# LIMITATIONS AT SHORT BUNCH LENGTH MONITORING

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#### Abstract

Analysis of main limitations on time resolution inhering in techniques for bunch length or bunch phase distribution monitoring with resolution in subpicosecond range, and also expressions for longitudinal aberrations of main optical elements in technique like a streak camera will be presented. Possibility of realization of the last with the resolution of about 0.01ps and less is discussed.

## 1 INTRODUCTION

Bunch phase distribution (BPD) monitoring with resolution in femtosecond range, that is required for the next linear collider, X-ray FELs, is a challenging problem for decision of which in the paper a new approach is proposed and validated through consideration of some basic limitations inhering in well-known techniques.

The monitoring implies a use of coherent or incoherent bunch radiation that, at present, corresponds to measurement technique operating in frequency- or time-domain respectively. The first limitation follows at once from the radiation wave length that is considered below. We note at once also that the name "coherent" is not very correct one because the BPD retrieving from its frequency spectrum demands the measurement in the entire domain of the BPD-spectrum including its incoherent part too.

## 2 TIME CONVERTING TECHNIQUE

The principle of coherent technique operation in the frequency-domain consists in retrieving the BPD from the square of its Fourier transformation  $F(\omega)$  that in turn is determined from the well-known expression [1]

$$P_c(\omega) = P(\omega)[N + N(N-1)F(\omega)],$$

where  $P_c(\omega)$  - measured frequency spectrum of power radiation and  $P(\omega)$  - the power spectrum emitted by a single electron and defined in the domain of the F-function. Consideration of the technical problems of the  $P_c$ -measurement one can find anywhere [2], but here we touch the basic problem dealt with the P-determination.

If  $P_c$  is the spectrum at the entrance of a spectrometer then P represents itself a transfer function of a vacuum chamber with a radiator.

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To determine  $F(\omega)$  correctly the function P has to be uniform over the entire spectrum of a bunch. It can be only in an idealized case. In the case of transition radiation, for example, it could correspond to the perfectly conducting and infinite screen, absence of another boundary conditions and movement of an electron beam with infinitesimal transverse sizes along its infinite emission length. In reality, it is not so and, moreover, it is impossible to determine the function P correctly by experimental way or through calculation.

In the general case, the measured spectrum Pc is too far from the power spectrum F. In a specific device we always have, for example, a finite emission length L. In the case, considering P as a function of L through the term [1 - cos(L/Z)] [3] only, where the formation zone Z  $\sim \gamma^2/\omega \sim \gamma^2\lambda$ , we obtain that P( $\omega$ )  $\sim \omega^2$  at L/Z << 1 and, as consequence, the broadening of the measured spectrum P. The last may lead to considerable compression of a retrieved BPD in comparison with real one. This effect is demonstrated in the papers [4,5] where the retrieved bunch length was nearly by an order less one obtain from to some extent idealized simulation of beam dynamics. In the paper [2] for specific measurement scheme, equipment and beam condition the spectrum P measured in very narrow band was divided by to fit the retrieved BPD obtained by "coherent" technique and more precise one together.

Hence, the "coherent" technique requires another one and may serve as an indicator for an accelerator maintenance in a specific regime.

To avoid the mentioned above frequency dependence of the P-spectrum we have to satisfy the condition L/Z >> 1, i.e.  $\lambda << L/\gamma^2$ . In reality, for relativistic beam (for example,  $\gamma > 100$ , L  $\sim 100$ mm) it leads us to use the radiation in the frequency domain of about visible light and higher, then we will have to use another time converting technique based on use of device like a streak camera.

In the case, taking the transition radiation as an example for consideration, the scheme of the monitoring could look as the following: the beam radiation from a foil-mirror passes through lens and is focused on the surface of photocathode of a streak camera. Impact of chromatic and spherical aberration on the focusing of ultrashort light pulses by lenses was researches rather well in the papers [6,7] where it was shown that the pulse broadening can be less than 10fs. Then the main limitation in this scheme of the monitoring will be defined by the resolution of a streak camera.

Below, it will be shown how can create the device, like a streak camera but realizing new principle of construction, with the resolution of 10fs and less.

## 3 TRANSIT TIME SPREAD

Using the method [8] for precise integration of motion equation one can obtain the following expressions for the chromatic aberration of the h-distance gap (being under the  $U_0$ -accelerating voltage) caused by an initial photoelectron energy spread, for example, from  $W_{01}$  to  $W_{02}$  for two cases: marked as  $\bf 1$  - for two electrode system formed by a cylindrical emitter with an optional radius R and plane electrode;  $\bf 2$  - the coaxial system with the internal electrode as the emitter

$$\Delta t_i \cong \frac{t_0}{k_i} \left\{ \sqrt{W_2} - \sqrt{W_1} - \left[1 + a_i \left(1 - \frac{1}{\sqrt{1 - \xi_i}}\right)\right] \frac{W_2 - W_1}{2} \right\},$$

where i=1,2;  $t_0=\left(2h/c\right)\sqrt{W_0/2U_0}$  - time of flight the h-distance in uniform field;  $k_i$  - coefficient of the field enhancement on the emitter in comparison with the uniform field and, respectively, equaled  $k_1=\sqrt{m(m+2)}\Big/ln\Big(m+1+\sqrt{m(m+2)}\Big),\,k_2=m/ln(m+1);$   $W_1=W_{01}/U_0;\,W_2=W_{02}/U_0;\,a_i\text{ equaled respectively }a_1=m/2\text{ and }a_2=m=h/R;\,\xi_1=2(k_1-1)/m\text{ and }\xi_2=(k_2-1)/m;$   $W_0=0.511\,10^6\text{ eV};\,c\text{-speed of light}.$ 

The main parameter here is m = h/R. Taking m > 10 one can enhance the field, decrease the effective length of the electron transit and, as consequence, decrease the aberration up to several fs and less for  $W_{02} = 1eV$ ,  $W_{01} = 0$  and h = 1mm at U < 8kV.

### 4 DIAPHRAGM ABERRATION

The hole of a diaphragm restricting the gap disturbs the accelerating field that in turn courses additional transit time spread of the photoelectrons. This diaphragm aberration in the unit of  $t_0$  as a function of the h-distance in the unit of semi-width of a slit or radius of a circular hole ( $\Delta x$ ) is shown in Fig.1 for the following cases: 1 - the aberration taking a spherical one into account for the slit in the field of the cylindrical emitter with m=100; 2 - id., but m=10; 3 - the slit diaphragm in the field of a plane electrode; 4 - the circular diaphragm in the field of the cylindrical emitter for m=100, but without the spherical aberration.

The dependencies have been determined as the FWHM of the transit time distribution of the electrons with uniform density of escaping along the emitter surface and for the diaphragm with infinitesimal thickness. Confidence probability for  $h/\Delta x > 10$  was more 0.8.

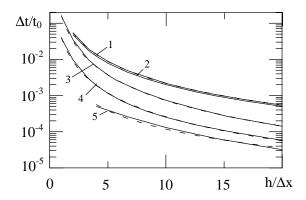


Figure 1: Diaphragm aberration as a function of the h-distance of the gap.

Approximating dependencies shown by the dashed line here are determined by the formula

$$\frac{\Delta t}{t_0} = \frac{A}{\left(h/\Delta x\right)^{\alpha}}$$

where values of A and  $\alpha$  for the mentioned above cases are: **1** - A = 0.2186,  $\alpha$  = 2.0; **3** - A = 0.1640,  $\alpha$  = 2.35; **4** - A = 0.0406,  $\alpha$  = 2.2; **5** - A = 0.006084,  $\alpha$  = 1.71.

Taking the coaxial geometry of the gap we eliminate the spherical aberration completely. In the case diaphragm aberration can be reduced to the negligible one.

### 5 RF-GAP

In the device for the BPD-monitoring the action of the rf-gap has to be independent of the photoelectron transverse position. For rf-deflector this basic requirement is not satisfied in principle, and, as consequence, there is an optimum field of the rf-deflector for the best resolution, i.e. the rf-field is restricted on its maximum magnitude here. It is quite different situation in the case of the longitudinal rf-modulation.

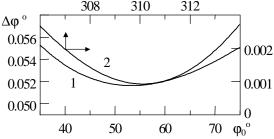


Figure 2: Phase resolution of coaxial resonator vs.  $\varphi_0$ 

The phase resolution of the rf-resonator in the case of the longitudinal modulation is defined as  $\Delta\phi=|[P(W_{02})-P(W_{01})]/(dP/d\phi_0)|,$  where the photoelectron momentum  $P(W_{0i})$  with its initial energy  $W_{0i}$  is determined at the resonator output,  $\phi_0$  - the initial phase at the electron

start. The resolution as a function of  $\phi_0$  for the coaxial resonator is shown in Fig.2 for two cases:  $\mathbf{1}$  - amplitude of the rf-voltage U=3kV, accelerating voltage  $U_0=4kV$ , h=1mm;  $\mathbf{2}$  - U=8kV,  $U_0=8kV$ , h=10mm. All dependencies were determined for  $W_{02}=1eV$ ,  $W_{01}=0$ , R=0.01mm and f=2.998 GHz.

#### 6 CAMERA

Combining the electrostatic accelerating field and the electron modulating rf-field in the coaxial resonator, where its internal conductor is the photocathode (or as a secondary electron emitter for SEM-monitor of the BPDmeasurement can be used) one can get ultra-fast camera for the BPD-monitoring (so named, maybe, for simplicity "troncamera" to retain the name "streak camera" for conventional one) if after the resonator a spectrometer operating in the regime of a spectrography will be installed. To get the resolution of about 100fs, 10fs or 1fs the camera will need the spectrometer with the relative momentum resolution equaled, respectively, 10<sup>-3</sup>, 10<sup>-4</sup> or 10<sup>-5</sup>. For the spectrometer with uniform magnetic field (it is shown in Fig.3) its well-known resolution is  $\Delta x/2R_m =$  $\alpha^2/2$  that allows us to get the camera resolution of 100fs and less. For getting the 10fs-resolution we will have to use, for example, an electrostatic prism with installation of a transaxial lens at its entrance and exit; for the 1fsresolution one may use the magnetic spectrometer with its field index n = 3/7.

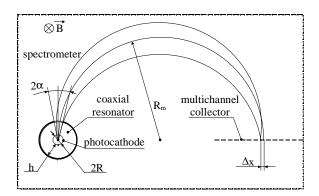


Figure 3: Scheme of camera

### 7 TIME EXPANDER

In Figure 4 the scheme of another device (tronexpader so named) for the BPD-monitoring is represented where the same coaxial resonator and instead of the spectrometer the 1-space free of field are used. The principle of its operation follows from its name directly. After the longitudinal rf-modulation the electrons fly within the angle  $\pm \alpha$ . The time expansion is defined here by the derivative

$$\frac{dt}{d\tau} = \frac{360^{\circ} f \cdot 1}{c \cdot \beta^{2} \gamma^{2} w} \cdot \frac{d(pc)}{d\varphi_{0}} ,$$

where t - the time of arrival at the collector for the electron with its time departure from the resonator  $\tau$ ; w - relativistic energy of the electron.

For l=100mm, f=3GHz, kinetic energy of the electron 1.8keV the magnitude of this derivative will be  $10^4$  at  $d(pc)/d\phi_0 = 1 \cdot 10^5$  eV/deg. The last derivative can be obtained, for example, at  $U_0 = 0.5\text{kV}$ , U = 10kV, h = 10mm, R = 0.01mm.

In means that by taking the recorder channel including the collector with entire frequency band of about 1GHz one can measure the BPD with the 100fs-resolution. It should be noted that for the same parameters mentioned above the estimation of the error of this method gives 1fs.

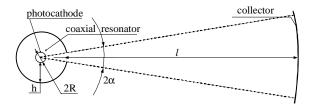


Figure 4: Scheme of expander.

## **8 CONCLUSION**

Proposed camera and expander face the limitations now following from the time spread of the photoelectron escaping and quantum mechanics, but all of them lay below 10fs here.

When a measuring system represents itself the tandem from the expander and the camera, i.e. when at the exit of the expander instead of its collector a plane rf-gap (close to that) with the transit hole for the electron and after that the spectrometer are installed, the resolution of the tandem will be determined through the time expansion in the expander and the time resolution of the camera so that entire resolution can reach  $10^{-18}...10^{-20}$  s. for solitary electron going from the photocathode.

# 9 ACKNOWLEDGMENTS

Author thanks I.G. Merinov for his codes and help to get the paper ready.

Work supported in part by the Russian Foundation for Basic Research

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