

AN ANALYSIS OF BNS DAMPING TECHNIQUES IN STORAGE RINGS AND COLLIDERS*

G. Penn, J.S. Wurtele, UC Berkeley / LBNL CBP, Berkeley, CA

Abstract

Transverse instabilities in linacs can be controlled by BNS damping, in which the transverse oscillation frequency is chirped from the head to the tail of the bunch. It has been suggested that this technique could be applied to quasi-isochronous rings, such as are proposed for a muon collider. We adopt a reduced phase space model, taking averages over transverse coordinates and with proscribed longitudinal synchrotron motion, to study the growth of transverse displacements of a bunch in the presence of synchrotron oscillations.

1 MOTIVATION

Proposed parameters for muon collider require transverse emittances to be very small. Muons do not have significant cooling from synchrotron radiation; in addition, very low momentum compaction factors will be used to limit the RF requirements. For these reasons, coherent ring instabilities need to be investigated in parameter regimes different from present-day accelerators and may require new techniques to control them. The beam dynamics resembles linac physics; however, total storage time in a 2Tev \times 2Tev muon collider will be comparable to the synchrotron period so the longitudinal motion of the particles cannot be neglected. In addition, collider designs may require beams to be very precisely targeted, a condition which will be increasingly difficult to achieve as the emittance of the beam is made smaller.

2 FORMALISM

This work considers a simplified formalism for the analysis of transverse instabilities. Key features of this formalism are the use of a reduced phase space, with only longitudinal coordinates, and suppression of time scales shorter than the synchrotron period and instability growth time. The transverse displacement of the beam is described in terms of a complex amplitude which isolates the dipole moment, although this can be generalized to higher order modes.

The dynamic variables for individual particles are z, δ, y , and p_y . Instead of time, the variable s is used, representing distance travelled in the ring. The quantity $\delta = \Delta P/P$ is the normalized momentum deviation, and p_y is the transverse angle dy/ds . The equations of motion can be summarized as

$$\begin{aligned} z' &= -\eta\delta, & \delta' &= z\omega_s^2/\eta c^2 \\ y' &= p_y, & p_y' &= -Ky + F_W \end{aligned} \quad (1)$$

The synchrotron frequency ω_s is taken to be constant, whereas $K = \omega_\beta^2/c^2$ is allowed to vary. The forcing term F_W is from the wakefield generated collectively by the particle bunch. Here, the symbol $'$ refers to differentiation with respect to s . We assume $\omega_s \ll \omega_\beta$.

We now define the complex transverse amplitude of a single particle. When there is no wakefield forcing term, the particle dynamics are given by

$$y = \Re \frac{A}{\sqrt{\beta}} e^{i\Phi}, \quad p_y = \Re \frac{1}{\sqrt{\beta}} \left(i + \frac{1}{2}\beta' \right) A e^{i\Phi} \quad (2)$$

Here, A is a constant, complex amplitude, Φ is a phase which satisfies $\Phi' = 1/\beta$, and the beta function satisfies the standard equation

$$2\beta\beta'' - (\beta')^2 + 4\beta^2 K = 4. \quad (3)$$

When $F_W \neq 0$, Eqs. 1 and 2 can still be used to describe A and its evolution:

$$A\sqrt{\beta} e^{i\Phi} = y - i(\beta y' - \beta' y/2) \quad (4)$$

$$\frac{i}{\sqrt{\beta}} A' e^{i\Phi} = y'' + Ky = F_W. \quad (5)$$

Note that the wake must be expressed in terms of A .

Collective dynamics are described by the Vlasov equation, $f' = Df/Ds = 0$, where $f(y, p_y, z, \delta, s)$ is the phase space density and D/Ds is the convective derivative. Taking $K = K(s, z, \delta)$ to be independent of the transverse coordinates, we form moments of the Vlasov equation by integrating over the coordinates y and p_y . This yields equations in a reduced phase space, with reduced density

$$F(z, \delta, s) \equiv \int dy dp_y f,$$

$$\text{so } 0 = F' = [\partial/\partial s - \eta\delta\partial/\partial z + (z\omega_s^2/\eta c^2)\partial/\partial\delta]F.$$

The ‘‘average’’ transverse displacement Y is defined by

$$Y(z, \delta, s) \equiv \frac{1}{F} \int dy dp_y y f$$

Note the linear number density is $\rho(z) = \int d\delta F$. The equation Y satisfies is

$$Y'' + KY = F_W,$$

where F_W is now the wakefield force from the dipole moment of the beam. We next define a collective oscillation amplitude A in terms of Y and Y' , in exactly the same way as for a single particle, and average over betatron oscillations.

* Supported by the U.S. DoE under grant No. DEFG-03-95ER-40936.

3 SUPPRESSION OF FAST TIME SCALES

The complex amplitude satisfies

$$A' = i \frac{r_0}{2\gamma L} \int d\tilde{z} W(z - \tilde{z}) \int d\tilde{\delta} \sqrt{\beta\tilde{\beta}} \tilde{F} \left[\tilde{A} e^{i(\tilde{\Phi}-\Phi)} + \tilde{A}^* e^{-i(\tilde{\Phi}+\Phi)} \right] \quad (6)$$

where the $m = 1$ wakefield forcing term has been expressed as

$$F_W(z) = -\frac{r_0}{\gamma L} \int d\tilde{z} W(z - \tilde{z}) \int d\tilde{\delta} \tilde{Y} \tilde{F}. \quad (7)$$

Here $\tilde{}$ refers to quantities evaluated at \tilde{z} , $\tilde{\delta}$, and W refers to the wakefield function integrated along the ring. The A and A^* terms arise from the real part of $A \exp(i\Phi)$. Note that $\tilde{\Phi} - \Phi$ is a slowly varying quantity, because the difference in phase only arises from effects like chromaticity. However, the term proportional to A^* is rapidly oscillating and will not contribute to the long-term evolution of the bunch.

An average over one turn results in

$$A' \simeq i \frac{r_0 c}{2\gamma L \omega_{\beta 0}} \int d\tilde{z} W(z - \tilde{z}) \int d\tilde{\delta} \tilde{F} \tilde{A} e^{i(\tilde{\Psi}-\Psi)} \quad (8)$$

where $\omega_{\beta 0}$ is a normalization related to an “average” beta function, and the head-tail phase averaged over a turn is

$$\Psi(z, \delta) \equiv \frac{1}{L} \oint ds [\Phi(s, z, \delta) - \Phi(s, 0, 0)]$$

Equivalently,

$$\Psi'(z, \delta) = \frac{\omega_{\beta 0}}{c} \frac{\Delta Q}{Q}$$

where Q is the total tune of the ring for the reference orbit, and we defined $\omega_{\beta 0} \equiv 2\pi Q c / L$.

In going from Eq. 6 to Eq. 8, we integrated the product of the wakefield function and the beta function around the ring. Thus, it is clear that the “total” wakefield function is actually weighted by the beta function; the local value of $\beta W(z - \tilde{z})$ is replaced by $(c/\omega_{\beta 0})W(z - \tilde{z})$ to smooth out the driving term over a single turn. Finally, we are left with quantities that only vary with the synchrotron period and the growth time.

4 HEAD-TAIL PHASE

We consider tune shifts ΔQ from chromaticities ξ , and BNS-like terms κz , (e.g., induced by RF quadrupoles)[1], that can vary across the bunch. Explicitly,

$$\frac{\Delta Q}{Q} \simeq \xi_0 \delta + \xi_1 \frac{z}{\sigma_z} \delta + \kappa z.$$

The head-tail phase is then given by

$$\Psi = -\chi_0 \frac{z}{\sigma_z} - \frac{1}{2} \chi_1 \frac{z^2}{\sigma_z^2} + \chi_{BNS} \frac{\delta}{\sigma_\delta}$$

where σ_z and σ_δ are the characteristic longitudinal dimensions of the bunch. Here, $\chi_{0,1} \equiv \omega_{\beta 0} \xi_{0,1} \sigma_z / \eta c$ and $\chi_{BNS} \equiv \omega_{\beta 0} \eta c \kappa \sigma_\delta / \omega_s^2$.

4.1 Unified treatment of detuning

Incorporating the effect of tune shifts into an overall head-tail phase allows for a unified treatment of ring dynamics, including chromaticity, variable focussing, and more general longitudinal dynamics. Aspects of the beam which cannot be directly modelled in this way are transverse amplitude dependent tune shifts, and coupling between longitudinal and transverse motion.

If we consider the weak beam limit where the synchrotron motion is the dominant effect, it is useful to use the longitudinal coordinates r_z , ϕ_z , where $z = r_z \cos \phi_z$, $\delta = (\omega_s r_z / \eta c) \sin \phi_z$. The longitudinal amplitude r_z is a constant of the motion, and the convective time derivative can be expressed as $D/Ds = \partial/\partial s + (\omega_s/c)\partial/\partial \phi_z$. In the weak beam limit, modes having the form $A = A_n(r_z) e^{in\phi_z}$ are only weakly coupled. For an “air-bag” beam model, where all of the particles have the same longitudinal amplitude, the mode frequency satisfies the equation

$$\omega_\ell - \ell \omega_s = -\frac{r_0 c^2 N}{8\pi^2 \gamma L \omega_{\beta 0}} \oint d\phi_z \oint d\tilde{\phi}_z W(z - \tilde{z}) e^{i(\tilde{\Psi}-\Psi)} \quad (9)$$

Without the head-tail phase, the wakefield only generates a frequency shift, and there is no instability until the modes strongly couple. This corresponds to the strong head-tail instability. However, when there is a head-tail phase, for example from chromaticity ($\xi_0 \neq 0$), we recover the weak head-tail instability[2].

4.2 Parity of the head-tail phase

In general, the effect of the head-tail phase depends strongly on the parity of the phase with respect to the energy deviation δ . Because the unperturbed beam is symmetric in δ , a perturbation which is odd in energy will give no net contribution to the wakefield. To lowest order in beam intensity, a head-tail phase which is odd in δ will only couple with odd modes and so will not lead to instability. We can see this explicitly by dividing the head-tail phase into even and odd parts, $\Psi \equiv \Psi_e + \Psi_o$. Then as an example, Eq. 9 can be rewritten as

$$\omega_\ell - \ell \omega_s = -\frac{r_0 c^2 N}{8\pi^2 \gamma L \omega_{\beta 0}} \oint d\phi_z \oint d\tilde{\phi}_z W(z - \tilde{z}) \times \cos \Psi_o \cos \tilde{\Psi}_o \exp(i\tilde{\Psi}_e - i\Psi_e) \quad (10)$$

The growth rate depends on $\sin(\tilde{\Psi}_e - \Psi_e)$, and there is only a weak beam instability if the head-tail phase has a term which is even in δ . Thus, chromaticity can lead to instability whereas variable focussing across the bunch is not by itself destabilizing. This suggests that bunch stabilization through varying chromaticity, as considered in Ref. [3], requires a larger tune shift across the bunch than through BNS damping. On the other hand, a time-varying chromaticity $\xi_0(s)$ would introduce an odd component into the head-tail phase and should have a different effect on the beam.

5 NUMERICAL RESULTS

Here, we evaluate linear growth rates for a weak beam. A broadband impedance model is used for the wakefield:

$$W(z < 0) = W_0 \exp\left(\frac{\omega_R z}{c} \sin \theta\right) \sin\left(\frac{\omega_R z}{c} \cos \theta\right) \quad (11)$$

where $\omega_R = c/b$. The simplest model might use b equal to the pipe radius, $\theta = \pi/6$, and $W_0 = 16\pi R_0/\mu_0 c b^2$, where R_0 is related to the shunt impedance.

Growth rates normalized to $W_0 r_0 c^2 N / 8\pi^2 \gamma L \omega_{\beta 0}$ are shown in Fig. 1 as a function of $\sigma_z c / \omega_R$, for the case where $\xi_0 = 0.1$. The lowest harmonics are shown. When $\sigma_z < c/\omega_R$, the broadband impedance model is qualitatively similar to the approximation of a constant wakefield[2] where the sign of the growth rate for the $n = 0$ mode is opposite to that of all other modes.

We now fix $\sigma_z/b = 13/3$. In Fig. 2, we consider a linear dependence in tune as a simple form of BNS damping, and compare it to a varying chromaticity. Both effects are taken to induce comparable tune shifts $\Delta\omega_\beta \simeq \omega_s$.

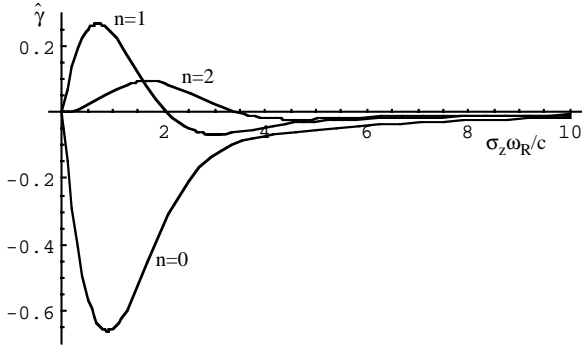


Figure 1: Normalized growth rates for $\chi_0 = 0.1$ as a function of σ_z/b . The lowest harmonics are shown.

6 LINAC MODEL

When the slip factor η is small, then the transverse dynamics become similar to the case of a linac. As a basic example, when Ψ is independent of energy (e.g., chromaticity effects), then we can define a local amplitude $A = \int d\delta F A$ which satisfies

$$\frac{\partial \mathcal{A}(z, s)}{\partial s} \simeq i \frac{r_0 c \rho(z)}{2\gamma L \omega_{\beta 0}} \int d\tilde{z} W(z - \tilde{z}) \mathcal{A}(\tilde{z}, s) e^{i\Psi(\tilde{z}) - i\Psi(z)} \quad (12)$$

This can be solved recursively to find the evolution of the beam, yielding the solution found in Ref. [2] when $\Psi \equiv 0$. When expressed in terms of the impedance Z , the time evolution operator has different contributions from the real and imaginary parts. While the real part leads to instability,

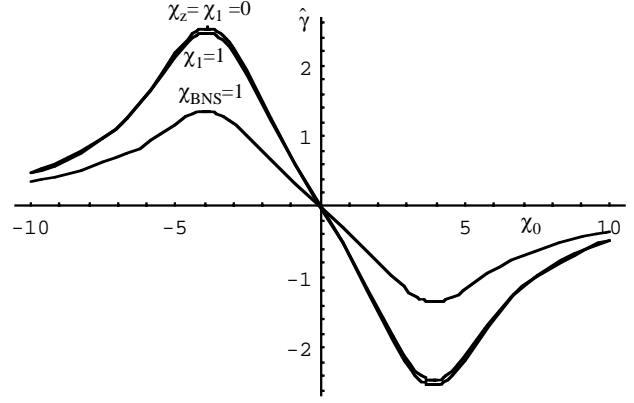


Figure 2: Normalized growth rates as a function of χ_0 for the lowest harmonic. Chromaticity alone, variable chromaticity with $\chi_1 = 1$, and BNS-like chirping with $\chi_{BNS} = 1$ are shown.

in the strong beam limit adding to $\Im m Z$ causes mixing of modes and can actually lower instability. For accelerators with intense beams, modifying the form of the impedance can be another way to stabilize the beam.

7 CONCLUSIONS

A formalism for treating transverse instabilities has been used to unify a wide range of physical effects on a beam, including variable chromaticity and BNS damping. The phase space has been reduced from four to two dimensions. The weak beam case reduces to the standard head-tail instability in a circular accelerator, while a strong beam with low slip factor is similar in form to beam dynamics in a linac.

Future work will focus on regimes where linac and ring physics have similarities, such as muon colliders. This analysis can be compared with existing numerical studies[4] for instabilities in a muon ring. In addition, the maximum displacement of the beam from wakefield forces can be calculated more efficiently in a reduced phase space, and adaptations of BNS damping evaluated on this basis.

8 REFERENCES

- [1] V.V. Danilov, "Increasing the transverse mode coupling instability threshold by RF quadrupole", Phys. Rev. STAB **1**, 041301 (1998).
- [2] A. Chao, *Physics of Collective Beam Instabilities in High Energy Accelerators*, John Wiley & Sons, NY, 1993.
- [3] W.-H. Cheng, A.M. Sessler, J.S. Wurtele, "Varying chromaticity: A damping mechanism for the transverse head-tail instability", Phys. Rev. E **45** 4695 (1997).
- [4] E.-S. Kim, A.M. Sessler, J.S. Wurtele, "Transverse instability in 50GeV \times 50GeV Muon collider ring", poster THP45, PAC99.