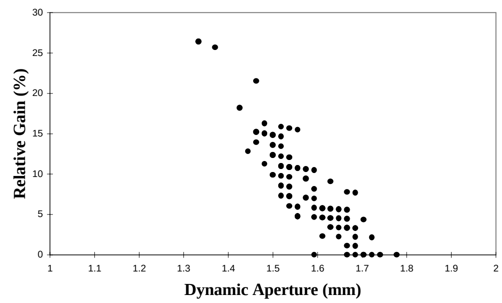

Figure 4: Relative gain as a function of the DA of the unsorted sequence of random errors for 100 realizations. All random errors.

4 CONCLUSIONS

We presented several sorting strategies and their application to a simplified LHC model with only random errors. The effect of sorting on the dynamic aperture is particularly significant for the worst realizations of the random errors, and improvements larger than 25% were found for the most complicated cases where all multipolar errors up to order 11 are included in the dipoles. The analysis of the robustness of these improvements is in progress in more recent LHC lattice models.

5 REFERENCES

[1] F. Willeke, DESY HERA **87–12** (1987).

[2] M. Giovannozzi, R. Grassi, W. Scandale, E. Todesco, Phys. Rev. **E 52**, pp (1995). 3093–3101

[3] W. Scandale, L. Wang, CERN SPS (AMS) **89–22** (1987).

[4] D.M. Ritson, CERN LHC Project Note **53** (1996).

[5] R. Bartolini, M. Giovannozzi, W. Scandale, E. Todesco, CERN LHC Project Note **38** (1995).

[6] M. Giovannozzi, E. Todesco, A. Bazzani, R. Bartolini, CERN PS (AP) **96-12** (1996).

[7] A. Bazzani, E. Todesco, G. Turchetti and G. Servizi, CERN **94–02** (1994).