MERGING BEAM-BEAM INTERACTION

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Abstract

New type of particle collisions is considered. In merging collisions two comoving beams intersect each other at small angle to provide low collision energy required for nuclear fusion experiments. Expression for luminosity is obtained. It is shown, that luminosity has a maximum value depending on particle velocity.

1 INTRODUCTION

Merging beam collisions is a new technique for study nuclear fusion processes [1]. Merging two radioactive isotope beams deliver low energy collisions just above the Coulomb barrier threshold that is difficult to be realized in other experimental methods. In this paper we analyze luminosity of merging beam-beam collision as a function of ring parameters and find optimal conditions to obtain maximum value of luminosity.

2 LUMINOSITY OF MERGING BEAM-BEAM COLLISIONS

Consider two bunched beams with particle densities ρ_1, ρ_2 and beam velocities $\vec{\beta}_1 = \vec{v}_1/c$, $\vec{\beta}_2 = \vec{v}_2/c$, crossing each other with angle 2φ (see Fig. 1). Assume beams have a Gaussian space charge density distributions

$$\rho_1 = N_1 \frac{\exp\left[-\frac{x_1^2}{2\sigma_{x1}^2} - \frac{y_1^2}{2\sigma_{y1}^2} - \frac{(z_1 - v_1 t)^2}{2\sigma_{z1}^2}\right]}{(2\pi)^{3/2}\sigma_{x1}\sigma_{y1}\sigma_{z1}}, \quad (1)$$

$$\rho_2 = N_2 \frac{\exp\left[-\frac{x_2^2}{2\sigma_{x2}^2} - \frac{y_2^2}{2\sigma_{y2}^2} - \frac{(z_2 - v_2 t)^2}{2\sigma_{z2}^2}\right]}{(2\pi)^{3/2}\sigma_{x2}\sigma_{y2}\sigma_{z2}}, \quad (2)$$

where N_1, N_2 are number of particles per each bunch, σ_{xi}, σ_{yi}, σ_{zi}, $i = 1, 2$ are root-mean square (rms) bunch sizes. Luminosity is defined as a ratio of interaction rate dN/dt to cross section σ of particle interaction $L = (1/\sigma)(dN/dt)$. According to invariant cross section formula [2], number of collisions δN in the volume dV during time dt is

$$\delta N = \sigma\sqrt{(\vec{v}_1 - \vec{v}_2)^2 - \frac{[\vec{v}_1 \times \vec{v}_2]^2}{c^2}}\,\rho_1\rho_2\,dV\,dt. \quad (3)$$

After two bunches crossed each other, total number of collisions, ΔN, is obtained via integration of Eq. (3) over volume of interacted bunches and time of interaction. In a ring N_b bunches collide per turn during time $T = 1/f_o$, where f_o is a particle revolution frequency. Therefore, luminosity of particle collider, $L = (1/\sigma)(\Delta N/T)$, is

$$L = N_b f_o \sqrt{(\vec{v}_1 - \vec{v}_2)^2 - \frac{[\vec{v}_1 \times \vec{v}_2]^2}{c^2}}\int_V\int_t \rho_1\rho_2\,dxdydzdt. \quad (4)$$

Transformation of coordinate system of every bunch (x_i, y_i, z_i), $i = 1, 2$ into laboratory coordinate system (x, y, z) is given by (see Fig. 1):

$$\begin{cases} z_1 = z\cos\varphi + x\sin\varphi \\ x_1 = -z\sin\varphi + x\cos\varphi \end{cases}, \quad \begin{cases} z_2 = z\cos\varphi - x\sin\varphi \\ x_2 = z\sin\varphi + x\cos\varphi \end{cases}. \quad (5)$$

Substitution of coordinate transformation (5) into Eq. (4) gives the following expression for luminosity:

$$L = G\frac{N_b f_o c N_1 N_2}{(2\pi)^3}\sqrt{\beta_1^2 + \beta_2^2 - 2\beta_1\beta_2\cos2\varphi - \beta_1^2\beta_2^2\sin^2 2\varphi}, \quad (6)$$

where integral over volume and time of interaction is

$$G = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{1}{\sigma_{x1}\sigma_{y1}\sigma_{z1}\sigma_{x2}\sigma_{y2}\sigma_{z2}}\;\cdot$$

$$\exp\left\{-x^2\left[\frac{\cos^2\varphi}{2}\left(\frac{1}{\sigma_{x1}^2} + \frac{1}{\sigma_{x2}^2}\right) + \frac{\sin^2\varphi}{2}\left(\frac{1}{\sigma_{z1}^2} + \frac{1}{\sigma_{z2}^2}\right)\right]\right.$$

$$-\frac{y^2}{2}\left(\frac{1}{\sigma_{y1}^2} + \frac{1}{\sigma_{y2}^2}\right) - z^2\left[\frac{\sin^2\varphi}{2}\left(\frac{1}{\sigma_{x1}^2} + \frac{1}{\sigma_{x2}^2}\right)\right.$$

$$\left. + \frac{\cos^2\varphi}{2}\left(\frac{1}{\sigma_{z1}^2} + \frac{1}{\sigma_{z2}^2}\right)\right] - z\,t\cos\varphi\left(-\frac{v_1}{\sigma_{z1}^2} - \frac{v_2}{\sigma_{z2}^2}\right)$$

$$-\frac{t^2}{2}\left(\frac{v_1^2}{\sigma_{z1}^2} + \frac{v_2^2}{\sigma_{z2}^2}\right) + x\left[z\sin\varphi\cos\varphi\left(\frac{1}{\sigma_{x1}^2} - \frac{1}{\sigma_{x2}^2} - \frac{1}{\sigma_{z1}^2} + \frac{1}{\sigma_{z2}^2}\right)\right.$$

$$\left.\left. - t\sin\varphi\left(\frac{v_1}{\sigma_{z1}^2} - \frac{v_2}{\sigma_{z2}^2}\right)\right]\right\}\,dx\,dy\,dz\,dt. \quad (7)$$

Assume, that collide bunches have the same longitudinal sizes $\sigma_{z1} = \sigma_{z2} = \sigma_z$ and same velocities $\beta_1 = \beta_2 = \beta$, but different transverse sizes $\sigma_{x1} \neq \sigma_{x2}$, $\sigma_{y1} \neq \sigma_{y2}$. Calculation of luminosity, Eqs.(6-7) results in expression
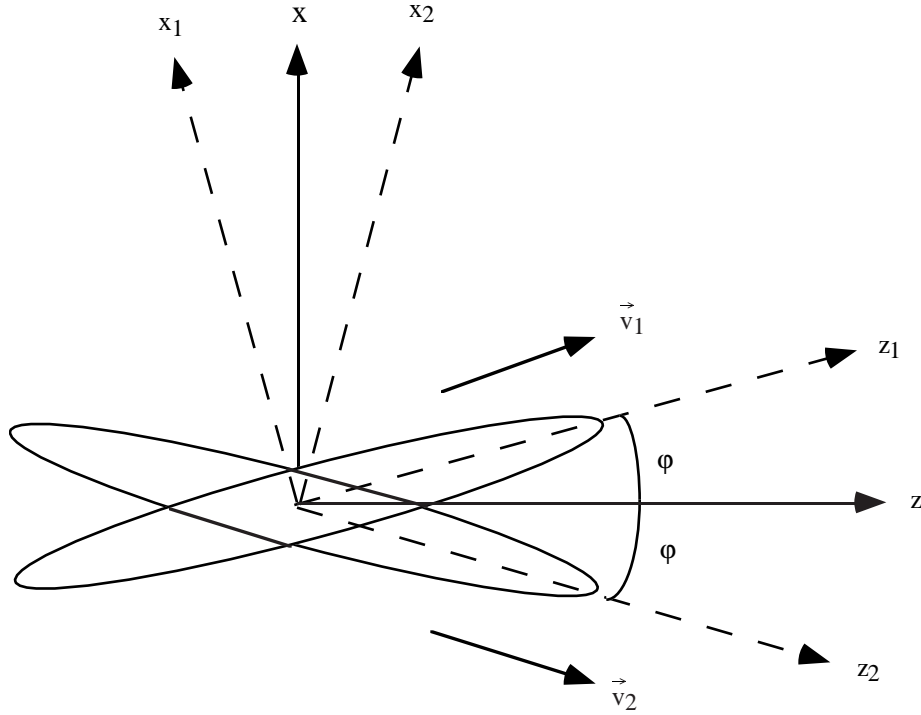
Fig. 1. Merging beam-beam interaction.

$$L = \frac{N_b f_o\, N_1\, N_2}{2\sqrt{2}\,\pi\,\sqrt{\sigma_{y1}^2 + \sigma_{y2}^2}\;\sigma_z \cos\varphi}\; \sqrt{\frac{1 - \beta^2 \cos^2\varphi}{1 + tg^2\varphi\,(\frac{\sigma_{x1}^2 + \sigma_{x2}^2}{\sigma_z^2})}} \;. \quad (8)$$

Formula (8) is derived assuming $\varphi \neq 0,\ \pi/2$. For typical parameters in the interaction point, $\varphi \ll 2\pi$, $\sigma_{x1,2} \ll \sigma_z$, the denominator factor in Eq. (8) can be neglected as a product of two small values:

$$tg^2\varphi\ (\frac{\sigma_{x1}^2 + \sigma_{x2}^2}{\sigma_z^2})\ \ll 1 \quad . \quad (9)$$

Luminosity, Eq.(8), is inversely proportional to longitudinal beam size, σ_z, and vertical beam size, $\sigma_{y\,1,\,2}$, and practically independent on horizontal beam sizes in the plane of particle collision, $\sigma_{x\,1,\,2}$.

Let us take into account, that revolution frequency of particle motion in a ring is given by $f_o = \beta c\, /\, C_r$ where $C_r = 2\pi R$ is a circumference and R is a mean radius of the ring. Therefore, luminosity as a function of particle velocity is:

$$L = \frac{c\, N_b\, N_1 N_2}{2\sqrt{2}\,\pi\,\cos\varphi\;\sqrt{\sigma_{y1}^2 + \sigma_{y2}^2}\;\sigma_z C_r}\;\beta\;\sqrt{1 - \beta^2\cos^2\varphi}\;. \quad (10)$$

Taking derivative of luminosity over particle velocity,

$$\frac{\partial L}{\partial \beta} = \frac{c\, N_b N_1\, N_2}{2\sqrt{2}\,\pi\,\cos\varphi\;\sqrt{\sigma_{y1}^2 + \sigma_{y2}^2}\;\sigma_z C_r}\;\frac{(1 - 2\beta^2\cos^2\varphi)}{\sqrt{1 - \beta^2\cos^2\varphi}} = 0\ ,\ (11)$$

the maximum luminosity, L_{max}, is obtained at the following optimal values of particle velocity, β_{opt}, and particle energy, γ_{opt} :

$$\beta_{opt} = \frac{1}{\sqrt{2}\ \cos\varphi}\ ,\qquad \gamma_{opt}\ = \frac{\sqrt{2}\ \cos\varphi}{\sqrt{2\cos^2\varphi\ -\ 1}}\ ,\qquad (12)$$

$$L_{max} = \frac{c\, N_b\, N_1\, N_2}{4\sqrt{2}\,\pi\,\cos^2\varphi\;\sqrt{\sigma_{y1}^2\ +\ \sigma_{y2}^2}\;\sigma_z\, C_r}\;. \quad (13)$$

In Fig. 2 luminosity as a function of particle velocity for angle $\varphi = 0.087$ is presented. Since the value of φ is small and $\cos\varphi \approx 1$, the maximum luminosity is obtained for $\beta_{opt} \approx 1/\sqrt{2}$, $\gamma_{opt} \approx 1.41$. At relativistic energies, $\beta \approx 1$, luminosity is

$$L\ (\beta \rightarrow 1) = \frac{c\, N_b\ N_1\, N_2}{2\sqrt{2}\,\pi\;\sqrt{\sigma_{y1}^2\ +\ \sigma_{y2}^2}\;\sigma_z\, C_r}\;tg\varphi\ . \quad (14)$$

Therefore, reduction of luminosity at relativistic particle energies $\gamma \gg \gamma_{opt}$ with respect to maximum value of luminosity is

$$\frac{L\ (\beta \rightarrow 1)}{L_{max}(\beta_{opt})} =\ \sin 2\varphi\ . \quad (15)$$
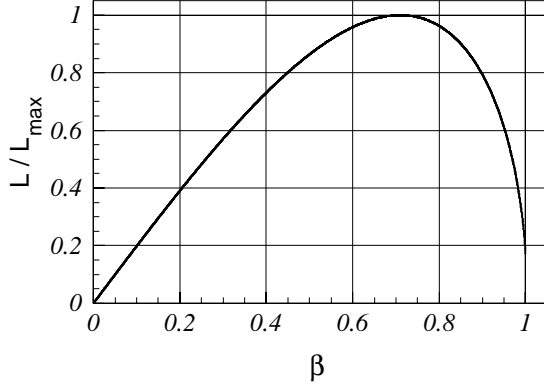
Fig. 2. Luminosity of merging beam-beam collisions for angle $\varphi = 0.087$ as a function of particle velocity.

3 COMPARISON WITH LUMINOSITY OF CROSSING ANGLE COLLISIONS

Let us compare maximum luminosity at merging collisions, Eq. (13) with that for head crossing angle collision with the same value of φ. Expression for luminosity with head crossing angle collision is obtained from general expression , Eqs. (6), (7), by change of $\vec{v}_2$ for $-\vec{v}_2$ [3]:

$$L = \frac{N_b\, f_o\, N_1\, N_2\, \sqrt{1 - \beta^2 \sin^2\varphi}}{2\pi\, \sqrt{\sigma_{y1}^2 + \sigma_{y2}^2}\, \sqrt{2\sigma_z^2 \sin^2\varphi + \cos^2\varphi\,(\sigma_{x1}^2 + \sigma_{x2}^2)}} \quad . \quad (16)$$

Ratio of two values of luminosity is:

$$\frac{L_{merge\ max}}{L_{cross\ angle}} = \frac{\sqrt{\sigma_{x1}^2 + \sigma_{x2}^2}}{2\sqrt{2}\,\sigma_z\,\beta\,\cos\varphi} \sqrt{\frac{1 + tg^2\varphi\, \dfrac{2\,\sigma_z^2}{(\sigma_{x1}^2 + \sigma_{x2}^2)}}{1 - \beta^2 \sin^2\varphi}}$$

$$\approx \frac{\sigma_x}{2\sigma_z} \quad . \quad (17)$$

For typical ring parameters $\varphi \approx 0$, $\beta \approx 1$, ratio of luminosity is of the order $\sigma_x/\sigma_z \approx 10^{-3}$. That means, that in merging collisions, achievable value of luminosity is several order of magnitude less than that in encountered crossing angle collisions, because merge beams are almost parallel to each other.

4 LUMINOSITY OF COMOVING BEAM COLLISIONS

From the previous section it is clear that merging collision is characterized by small value of luminosity. V.V.Parhomchuk (BINP, Novosibirsk) pointed out [4]

that, if collisions is performed on-axis, luminosity does not depend on particle velocities. Consider two bunched beams, which move in the same direction with different velocities $v_1 \neq v_2$. Luminosity of such comoving collisions is obtained from general formulas (6), (7) by change of $\vec{v}_2$ for $-\vec{v}_2$ and assuming $\varphi = 0$:

$$L = \frac{N_b\, f_o\, c\, N_1\, N_2}{(2\pi)^3}\, \frac{|\beta_1 - \beta_2|}{\sigma_x^2\, \sigma_y^2\, \sigma_z^2} \cdot G \ , \quad (18)$$

$$G = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \exp\left[-\frac{x^2}{\sigma_x^2} - \frac{y^2}{\sigma_y^2} - \frac{z^2}{\sigma_z^2} - \frac{t^2}{2}\frac{(v_1^2 + v_2^2)}{\sigma_z^2} \right.$$

$$\left. + z\cdot t\frac{(v_1 + v_2)}{\sigma_z^2} \right] dx\, dy\, dz\, dt \ . \quad (19)$$

After evaluation of integral (19), the value of luminosity is:

$$L = \frac{N_b\, f_o\, N_1\, N_2}{4\pi\, \sigma_x\, \sigma_y} \quad . \quad (20)$$

Eq. (20) gives the same result as luminosity of head-on collision. Integration in Eq. (19) is performed in the infinite limits. It means, that bunches of both beams have to pass through each other completely. Bunch length is $l \approx 4\,\sigma_z$. To perform interaction, collision region, l_{coll}, has to be as long as [4]

$$l_{coll} = l\, \frac{\beta_2}{|\beta_1 - \beta_2|} \quad . \quad (21)$$

Typical value of ratio in Eq. (21) is $\beta_2/\Delta\beta \approx 10$. To obtain the same value of luminosity in comoving collisions as that for head-on collisions, it is necessary to provide long interaction region with short bunches [4]. Merging beam collisions deliver smaller value of luminosity with short interaction region.

5 REFERENCES

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