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TUNE-SHIFT WITH AMPLITUDE DUE TO NONLINEAR KINEMATIC EFFECT*

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Abstract

Tracking studies of the Muon Collider 50 on 50 GeV collider ring [1] show that the on-momentum dynamic aperture is limited to around 10σ even with the chromaticity sextupoles turned off. Numerical results from the normal form algorithm show that the tune-shift with amplitude is surprisingly large. Both analytical and numerical results are presented to show that nonlinear kinematic effect originated from the large angles of particles in the interaction region is responsible for the large tune-shift which in turn limits the dynamic aperture. A comparative study of the LHC collider ring is also presented to demonstrate the difference between the two machines.

1 INTRODUCTION

Nonlinear effects have been studied in detail throughout the history of alternating gradient accelerators. Among the different approaches [2], the canonical perturbation theory developed and introduced to the accelerator community by Moser [3] has become the *de facto* standard method to treat resonances analytically (for early reviews, see references [4, 5, 6]). Over the years, calculations of first [6], second [4, 5, 8] and, at least in one instance [7], third order perturbations were carried out. Among all the works mentioned above, one common assumption is paraxial approximation, which assumes that $p_{x,y}/p_0 \ll 1$ and keeps only quadratic terms of $p_{x,y}/p_0$ in the Hamiltonian. Until recently paraxial approximation, together with the canonical perturbation theory, has proven itself rather effective to describe single particle dynamics of high energy accelerators. One of the most important prediction of paraxial approximation is that, free of nonlinear field, the motion of a particle is purely linear, which entails that stability, once established, is global.

With the advance of the idea of muon colliders [1, 9], new constraints are placed on the collider rings due to the finite muon life time. In order to reach the desired luminosity, β^* and circumference have to be as small as possible. To remove detector background from muon decays and subsequent showers, the inner triplet of the low-β insertion has to be kept at a distance to the interaction point (IP). For example, the β^* and the distance from the IP to the inner triplet of the 50 on 50 GeV muon collider are 4 cm and 4.5 m, respectively. As a comparison, those of the Large Hadron Collider are 0.5 m and 23 m. In this report, numerical results are presented to demonstrate that paraxial approximation breaks down in the case of the 50 on 50 GeV muon collider ring. A first-order perturbation theory beyond paraxial approximation is developed to explain the simulation results and establish algebraic relations between tune-shift with amplitude and the parameters mentioned above. Suggestion of a possible correction scheme is made in Section 4.

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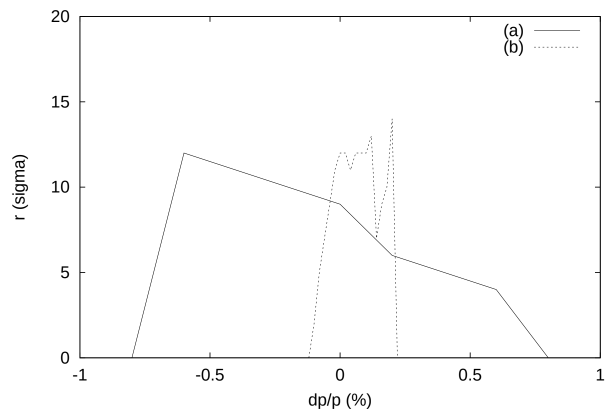

Figure 1: Dynamic aperture of the 50 on 50 GeV collider ring, with sextupoles (a) on and (b) off. Tracking was done using the code COSY INFINITY [14]

2 NUMERICAL EVIDENCE

It is well known that an accelerator free of errors can have a very large dynamic aperture for on-momentum particles. The reason is that the only nonlinear elements present, i. e. the chromaticity sextupoles, start to contribute to the tune-shift with amplitude only through the second-order perturbation, although they drive third-order resonances directly through first-order perturbation. Therefore, the tunes of the particles can be far away from major resonances at very large amplitude, when, for most purposes, the center tunes are so chosen. Taking again the Large Hadron Collider as an example, the dynamic aperture of its ideal collision lattice is well above 30 σ. Hence it was rather unexpected to learn that the dynamic aperture of the on-momentum particles is always below 10 σ, independent of the actual layout of the machine [10]. Furthermore, this is qualitatively true even when the sextupoles are turned off (see Fig. 1). Similar behavior was observed by the author using rather different arcs and Ohnuma [12] using a Runge-Kutta integrator for the inner triplet and a linear matrix for the rest of the ring. This study shows definitively that the limit of dynamic aperture lies in the interaction region.

An other indicator of the strength of nonlinearity is the tune-shift with amplitude. Fig. 2 depicts the tune-shift with amplitude obtained from DA normal form calculations. It is rather clear that even the lowest order contribution to the tune shift is very large, to the extent that it is hard to be-

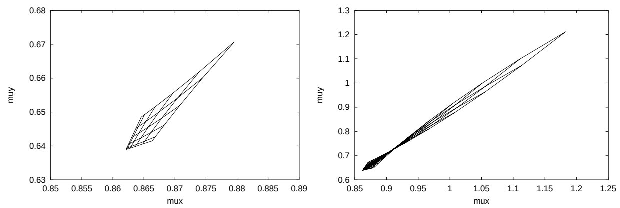

Figure 2: Tune footprint of the 50 on 50 GeV collider ring obtained from DA normal form calculations using the third-order one-turn map. Left plot: The lower left corner of the diagram represent the center tunes; the upper right corner represent the tunes of the particle with $x, y = 5\sigma$, $p_x, p_y = 0$; the lower right corner represent the tunes of the particle with $x = 5\sigma$, $y, p_x, p_y = 0$. Right: Same as the left one except that the maximum displacement is 10σ.

lieve that it actually reflects reality. In fact, it appears from the numerical results that the Taylor serieses of the tunes as functions of normal coordinates are divergent, which shows how strongly paraxial approximation has been violated.

3 FIRST-ORDER PERTURBATION THEORY

Since there is no nonlinear field in the interaction region, the only source of nonlinearity is the high-order terms in $P_s = \sqrt{P^2 - P_x^2 - P_y^2}$, which demonstrates that for the 50 on 50 collider ring paraxial approximation is no longer valid. To move beyond paraxial approximation, we go back to the exact Hamiltonian, which is, following the notation in [11],

$$H = -p_s = -\frac{e}{c}A_s - \left(1 + \frac{x}{\rho}\right)\sqrt{(P^2 - P_x^2 - P_y^2)}, \quad (1)$$

where p_s and A_s are the longitudinal momentum and the vector potential, respectively. Note that Eq. (1) is exact only when there is not electric field present and fringe field is neglected. Since only the interaction region is of our interest, $\rho \to 0$. Furthermore, P is set to P_0, for only on-momentum particles are considered. Finally, after normalizing P_x, P_y and H by P_0 and expanding P_x and P_y to the fourth power, the Hamiltonian becomes

$$H = \frac{1}{2}P_x^2 + \frac{1}{2}P_y^2 - \frac{1}{2}K_1(s)x^2 + \frac{1}{2}K_1(s)y^2 + \frac{1}{8}\left(P_x^2 + P_y^2\right)^2. \quad (2)$$

To develop a canonical perturbation theory, it is more convenient to use action-angle variables, which transform the Hamiltonian to

$$H_1 = \frac{J_x}{\beta_x} + \frac{J_y}{\beta_y} - \frac{1}{2}\left[\frac{J_x}{\beta_x}(\sin\theta_x + \alpha_x\cos\theta_x)^2 + \frac{J_y}{\beta_y}(\sin\theta_y + \alpha_y\cos\theta_y)^2\right]^2. \quad (3)$$

After straightforward derivation, the angle independent part

	LHC	MC
L (m)	46	9
β^* (m)	0.5	0.04
Lγ^{*2} (1/m)	184	5625
ϵ_{rms} (π mm mrad)	0.168	0.0005
$\Delta\nu_x$ (10σ)	2.7e-7	0.0028

Table 1: Tune shifts are calculated assuming $x = 10\sigma$ and the other coordinates equal 0.

of the Hamiltonian is obtained, which is

$$H_{10} = \frac{J_x}{\beta_x} + \frac{J_y}{\beta_y} + \frac{1}{2}\left(\frac{3}{8}\gamma_x^2 J_x^2 + \frac{1}{2}\gamma_x\gamma_y J_x J_y + \frac{3}{8}\gamma_y^2 J_y^2\right). \quad (4)$$

As a result, the tunes are

$$\nu_x(\vec{J}) = \nu_{x0} + \frac{1}{2\pi}\int_0^C \left(\frac{3}{8}\gamma_x^2 J_x + \frac{1}{4}\gamma_y^2 J_y\right), \quad (5)$$

$$\nu_y(\vec{J}) = \nu_{y0} + \frac{1}{2\pi}\int_0^C \left(\frac{1}{4}\gamma_x^2 J_x + \frac{3}{8}\gamma_y^2 J_y\right), \quad (6)$$

where C is the circumference of the machine.

Since γ is inversely proportional to β, the drift space between the inner triplets may contribute significantly to the tune-shift with amplitude. Although numerical results show that it constitutes a small fraction of the total tune footprint, it can be used as a parameter to roughly estimate the validity of paraxial approximation. Since γ remains constant throughout a drift space, the tune-shift with amplitude given by Eq. (6) is greatly simplified, which are

$$\Delta\nu_x = \frac{1}{2\pi}\left(\frac{3}{8}\gamma_x^2 J_x + \frac{1}{4}\gamma_y^2 J_y\right)L, \quad (7)$$

$$\Delta\nu_y = \frac{1}{2\pi}\left(\frac{1}{4}\gamma_x^2 J_x + \frac{3}{8}\gamma_y^2 J_y\right)L, \quad (8)$$

where L is the length of the drift. Table 3 illustrates the drastic contrast between the Large Hadron Collider and the 50 on 50 GeV muon collider. Note that the tune shift of the muon collider listed here is comparable to the beam-beam tune shift of the Large Hadron Collider. [**14]**

4 CONCLUSION

Numerical simulations show that paraxial approximation is not valid for the 50 on 50 GeV muon collider. First-order canonical perturbation theory is developed taking into account the next to leading order terms in P_x and P_y. With respect to the correction of the first-order tune shift, octupole pairs can be placed at high β locations with 45° phase separation, same polarity and strengths adjusted properly to cancel the tune-shift without driving resonances. It is most desirable to place the pair at locations where β_x/β_y are equal, because all resonances driven by octupoles can be completely suppressed. Otherwise only partial suppression can be achieved.

5 ACKNOWLEDGEMENTS

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