HALO STUDIES IN SPACE CHARGE DOMINATED BEAMS

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Abstract

We studied bounded thermal equilibrium of Vlasov-Maxwell in space charge dominated beams. These meta-equilibrium, which have a life time smaller than the short-range collision time and are in the region of the transit time in a Linac, can be used efficiently to avoid or to control the halo generation. Therefore, it is fundamental to understand their inner structure, and to analyze the basic mechanisms which can drive the beams to such equilibrium, or explain the way some known processes can contribute to their destruction.

1 INTRODUCTION

In the development of new accelerators requiring intense ion beams (40-150 mA), for the industrial production of tritium (APT, TRISPAL), for the generation of neutron intense sources (ESS) or the nuclear waste transmutation, the activation problem of the structures is crucial.

It is generally admitted that beam losses must be below 1 ppm, and comes from a peculiar profile of the radial beam density which goes very far and arrives to touch the walls; this profile is called "halo", to give the precise idea of a central core, surrounded by a diffuse tail [5].

For a better understanding of the halo formation, we limited our study to a continuous beam, and to the part of the machine where the beam can effectively activate the walls (5-1000 MeV); this part going from the DTL entrance to the CCL output, is represented by an axi-symetric focusing channel.

In these conditions, where the vacuum is good (10^{-5} - 10^{-7} torr), the beam is governed by the Vlasöv-Maxwell system of equations which has theoretically a lot of mathematical solutions, like the Maxwellian which is not bounded, or some Meta-equilibria [1] which depend of the Hamiltonian and are bounded in the phase-space.

This propriety is interesting because we can hope that if the beam has not a maxwellian profile at the DTL entrance, it might be transported with a bounded profile.

But things begin to be wrong, when we try to determine if the beam converges really to one of these probable equilibrium states.

Firstly, in the absence of a residual gas, beam particles do not undergo elastic and inelastic binary collisions which could give them a thermal component.

Secondly, it is easy to verify that the Hamilton-Maxwell equations which govern the particle motion are deterministic and apparently invariant with time reversibility.
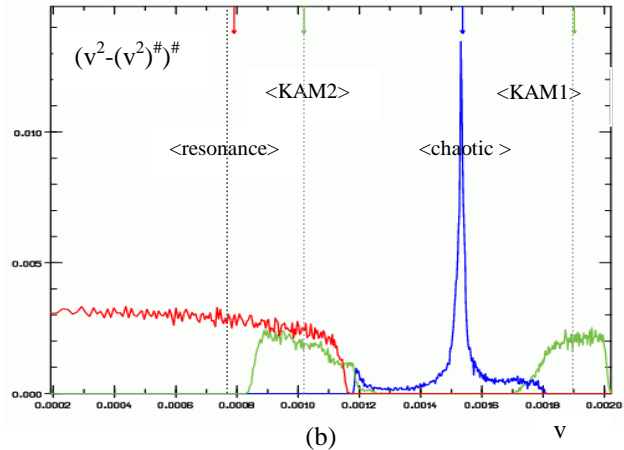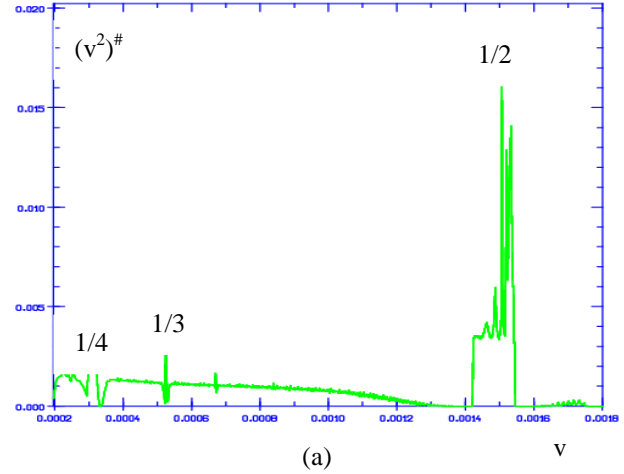

(a)

(b)

Figure 1 : ergodicity test with parameters r_0=3mm, η=0.6, μ =0.88, N=20; fig 1.a represents $(v^2)^{\#}$ for the whole beam and fig 1.b represents $(v^2-(v^2)^{\#})^{\#}$ for the resonance basin ½.

In these conditions is it reasonable to imagine or to hope that the beam can relax to a macroscopic equilibrium following an irreversible way?

This is a forty years old discussion [1-2-3], and answers to this question are not straightforward, because in the absence of binary collisions, the non-linearities become predominant in the Vlasöv-Maxwell or Hamilton-Maxwell equations, and complicate considerably the analyze of the problem.

In the following sections, we present some ideas which could help to clarify the discussion about topics like beam "irreversibility", "ergodicity", "mixing", and finally "stability".

2 IRREVERSIBILITY

Practically, the time reversibility of a dynamical system is not trivial to obtain without a large energy expense: the charged particle number is so large that it is out of mind to achieve the motion control of each particle at microscopic level, and then to proceed a reversible flash-back of the beam to its previous macroscopic state.

Therefore, the beam evolution is really time irreversible, even if the particle motion is deterministic; now, we have an indication of the time irreversibility, but we do not know, if the dynamical system is chaotic or if it can be described by a macroscopic state which verifies the statistic laws and more, if the dynamical system converges to a unique macroscopic equilibrium state.

Krilöv [10-11] studied this problem and concluded that, for the rapid establishment of any macroscopic state the system must be "ergodic", but for the relaxation to a unique macroscopic state the system must be "mixing".

3 ERGODICITY

The concept of ergodicity is introduced each time it is tried to split a system in sub-domains independent from the dynamic viewpoint.

In each sub-domain, we must verify the ergodic theorem proposed by Birkhoff [11] : the time average $(G)^{\#}$ of any defined observable is equal to the space average $<G>$ of the same observable; in fact, there is a less restrictive version of this theorem, which shows that in general the time average exists nearly anywhere in the concerned sub-domain and is distributed around the space average.

We verified the ergodicity concept using a PCM[4] code, and calculating the time average of the squared velocity $(v^2)^{\#}$ of the sampled particles, during N core oscillations.

We find first in fig 1.a that the beam is not ergodic as a whole, but it can be split in separated sub-domains which correspond exactly to the resonances 1/2, 1/3, 1/4,… [6].

In any sub-domains, the phenomena are blurred by the periodic motion associated to the core-breathing; this problem can be easily avoided by calculating the time average of an observable like the temperature $(v^2-(v^2)^{\#})^{\#}$.

In each basin of resonances (1/2 for example), the phase space sub-domain is limited by bundles of invariant torus (KAM1 and KAM2 - figure 2.a); the complementary part contains stochastic particles and resonances and its size increases with the perturbation.

The results (see figure 1.b) are very surprising, because we find that all the components of the resonance basin are ergodic : these results are in accord with the PCM code, but are not realistic, and might be considered with care; even if the PCM code helps greatly in this analyze, we touch there one of the limits of this code : the Landau damping is not reproduced.

Therefore, the results are true for the KAM surfaces and the chaotic particles, but are false for the resonances which carry the energy of perturbation and might disappear with time, by Landau damping.

Now, if we forget the resonances, figure 1.b gives an exact description of the ergodic components in each basin if attraction; the beam is split in ergodic and embodied sub-domains, which are limited by KAM surfaces and contain chaotic particles…

4 MIXING

The mixing is generally very difficult to prove rigorously, but it is often associated with non-linear interactions which transform the phase space as a "backer rolling"; thus the Vlasöv-Maxwell and Hamilton-Maxwell equations have two important proprieties :

-an extrem sensibility versus weak perturbations of the forces or small changes in initial conditions, and even the existence of parametric instabilities,

-quasi-periodical and ellipsoidal trajectories in the phase space which still increase the mixing rapidity, since the same particle can travel alternatively in the core and in the tail of the beam.
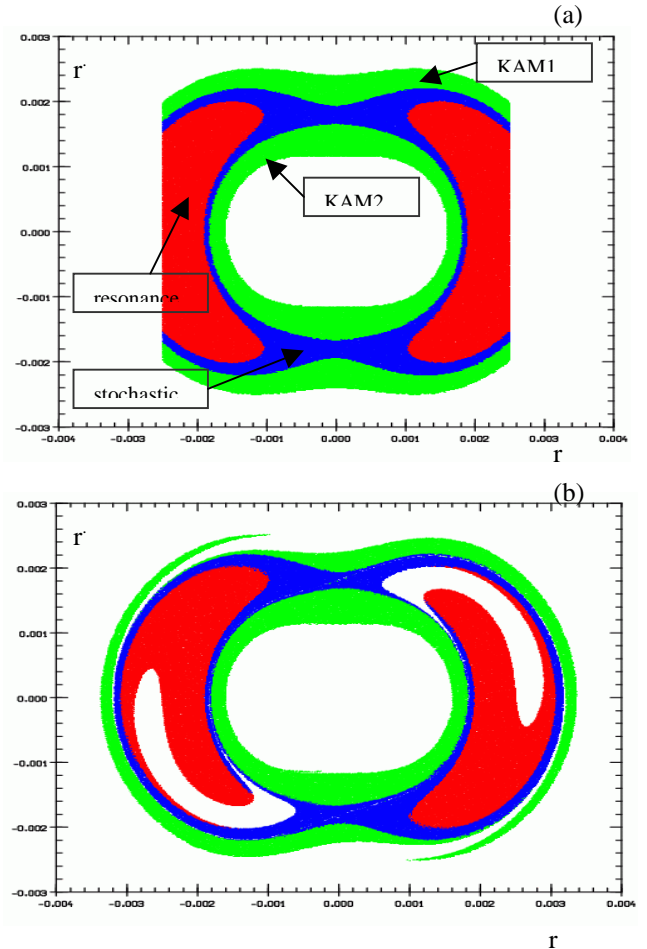

Figure 2 : mixing test with parameters r_0=3mm, η=0.6, μ =0.88; fig 2.a represents the initial perturbation and fig 2.b gives (r-r') for N=20.

We verified the mixing propriety in the resonance basin 1/2; the initial perturbation is a violent cutoff of the phase space done by a scraper and is represented in figure 2.a.

We can observe in figure 2.b, the rapid reconstruction of the sub-domain and we must conclude that a local equilibrium exists in this sub-domain; this could be verified for all sub-domains, and we would find the same result for each of them.

Therefore, the beam does not relax to a unique equilibrium state, but it is continuously in balance between local and self-similar equilibrium which are very stable and strong.

5 DISCUSSION

From the preceding studies about ergodicity and mixing, two classes of organized structures appeared :

-the static structures, or KAM surfaces, which limit self – similar sub-domains in the phase space,

-the dynamical structures, or the resonances, the hyperbolic fixed points,…, which exist potentially in each sub-domain, and depend directly of the space charge η and the perturbation μ parameters [5].

This drives finally to very simple notions :

-a charged particle beam can be described by a balance between many self-similar local equilibrium,

-these local equilibrium have a fractal structure that we can find in the phase space, position and Fourier spaces,

-the fractals are governed by scale invariants which depend directly from the system parameters.

From these considerations, scale invariants and density profiles were calculated analytically, at equilibrium($\mu=1$) and as a function of the space charge η (figure 3).
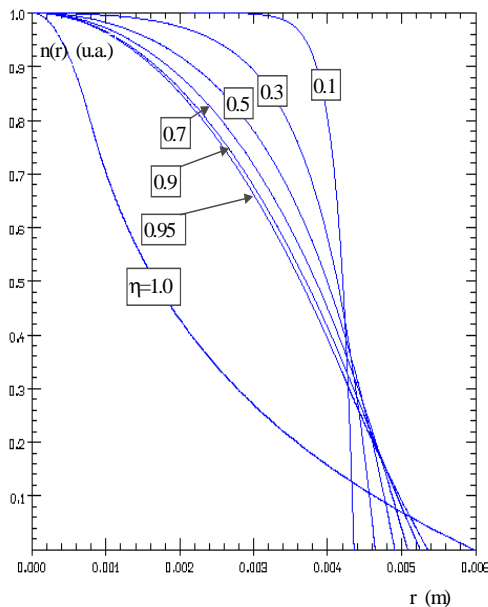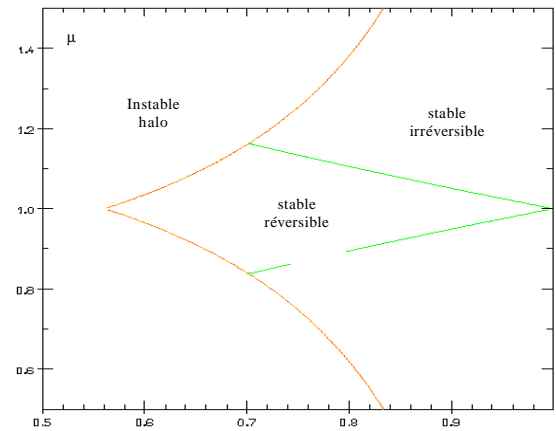

Figure 4 : stability graph as a function of the space charge η and the mismatch μ. parameters.

Now, we know that these local equilibrium are strong and rapidly constituted and we can calculate their stability domain (figure 4).

As long the beam stays in its stability domain, the balance between local equilibrium is respected, and all is right; but this does not signify that the balance is indes-tructible!

If we try to transport the beam in conditions which are out off its stability domain, this one will try to reorganize itself to achieve a new balance of local equilibrium :

-if this new situation exists in the machine, the beam will suffer an emittance growth, but it's all !

-if this new situation does not exist, the beam will try to reorganize itself continuously, and will touch finally the walls of the machine.

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Figure 3 : radial density profiles calculated as a function of the space charge parameter η.