RESONANT BEHAVIOUR OF HEAD TAIL MODES

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Abstract

Bunched beams in synchrotrons suffer from synchro-betatron resonances. They are produced by the coupling of the longitudinal motion into the transverse plane via dispersion and/or off-centre orbits in the accelerating cavities. These resonances are incoherent. The wake fields from bunches on central orbits, provoke head tail modes, which at a certain intensity will lead to the mode coupling instability. Besides head-tail modes, these wake fields create also new resonant conditions for coherent motion. In LEP, these coherent resonances, which are also present for central orbits, start to dominate the beam behaviour as of a certain bunch current and the mode coupling instability limit can only be reached for well defined betatron tunes

1 INTRODUCTION

In LEP the intensity is normally limited by the transfer mode coupling instability (TMCI). The threshold for this instability increases proportional with Qs (synchrotron tune). When more RF power became available for the LEP2 stage , higher Qs values could be used at injection. It was found that for Qs values above 0.15 the TMCI threshold could not be reached anymore (fig 1), and the intensity limit became very strongly dependent on the tune, suggesting a limitation due to synchro-betatron resonances. Incoherent synchro-betatron resonances could be observed in LEP [1], but they were never considered to be a big problem since there is enough comfortable tune space left and the resonances due to the Sundelin effect (which is intensity depending) were found to be rather weak. On the other hand, coherent resonances could be observed. For an intensity higher than 0.4 mA per bunch head tail modes become visible in the tune spectra and they show clearly a resonant behaviour [2],[3]. In order to understand the resonant behaviour of this modes, and their relationship to the "normal" synchro-betatron resonances calculations were performed using a two particle head-tail model. Such a model was already successfully used in order to explain the effect of the beam-beam interaction on head tail modes in LEP [4]. The two particle head tail model describes very well the behaviour of the dipole and quadrupole modes and, since the bunches in LEP are very short (1 to 2 cm at injection), higher modes can be ignored.

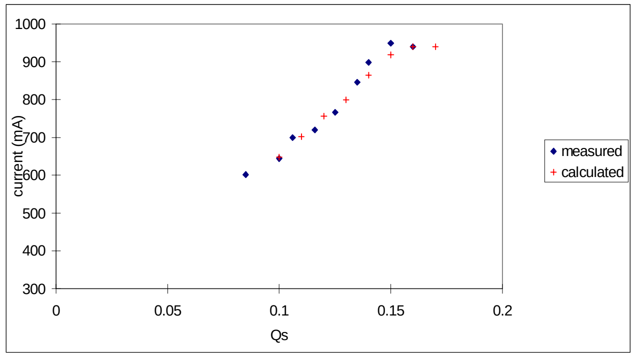

Fig 1 : The TMCI threshold can normally be increased by increasing Qs. At a certain Qs value however, coherent synchro-betatron resonances take over and limit the current to a value lower than the TMCI. In 1996 there was no gain for the LEP bunch current when pushing Qs above 0.15. The limit was then 0.95 mA/bunch. In 1998 this limit was increased to 1.05 mA/bunch by reducing the impedance (taking out copper cavities).

2 THE CALCULATION

The transverse coordinates of the two macro particles are represented by x and y. The equation of motion can be written as the equations of two coupled oscillators and an external oscillator describing the synchro betatron coupling :

$$x''+\omega^2 x = k_1 x + g(t)y + \delta \sin \omega_s t$$
$$y''+\omega^2 y = k_1 y + g(-t)x - \delta \sin \omega_s t$$

With :

ω : unperturbed betatron tune
k1 : describes the wake field from head on head or tail on tail
δ sin(ω_st) : describes coupling from synchrotron motion

The coupling coefficient g(t) describes the wake field form head on tail . It describes the fact that the wake field can only be "felt" when the particle is behind the other one and it can be represented as a square pulsed function with ω_s (the synchrotron frequency), as fundamental frequency. This block function can be developed in a fourier series of odd multiples of ω_s. For our calculation we will at first take only the fundamental mode into account.

$$g(t) \quad \frac{g_0}{2} + \frac{2}{\pi} g_0 \sum_{n\ 0}^{\infty} \frac{1}{(2n+1)} \sin((2n+1)\omega_s t)$$

putting : $S = x+y$ and $D = x-y$

$$w_1^{\,2} = \omega^2 - k_1 - g_0/2$$

$$w_2^{\,2} = \omega^2 - k_1 + g_0/2$$

$$k = g_0/\pi$$

(k_1,g_0 and k are proportional to the bunch current)
and keeping only the first harmonic of g(t) one gets :

$$S'' + w_1^{\,2} S \quad -2k \sin(\omega_s t) D$$
$$D'' + w_2^{\,2} D \quad 2k \sin(\omega_s t) S + 2\delta \sin(\omega_s t)$$

The variable S describes the centre of mass motion of the two particles (dipole mode) and the variable D describes the beam size (quadrupole mode). On a normal pick-up only the centre of mass motion is visible.

The fourier transform of the two equations looks like :

$$\tilde{S}(\Omega) \quad \frac{ik\tilde{D}(\Omega+\omega_s) - ik\tilde{D}(\Omega-\omega_s)}{w_1^{\,2} - \Omega^2}$$

$$\tilde{D}(\Omega) \quad \frac{ik\tilde{S}(\Omega-\omega_s) - ik\tilde{S}(\Omega+\omega_s)}{w_2^{\,2} - \Omega^2} + 2\delta S\tilde{Y}(\Omega)$$

The system can be described as two resonators (S and D) with frequencies w_1 and w_2. The two resonators are coupled trough a frequency shifter (+- ws) and the gain of the coupling is proportional to k i.e. the bunch intensity (fig 2). The external driving term from the synchro betatron resonance is coupled to the D resonator, because it has opposite phase for head and tail. In order to calculate the eigenfrequencies on cane inject white noise in the system e.g. at the entrance of S and calculate how it evolves following the arrows. The S modes represent the motion of the centre of mass and hence are visible on a normal pick up. The D modes represent the quadrupole mode and are invisible.

What has been left out in the formula for simplicity is the coupling to the systematic integer resonance which acts on the S resonator since it has the same phase for both particles. This coupling leads to the same conclusions as the synchro-betatron coupling.

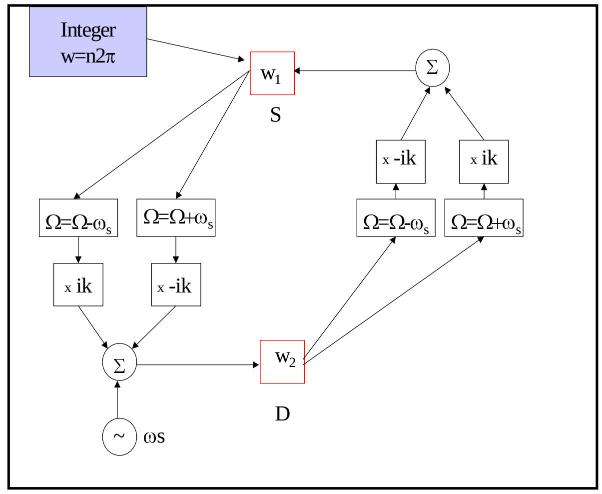

Fig 2 : Schematic representation of the two coupled oscillators S and D.

Only by looking at fig 2 one can draw some very interesting conclusions :

- w_1 and w_2 are changing with intensity in a different way . Once w_1 comes close to w_2+-ω_s the system starts to become unstable. This is the so called transfer mode coupling instability.
- The ωs which injected at the D resonator gets shifted by +- ω_s when it gets to the S resonator and the shifted again when going to, the D. This has two consequences :
  1) Even if the Qs oscillator is linear i.e. has no higher harmonics, the system will see higher harmonics of Qs.
  2) The S oscillator sees only the even multiples of Qs, the D oscillator sees the odd harmonics of Qs
- The systematic integer resonance is injected at the S resonator since it has the same phase for head and tail. The consequences are exactly the same as for the synchro-betatron coupling.
- VERY IMPORTANT : even in the absence of synchro-betatron coupling in the classical way (Sundelin or dispersion in cavities), the Qs harmonics will still be present. They are injected in the system from the systematic integer resonance which is frequency shifted by +-nws, n being even for the S resonator and odd for the D oscillator.
- Any frequency in S that comes close to w_2+-ωs will lead to a resonance.
- Any frequency in D that comes close to w_1+-ωs will lead to a resonance.
- The feedback from S to D is proportional to k (impedance x intensity), so the resonances become stronger for higher intensities.

3 RESULTS

Fig 3 and 4 show the calculated spectrum of S and D modes. In order to avoid singularities, a damping term (iλS' and iλD') was added to the equation of motion, λ being the damping constant at injection in LEP. The S modes can be compared to the measured spectrum in fig 5.

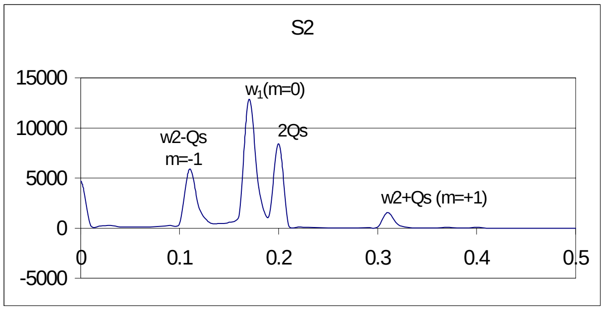

Fig 3 : calculated S-modes (visible modes)

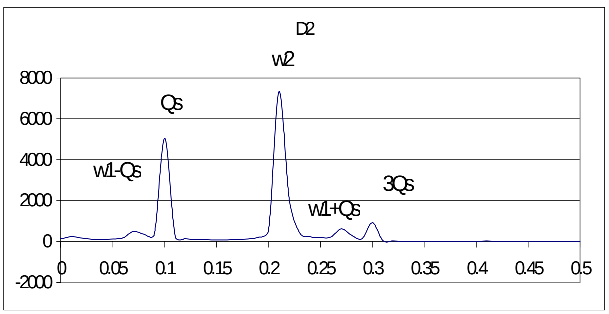

Fig 4 : calculated D modes (invisible)

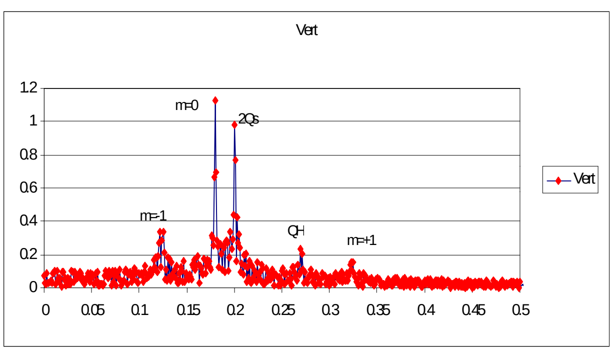

Fig 5 : measured vertical spectrum in LEP. Ibunch=0.5 mA.

In fig 6 the growth rate of the individual particle S and D modes combined as a function of the unperturbed tune (I=0.5 mA) . The dark lines are de visible modes, the faint lines are the invisible modes. The horizontal lines correspond to the multiples of Qs (Qs=0.15). Resonances occur when lines of the same colour cross. In fig 7, the same calculation was done for a bunch current of 0.8mA. The resonances start to become so strong that not much stable tune space is left.

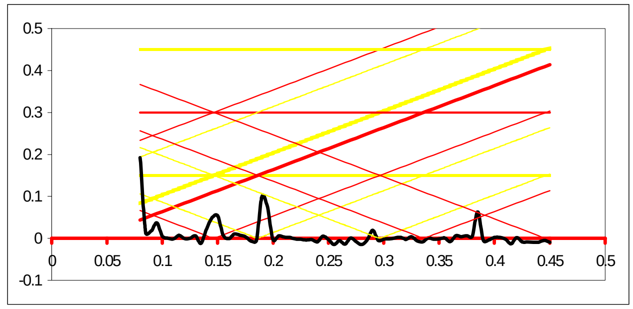

Fig 6

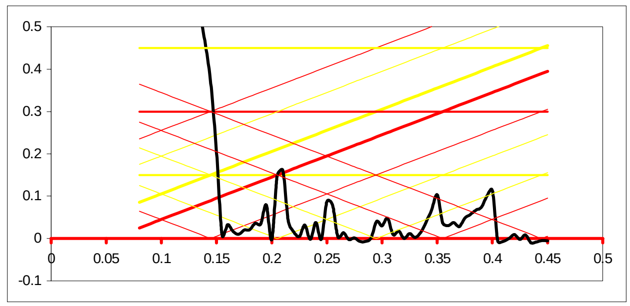

Fig 7 :Growth rate / damping for an intensity of 0.8 mA per bunch and a Qs of 0.15.

4 REFERENCES

[1] P. Collier, K. Cornelis, A. Hofmann, S. Myers, H. Schmickler, *Experimental observations of Synchro-Betatron resonances*, proceedings EPAC London, 1994.

[2] K. Cornelis, *Observations in LEP on coherent Synchro-betatron Resonances*, Proceedings of the ICFA workshop, Madeira, 1993

[3] K. Cornelis, M. Meddahi, H. Schmickler, *Resonant behavior of head tail modes*, SL-MD note 145, CERN, 1994.

[4] K. Cornelis, Guangxiao Li, *Calculation of TMC Threshold in the precence of Beam-Beam Force*, proceedings EPAC London, 1994.