LONGITUDINAL COUPLED BUNCH INSTABILITIES IN THE JHF 50GeV MAIN RING

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Abstract

We have studied coupled bunch instabilities in the JHF proton synchrotron. We are particularly interested in the Q-dependence of the growth rate because we plan to have a newly developed broad band cavity. Results show that a lower growth rate can be achieved by broad band cavity.

1 INTRODUCTION

In the multi-bunch operation, each individual bunch can be coupled through RF cavities. This interaction causes the divergence of longitudinal and transverse bunch oscillations, called Coupled Bunch Instabilities (CBI). These instabilities are very serious problem for high intensity synchrotrons where the beam induced wake fields are large. We discuss CBI only in longitudinal motion because it is expected to be worse than the transverse counterpart.

The wake field which causes CBI comes from parasitic resonance at frequency larger than a few times of RF frequency. One way to avoid CBI is to use a feedback system and cancel the wake field.

Another possible way to avoid CBI is to use a broad band RF cavity (BBC), which is expected to damp out the wake field quickly. The BBC can be realized by using a magnetic core with a low Q (quality factor) value like FINEMET [3] whose Q is about 1. We have calculated the Q dependence of CBI to prove that the BBC is effective to CBI.

2 LINEAR THEORY OF COUPLED BUNCH INSTABILITIES

2.1 Pointlike bunch

Longitudinal coupled bunch motion can be represented by the superposition of N_B independent modes, where N_B is the number of bunches. If all bunches are equally populated in the ring, the longitudinal motion of N-th bunch in the μ-th mode can be written as

$$d\phi = \exp\left(i\mathbf{\Omega}^{(\mu)}t + 2\pi i\mu\frac{N}{N_B}\right). \tag{1}$$

Here $d\phi$ is the displacement of bunch position in longitudinal direction, $\mathbf{\Omega}^{(\mu)}$ the synchrotron frequency in the presence of coupling between bunches, -Im$\mathbf{\Omega}^{(\mu)}$ the growth rate of CBI. We assumed that the amplitude of synchrotron oscillation is very small.

Equations of longitudinal bunch motion are

$$\frac{d\phi}{dt} = -\eta\omega_{rf}\frac{\delta P}{P_0} \tag{2}$$

$$\frac{dU}{dt} = \frac{\omega_{rf}}{2\pi h}(eV_{rf}\sin(\phi_0+\delta\phi)+U_{wake}) \tag{3}$$

which lead to the solution

$$-\mathrm{Im}\mathbf{\Omega}^{(\mu)} = -\frac{e^2N\eta}{2\beta^2U_0\omega_sT_0^2}\times \mathrm{Re}\left\{\Delta\omega^{(\mu)}Z(\Delta\omega^{(\mu)}) + \sum_{k=1}^{+\infty}\left(\omega_k^{(\mu)+}Z(\omega_k^{(\mu)+}) - \omega_k^{(\mu)-}Z(\omega_k^{(\mu)-})\right)\right\}. \tag{4}$$

Here $Z(\omega)$, the impedance, is the Fourier transformation of wake function $W(t)$, β the speed of particle, U_0 the synchronous energy, ω_s the synchrotron frequency, T_0 the revolution period, η the slippage factor, N the number of particles in the whole ring and $\omega_k^{(\mu)\pm}$ is the coupled bunch sideband at k times RF frequency, which is defined by

$$\begin{aligned}\omega_k^{(\mu)\pm} &= k\omega_{rf}\pm\Delta\omega^{(\mu)} \\ &= k\omega_{rf}\pm(\mu\omega_0+\omega_s)\end{aligned} \tag{5}$$

We will study the Q-dependence of growth rate with equation (4).

We assume the impedance of RF cavity to be that of LCR circuit, which is

$$Z_0(\omega) = \frac{R}{1+iQ(\omega_\mathrm{r}/\omega-\omega/\omega_\mathrm{r})}, \tag{6}$$

where ω_r is the resonant frequency and Q is quality factor. Then the wake field excited by a passage of pointlike charge at the light speed is

$$W_0(t) = 2e\alpha Re^{-\alpha t}\left(\cos\overline{\omega}t + \frac{\alpha}{\overline{\omega}}\sin\overline{\omega}t\right), \tag{7}$$

where e is the unit charge, $\overline{\omega}$ and α are real part and imaginary part of complex resonant frequency $\boldsymbol{\omega}_\mathrm{r}$ of the cavity, respectively.

2.2 Bunch with a Gaussian distribution

In previous subsection, we assumed the bunches as point charges. However, a real bunch has a finite length both in the longitudinal and transverse direction. If we take the longitudinal bunch length into account, we have to calculate the growth rate of CBI using effective impedance which is roughly obtained by the product $Z_0(\omega)\tilde{I}^2(\omega)$, where $\tilde{I}(\omega)$ is the bunch spectrum. For a Gaussian bunch, it is

$$Z(\omega) = Z_0(\omega)e^{-\omega^2\sigma^2}, \tag{8}$$

where σ is bunch length and

$$\omega_p = pM\omega_0 + \mu\omega_0 + \omega_s.$$

We assume that the bunching factor is 0.3, so that σ is about 35nsec. Figure 1 shows the real part of $\omega Z(\omega)$, for $Q = 1$ and $Q = 30$. The peak height of impedance is gradually falling down with ω. Thus, the parasitic resonance at sufficiently high frequency is not important.

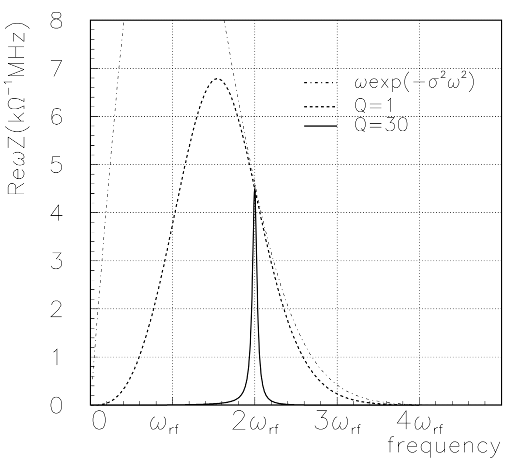

Figure 1: The real part of $\omega Z(\omega)$ for the parasitic resonance at two times of ω_{rf}, where Z is the effective impedance, calculated from Eq. (8). The bunching factor is 0.3.

kinetic energy	$K = U_0 - M = 3GeV$
number of particles	$N = 2 \times 10^{14} ppp$
harmonics	$h = 17$
RF voltage	$V_{RF} = 300kV$
RF frequency	$\omega_{rf}/2\pi = 3.43MHz$
slippage factor	$\eta = -0.0577$
synchrotron freq.	$\omega_s/2\pi = 717Hz$

Table 1: The parameters of JHF 50GeV ring at flat bottom.

3 RESULTS

First, we investigate the $\omega_{\rm r}$ dependence of the growth rate of longitudinal CBI when $Q = 1$, 5 and 30 (Fig. 2) where the parameters of the JHF 50GeV ring at the injection energy as tabulated in Table 1 are used. We assumed the bunching factor to be 0.3. The figure shows, for example, the parasitic resonance with $R = 10k\Omega$ at about 3 times of RF frequency 'causes CBI with of 10sec^{-1} when $Q = 30$ and 0.1sec^{-1} when $Q = 1$, respectively.

Secondly, we investigate the Q-dependence. The maximum value among the growth rate for each region of ω_r is shown in Fig. 3. It is clear that the growth rate of the CBI is drastically reduced when the Q value of the parasitic resonance is less than 5.

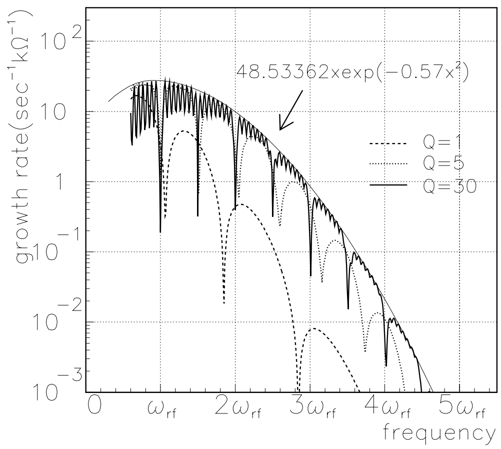

Figure 2: The growth rate of the longitudinal CBI vs frequency of parasitic resonance. ω_{rf} is the RF frequency.

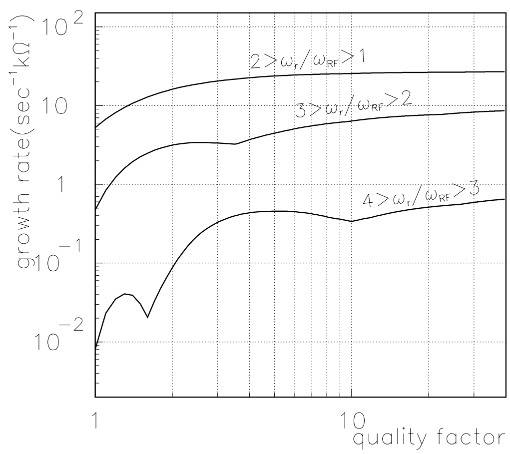

Figure 3: The growth rate of the longitudinal CBI as a function of quality factor. We assumed a parasitic resonance at each ω_r region and plotted the envelope.

4 CONCLUSION

We have calculated a growth rate of longitudinal coupled bunch instability for the JHF 50GeV proton synchrotron. It is found that in order to avoid CBI, the BBC is very useful. The BBC can be realized with FINEMET.

In the case of the flat bottom of JHF main ring, the growth rate becomes large and constant for the region $Q > 5$, but becomes lower quickly at $Q < 5$.

5 REFERENCES

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[3] M. Fujieda *et al*: Studies of Magnetic Cores for JHF Synchrotorns, these proceedings.