ENERGY RESOLUTION AT INTERACTION POINT FOR ASYMMETRIC BEAMS

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Abstract

The monochromatic scheme adopted in the tau-charm factories uses a large dispersion at interaction point to obtain a very high collision energy resolution.

Due to the presence of dispersion, however, the beam-beam interaction induces the synchro-betatron coupling. Even if in the linear regime, it can lead to potentially serious effects. One of the most serious is the rapid decrease of the energy resolution when the beam-beam parameter increases, even if the motion is stable. In this paper we study this effect in detail either for symmetric and asymmetric beams and give formula for the energy resolution.

1 INTRODUCTION

Recently the monochromatization has been considered seriously for future tau-charm factories [1], where a rather large dispersion exists at the IP with opposite signs for both beams. In this case, the dispersion effects can no longer be discussed in the perturbative sense as in the conventional colliders [2, 3]. In a previous paper [4] we discussed the effects of the dispersion at the IP paying enough attention to the mutual interaction between the betatron and the synchrotron degrees of freedom, and the possible problems associated with the monochromatization within the linear approximation of the beam-beam force.

The purpose of the monochromatization is to make the spread σ_ε of the collision energy much smaller than the nominal one σ^0_ϵ. Thus this quantity gives the figure of merit of the machine as well as the luminosity. The monochromatic scheme uses beams with opposite dispersion D, and realizes a distribution (centered at the IP) in vertical position y analogue (Gaussian) to that one (centered at E_0) in energy E. In this way the particles with energy $E_0 - \epsilon$ collide with those at the same quote in the other beam but with different energy $E_0 + \epsilon$, then getting as collision energy $E_{CM} \sim 2\, E_0$.

2 ENERGY RESOLUTION

For simplicity we consider the synchrotron motion and one betatron (vertical) oscillation degree of freedom only. The physical variables of a particle for the betatron and synchrotron motions are $\mathbf{x}_\pm = (y_\pm, p_{y\pm}, z_\pm, \epsilon_\pm)$, where $y_\pm$ is the vertical coordinate, $p_{y\pm}$ the vertical momentum normalized by the (constant) momentum p_0 of the reference particle, $z_\pm$ the time advance relative to the reference particle multiplied by the light velocity c, and $\varepsilon_\pm = (E_\pm - E_0)/E_0$ is the energy deviation from the nominal value E_0 and normalized by it.

In the monochromatization, as well as the luminosity L, the spread σ_w of the collision energy $w - 2E_0 \equiv \varepsilon_+ + \varepsilon_-$ is important. The luminosity can be expressed as

$$L = \mathrm{const} \times \int f_+(y,\varepsilon_+) f_-(y,\varepsilon_-) dy d\varepsilon_+ d\varepsilon_- . \quad (1)$$

Here $f_\pm(y_\pm, \varepsilon_\pm)$ is the distribution function in the $(y_\pm, \varepsilon_\pm)$ space. If we assume

$$f_\pm(y,\varepsilon_\pm) = \frac{\exp\left(\dfrac{-A_{22}{}^{\pm} y^2 + 2A_{12}{}^{\pm} y\varepsilon_\pm - A_{11}{}^{\pm}\varepsilon_\pm^2}{2\det A_\pm}\right)}{2\pi\sqrt{\det A_\pm}}, \quad (2)$$

with A being

$$A_\pm = \begin{pmatrix} \langle y^2\rangle_\pm & \langle y\varepsilon\rangle_\pm \\ \langle y\varepsilon\rangle_\pm & \langle \varepsilon^2\rangle_\pm \end{pmatrix}, \quad (3)$$

we get

$$L = \mathrm{const} \times \frac{1}{\sqrt{2\pi}} \frac{1}{\sqrt{A^+_{11} + A^-_{11}}} . \quad (4)$$

The luminosity density with respect to w is defined as [10]

$$\Lambda(w) = \frac{1}{L}\int f_+(y,\varepsilon_+) f_-(y,\varepsilon_-)\delta(w - \varepsilon_+ - \varepsilon_-) dy d\varepsilon_+ d\varepsilon_- . \quad (5)$$

Here w stands for the deviation of the collision energy from the nominal one ($2E_0$). Then we have

$$\Lambda(w) = \frac{1}{\sqrt{2\pi\sigma_w^2}} \exp\left(-\frac{w^2}{2\sigma_w^2}\right), \quad (6)$$

and the formula for the energy resolution for asymmetric beams is

$$\sigma_w^2 = \frac{\mathcal{A}\det A_+ \det A_-}{A^+_{11} + A^-_{11}} . \quad (7)$$

with

$$\mathcal{A} = \left(\frac{A^+_{11}}{d_+} + \frac{A^-_{11}}{d_-}\right)\left(\frac{A^+_{22}}{d_+} + \frac{A^-_{22}}{d_-}\right) - \left(\frac{A^+_{12}}{d_+} - \frac{A^-_{12}}{d_-}\right)^2 . \quad (8)$$

If we assume that two beams are modified symmetrically, we have $A^+_{11} = A^-_{11}$, $A^+_{22} = A^-_{22}$, and $A^+_{12} = -A^-_{12}$, $A_\pm = A$ and the formula for the energy resolution becomes

$$\sigma_w^2 = \frac{2\det A}{A_{11}} . \quad (9)$$

In the absence of collision ($\xi_0 \simeq 0$) we can assume the following:

$$A_\pm = \begin{pmatrix} \beta^0_y \epsilon^0_y + D_0^2(\sigma^0_\varepsilon)^2 & \pm D_0(\sigma^0_\varepsilon)^2 \\ \pm D_0(\sigma^0_\varepsilon)^2 & (\sigma^0_\varepsilon)^2 \end{pmatrix} \quad (10)$$

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which implies [10]

$$\sigma_w^2 = \frac{2(\sigma_\varepsilon^0)^2}{1+\frac{D_0^2(\sigma_\varepsilon^0)^2}{\beta_y^0\epsilon_y^0}}. \qquad (11)$$

3 EQUILIBRIUM ENVELOPE

The equilibrium value of the envelope matrix σ, where

$$\sigma_{ij} = \langle (x_i - \bar{x}_i)(x_j - \bar{x}_j)\rangle, \qquad (12)$$

is determined by the following equation [9]:

$$\sigma = M_{bb}^{1/2}[\bar{\Lambda}M_{arc}\sigma(\bar{\Lambda}M_{arc})^t + (I-\bar{\Lambda}^2)\bar{E}](M_{bb}^t)^{1/2}, \qquad (13)$$

where

$$\bar{\Lambda} = H_0B_0\Lambda(H_0B_0)^{-1}, \quad \bar{E} = H_0B_0E(H_0B_0)^t, \quad (14)$$

$$\Lambda = diag(\lambda_y,\lambda_y,1,\lambda_z^2), \quad E = diag(\epsilon_y^0,\epsilon_y^0,\epsilon_z^0,\epsilon_z^0). \qquad (15)$$

$\lambda_{y,z} = \exp(1/T_{y,z})$ being the damping constants and $\epsilon_{y,z}^0$ the nominal emittances. The one turn matrix from IP ($s = 0$) to IP is [5]:

$$M = M_{bb}^{1/2}M_{arc}M_{bb}^{1/2} \qquad (16)$$

where:

$$M_{arc} = M(0_-,0_+) = H_0B_0\hat{M}_{arc}\,B_0^{-1}H_0^{-1}, \qquad (17)$$

$$\hat{M}_{arc} = diag(r(\mu_y^0), r(\mu_z^0)), \quad B_0 = diag(b_y^0, b_z^0), \quad (18)$$

with

$$r(\mu_{y,z}^0) = \begin{pmatrix} \cos\mu_{y,z}^0 & \sin\mu_{y,z}^0 \\ -\sin\mu_{y,z}^0 & \cos\mu_{y,z}^0 \end{pmatrix}, \qquad (19)$$

$b_{y,z}^0 = diag(\sqrt{\beta_{y,z}^0}, 1/\sqrt{\beta_{y,z}^0})$,

$$H_0 = \begin{pmatrix} I & h_0 \\ h_0 & I \end{pmatrix}, \qquad h_0 = \begin{pmatrix} 0 & D_0 \\ 0 & 0 \end{pmatrix}. \qquad (20)$$

$\mu^0 = 2\pi\nu^0$, ν^0 being the nominal tune, $\beta_{y,z}^0$ the nominal betatron functions at IP ($\beta_z^0 \equiv \sigma_z^0/\sigma_\varepsilon^0$, σ_z^0 being the nominal bunch length), and D_0 the dispersion at IP. Note that H_0, B_0, and $\hat{M}_{arc}$ are symplectic. Finally the beam-beam interaction is described as a linear kick

$$M_{bb} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -4\pi\xi_0/\beta_y^0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \qquad (21)$$

with ξ_0 being the vertical (nominal) beam-beam parameter

$$\xi_0 = \frac{2\pi\gamma r_e N\beta_y^0}{2\pi\gamma\sigma_y^0(\sigma_y^0+\sigma_x^0)}. \qquad (22)$$

Note that the nominal synchrotron tune ν_z^0 is negative for conventional electron machines with positive momentum compaction factor α_p, and we have assumed that there is only one IP which is a symmetric point with respect to betatron and synchrotron motions. We have also implicitly assumed that dispersion does not exist in cavities.

4 DISCUSSIONS

In Fig. 1 we show that the energy resolution σ_w increases rapidly with ξ_0 and approaches its nominal value σ_ϵ^0, then making the monochromatization less effective or even useless. This, obviously, gives a more stringent limit for the maximum value of ξ_0 than the single particle instability [4].

Furthermore, from Fig. 1, it is clear that this effect weakly depends on the betatron tunes and is almost indipendent on the synchrotron ones.

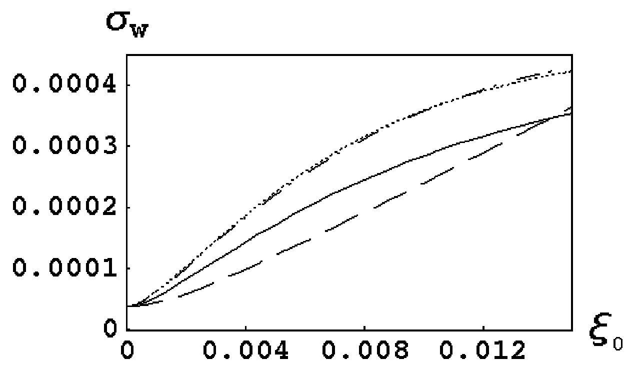

Figure 1: The energy resolution σ_w versus ξ_0 with $D_0 = 0.4$m and other parameters are in Table 2.

Table 1: Legenda of Figure 1

line	ν_y^{0+}	ν_y^{0-}	ν_z^{0+}	ν_z^{0-}	beams
dash-dot	0.05	0.05	0.03	-0.03	asymmetric
dash	0.1	0.05	0.03	-0.08	asymmetric
solid	0.1	0.05	0.03	0.03	asymmetric
dot	0.05	0.05	0.03	0.03	symmetric

Table 2: Parameters for both beams

β_y^0	0.03m	β_z^0	26.3m
ϵ_y^0	410^{-9}m	ϵ_z^0	3.810^{-6}m
σ_ε^0	3.810^{-4}	σ_z^0	0.01m
T_y	1000	T_z	500

The reason of the rapid growth of the energy resolution with ξ_0 is due to one of σ_{11} and in particular of σ_{22} [8], hence of the vertical and longitudinal emittances respectively. They increase in a similar manner regardless of the sign of ν_z^0, even if for negative ν_z^0 the increase might be easier to understand, because the system is unstable [4].

The emittances are obtained from the envelope matrix σ as follows:

$$\text{Eigenvalues}[J\sigma] = \{i\epsilon_y, -i\epsilon_y, i\epsilon_z, -i\epsilon_z\}. \qquad (23)$$

In Fig.2 we plot the emittances $\epsilon_{y,z}$ as functions of ξ_0, for $\nu_z^0 = 0.08$. This shows that the longitudinal emittance $\epsilon_z$is influenced considerably by the the beam-beam force. This effect has been usually overlooked in the literature where the synchrotron oscillation is assumed to be unaffected. Also, the vertical emittance ϵ_y grows up quite rapidly.

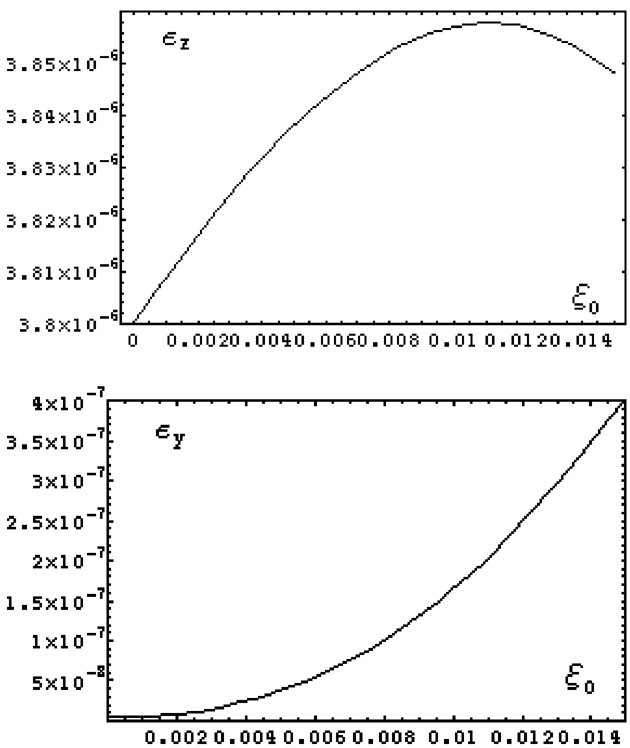

Figure 2: The synchrotron emittance ϵ_z (top) and the betatron one ϵ_y (bottom) as functions of ξ_0 with $\nu_{z0} = 0.08$ and all other parameters are listed in Table 2.

As conclusion, within the linear analysis used in this paper, it seems difficult to avoid this dangerous effect. However there can be some other effects in nonlinear regime which is worth to study. In deciding the machine parameters one should pay enough attention to the energy resolution.

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