An Advanced Electromagnetic Eigenmode Solver for Vacuum Electronics Devices - CTLSS *

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Abstract

The Cold-Test and Large-Signal Simulation code (CTLSS), a design tool for vacuum electronics devices, is presented. The prototype tool is a three-dimensional, frequency-domain cold-test code that operates on a rectangular structured grid. It uses a generalisation [1] of the Jacobi-Davidson algorithm [2] that has proven effective in solving for eigenmodes in problems having sharp-edged structures with materials having large dielectric constants and loss tangents as high as 100%. We present the CTLSS algorithm and code features that are useful for vacuum electronics design. Analysis of both closed cavities and periodic slow-wave structures are given. Tests indicate that the CTLSS algorithm can determine mode frequencies to well below 0.1% accuracy for all modes computed. A new formulation has been implemented to compute the complex axial wavenumber, k_z, in a periodic waveguide, as the eigenvalue for a specified real frequency, and test results will be presented. This code is being extended to include an unstructured mesh for the conformal representation of structures using high order elements [3].

1 INTRODUCTION

CTLSS provides fully 3-dimensional eigenmode analysis of complex electromagnetic structures, in particular where strongly absorbing dielectric materials are present. This requirement arises in the design of some vacuum electronic components and devices, and in wider fields where control of mode structure and attenuation in waveguides and cavities is important. Our implementation of the Jacobi-Davidson algorithm for determining the eigenfrequencies underlies the ability to handle lossy systems, and previous tests [1, 4] have indicated that material losses as large as $\tan\delta$=1 are permissible. We summarise here the algorithms used by CTLSS, and describe recent developments.

2 ELECTROMAGNETIC MODEL

We use the following field eigenvalue equation, derived directly from Maxwell's equations for oscillatory fields,

$$\left\{\mathrm{curl}\,\mu^{-1}\mathrm{curl} - \omega_n^2\,\epsilon\right\}\vec{E}_n = 0 \qquad (1)$$

The solutions of this equation includes infinitely-degenerate, zero-frequency solutions having electric field eigenvectors that may be expressed as the gradient of an arbitrary scalar field. These solutions have proved problematic historically, and care must be taken to ensure that they remain completely decoupled from the desired solutions. The discretisation process that we use is derived so as to ensure that the corresponding eigenvalues of the discretised problem remain exactly zero.

2.1 Structured grid cold-test model

The present implementation of CTLSS uses a structured orthogonal grid to optimise the matrix computation, since it is the time spent computing the action of the operator on a field vector that dominates the computation. We use the method of contour-path integration to discretise the continuum field equations in local cell-coordinates, for which it corresponds to the Yee formulation. This method permits us to use non-uniform grids in arbitrary orthogonal coordinate systems to best match the geometry being modelled.

2.2 Non-Hermitian matrices

When all materials are lossless, the matrices that result are Hermitian for any phase advance of a periodic system, and therefore the eigenfrequencies are real-valued and the eigenfields are orthogonal. However, if absorbing materials exist in the computational domain, represented by complex values of the material constants, ϵ and μ, then the matrix equations are non-Hermitian. The eigenvectors are not orthogonal and the eigenfrequencies are complex-valued. For the particular cases of 0° or 180° phase advance, or if the system is a closed cavity, the matrices are complex-symmetric. However, if arbitrary phase-advance boundary conditions are specified, then the matrices have no explicit symmetry. In any case, methods of solution that can handle non-Hermitian systems are necessary. We have adapted the Jacobi-Davidson algorithm to solve for the eigenfrequency solutions of problems of this type.

3 JACOBI-DAVIDSON ALGORITHM

In outline, the Jacobi-Davidson method is an iterative subspace method, in which the large matrix problem to be solved is projected onto smaller subspaces to obtain estimated eigensolutions at each iteration. The subspaces are extended by applying an orthogonal correction procedure to selected eigensolution estimates. The correction vectors are used to *extend* the subspaces, to promote an improved

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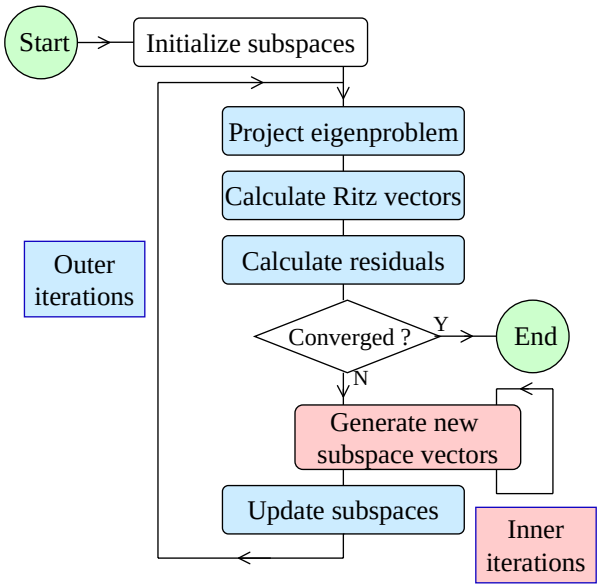

Figure 1: Jacobi-Davidson algorithm

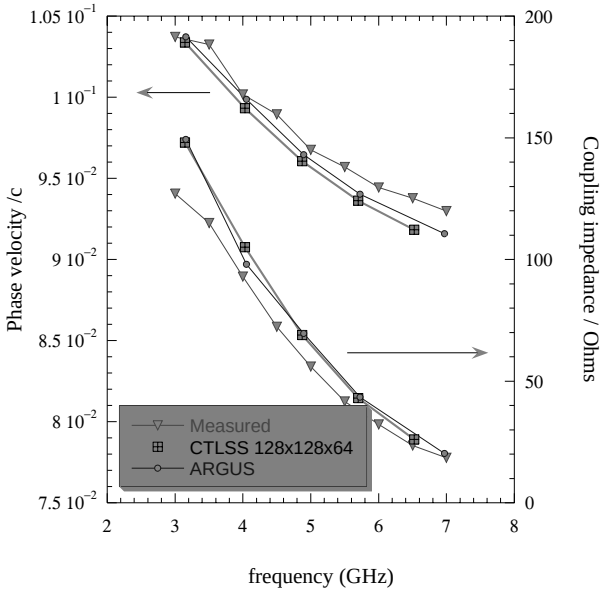

Figure 2: Helix-waveguide dispersion and coupling impedance data, compared with experimental data and with calculations using the ARGUS code

solution estimate at the next iteration. Typically, this procedure is restarted after some fixed number of iterations to limit the maximum subspace size that must be stored. After a number of iterations, the subspace is contracted to include only a few of the solution estimates that lie closest to the desired eigensolutions, and the process of subspace expansion recommences. In place of this approach, we employ a strategy in which eigenvectors associated with the eigenvalue estimates furthest from a target eigenvalue are removed from the subspaces at each iteration and replaced with the update vectors derived from the closest estimates. This maximises the retention of information in the subspace that can contribute to the convergence of the algorithm.

We may choose a target frequency in the centre of our frequency band of interest, and are therefore not limited to finding the lowest frequency eigensolutions. This enables us to exclude all of the zero-frequency eigensolutions from the computation, and also to find just a few selected solutions from within a complex spectrum.

Figure 1 summarises the iterative procedure implemented by the Jacobi-Davidson algorithm. The large eigenproblem is projected onto a pair of subspaces (represented by small sets of orthogonal vectors), and the resulting reduced eigenproblem is solved for eigenvalue and eigenvector estimates. The Ritz vectors are these vectors projected back into the full solution space, and the residual error vectors characterise the deviation of the Ritz vectors from true solutions of the full eigenproblem. The vector subspace corrections are derived from the residuals associated with solutions closest to a target frequency, using an orthogonal correction step [2]. This step requires an approximate solution of a linear system of equations, and may be implemented as a few iterations of an iterative procedure, for which we use the quasi-minimal residual (QMR) method [5]. The correction vectors then replace the unwanted Ritz vectors in the subspaces, and the procedure is repeated until convergence is obtained.

For systems that are not Hermitian, the left and right eigenvectors corresponding to an eigenvalue will in general not be equal, and separate subspaces should in principle be maintained and updated. For complex-symmetric matrices, the left-hand subspace may be taken as the complex conjugate of the right-hand subspace, and the symmetry is maintined throughout the calculation to reduce the associated memory requirement. For fully non-symmetric matrices, it is still possible for this type of problem to use a single subspace, but it is necessary to use a non-symmetric iterative solver for the linear system. The transpose-free variant of the QMR routine (TFQMR) was found to be ideal for this purpose.

Figure 2 shows the results of a calculation modelling a single period of a helix-waveguide structure in which a helical wire is supported by dielectric vanes inside a cylindrical waveguide to create a slow-wave structure. The calculation using CTLSS of the dispersion of the phase velocity, ω/k_z, of this system is shown in comparison with experimental values, provided by Northrop-Grumman, and with a similar calculation using the ARGUS code. The second set of curves shows the coupling impedance parameter derived from the lowest frequency eigenfield solution.

4 DRIVEN FREQUENCY ALGORITHM

An alternative formulation has been implemented to determine the eigenmodes of a waveguide. Instead of finding the eigenfrequencies for known values of k_z, it is useful when losses are present to find instead the complex valued $k_z(\omega)$

at given real frequencies.

In this method, we excite the waveguide with a trial source current $\vec{J}(\omega, k_z) \propto \exp(ik_z z)$ representing the charged particle beam with a fixed interaction frequency and spatial variation. We can solve directly for the electric field distribution in the waveguide by discretising the equation

$$\{\text{curl}\ \mu^{-1}\ \text{curl} - \omega^2\ \epsilon\}\ \vec{E} = i\omega\vec{J} \qquad (2)$$

and using QMR to solve for the field $\vec{E}$. From this solution, we define an impedance function in terms of the fields, for total beam current I,

$$Z(\omega, k_z) = \frac{1}{I^2}\int dx^3 \vec{J}^* \cdot \vec{E} \qquad (3)$$

This function has a pole when k_z approaches the resonance eigenvalue, k_z^*, according to

$$Z(\omega, k_z) = \frac{N}{k_z - k_z^*} + C + O(k_z - k_z^*) \qquad (4)$$

A search over k_z for the roots of Z^{-1} therefore provides the eigenwavevector solution. In addition, the coupling impedance may be derived from the value of N, while the value of the constant C characterises the coupling to the space-charge field. These parameters may be determined by evaluating Z close to the resonance, and passed to a large-signal code for a full non-linear analysis of the particle/wave interaction. This method offers an alternative means of solving the eigenwave problem that could offer a speed advantage over the standard matrix eigenmode analysis for this type of problem.

Figure 3 shows the function Z^{-1} evaluated for a range of phase-advance angles across each period, $\theta = k_z L$. It is clear that the function is smoothly varying, and therefore amenable to automated root-finding. The lower graph shows dispersion data $\omega(k_z)$ calculated using the Jacobi-Davidson method. The roots of Z^{-1} above correspond to the propagating modes at the frequencies chosen.

5 LARGE-SIGNAL MODEL

The 3-D eigensolver module of CTLSS has been linked to the non-linear parametric simulation code CHRISTINE [6]. Using a 1-dimensional parametric model ideal for rapid design optimisation, this code calculates the non-linear interaction between a travelling electromagnetic wave and a beam of electrons that is the basis for wave amplification in a class of vacuum electronic devices. From the optimum 1-D solution, a trial structure is generated automatically for full 3-D analysis using CTLSS that may be adjusted to obtain the realistic travelling wave dispersion and coupling impedance characteristics closest to the design values. These values may be fed back to the non-linear code to predict the actual device efficiency and complete an cycle of the design procedure. This methodology provides a means to predict the true non-linear characteristics of a complex 3-dimensional device. This application of the CTLSS code is presently being tested.

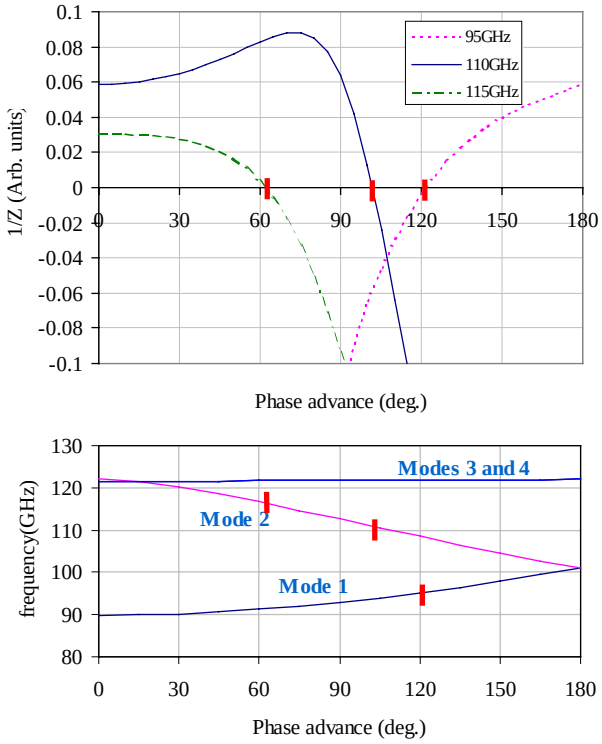

Figure 3: Correspondence between the eigenmode dispersion solution and the poles of the driven frequency impedance function for a coupled-cavity waveguide

6 CONCLUSION

The CTLSS code has been demonstrated as a useful design tool in the simulation of complex electromagnetic structures for applications in particle beam devices.

7 REFERENCES

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