TRANSVERSE INSTABILITY IN A 50GEV×50GEV MUON COLLIDER RING *

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Abstract

The intense bunch which is called for in the muon collider design will be subject to transverse instability. It has been suggested that tune spread due to BNS damping be used to control the instability. The transverse dynamics in a 50GeV×50GeV muon collider ring is examined numerically using a broad-band impedance. In addition to the BNS damping, tune spreads due to chromaticity, amplitude dependent tune shift and the beam-beam interaction are taken into account. It is shown that each of these tune spreads adequately stabilize the transverse dynamics.

1 INTRODUCTION

The design of a 50 GeV muon collider ring, from the view of collective effects, has some features which need to be examined. The bunch has a large charge: N=4×10^{12}. The bunch length is long: σ_z=13cm. The momentum compaction η_1 is very small: $\eta_1 \sim -10^{-6}$: a muon would undergo 1 synchrotron oscillation in 20000 turns. Muons have a very short lifetime: $\tau_\mu \simeq$ 1.1 ms at 50 GeV, corresponding to 1000 turns in a ring with the circumference of 300 meters. These parameters lead us to careful analysis in the ring operation: the intense bunch may cause instabilities and the very small η_1 gives dynamics similar to a linac.

This paper presents investigations on the collective transverse beam dynamics in a 50GeV×50GeV muon collider ring. To limit rf, it has been proposed to operate the ring close to the transition (η_1=-10^{-6}), such that the synchrotron motion is almost frozen over the storage time. This will require the use of two rf cavities to compensate for the energy spread induced through the longitudinal wakefield[1]. Furthermore, because only a small rf voltage is required to compensate the longitudinal wake, the impedances of the two rf cavities are negligible compared to the ring impedance. In this paper, the transverse force on the muons was modeled by a broadband impedance, corresponding to an averaged wakefield that does not include the rf cavities. The wakefield in the simulation is calculated by using a technique, described in Ref. [1], where bins of variable width increase the accuracy and reduce computing time.

Since synchrotron radiation damping and Landau damping are negligible, they do not help in stabilizing against collective instabilities. It was previously shown that a BNS tune spread damps the transverse instability [2]. In this paper, it is shown through a numerical simulation, that the transverse instability is also damped by tune spreads due to chromaticity, amplitude dependent tune shifts, and the beam-beam interaction.

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2 THE COMPUTATION OF THE WAKEFIELD AND MACROPARTICLE EQUATIONS

The transverse wake generated by a beam interacting with discontinuities of components in the ring is approximated by a broad-band impedance with quality factor of the order of unity. The transverse wake function is simply related to the longitudinal impedance [3],

$$Z_1^\perp \sim \frac{2c}{b^2 w} Z_0^{||}, \tag{1}$$

$$W_1(z) = \frac{-i}{2\pi} \int_{-\infty}^{\infty} Z_1(w) e^{iwz/c} dw, \tag{2}$$

With this definition and a broadband resonator model for the longitudinal impedance, the transverse wake function $W_1(z)$ for the broad-band impedance is given by

$$W_1(z<0) = \frac{2cR_s\omega_R}{b^2 Q\bar{\omega}} e^{\alpha z/c} \sin\frac{\bar{\omega}z}{c}, \tag{3}$$

where $\alpha = \omega_R/2Q$ and $\bar{\omega} = \sqrt{w_R^2 - \alpha^2}$, Q=$R_s\sqrt{C/L}$ is the quality factor, $w_R = 1/\sqrt{LC}$ is the resonant frequency and $R_s = C/(2\pi b)Z_{||}/n_h$ is the shunt impedance. For the broad-band model, $Q = 1$ and $\omega_R = c/b$, where b is the vacuum pipe radius. Here C is the ring circumference and n_h is the harmonic number.

The transverse equation of motion is

$$\begin{aligned} &y''(z,s) + (\omega_\beta^2/c^2)\, y(z,s) \\ &= -(r_0/\gamma C) \int_z^{\infty} dz' \rho(z') W_1(z-z') y(z',s), \end{aligned} \tag{4}$$

where w_β is the betatron frequency, r_o=$e^2/m_\mu c^2$, $\gamma = E/m_\mu c^2$ and $\rho(z)$ is the particle distribution function.

Macroparticles are tracked in phase space with equations of motion which include a transverse wake due to a broad-band impedance and betatron oscillation. Each macroparticle i has transverse coordinates (y_i, y_i') and is tracked for 1000 turns. $' = d/ds$, where s measures distance around the ring. The transverse dynamics from the betatron motion for the i^{th} macroparticle on any turn n are derived from its coordinates on turn $n-1$ by:

$$\begin{pmatrix} y_i(n) \\ y_i'(n) \end{pmatrix} = \begin{pmatrix} \cos(\frac{\omega_\beta C}{c}) & \sin(\frac{\omega_\beta C}{c}) \\ -\sin(\frac{\omega_\beta C}{c}) & \cos(\frac{\omega_\beta C}{c}) \end{pmatrix} \begin{pmatrix} y_i(n-1) \\ y_i'(n-1) \end{pmatrix} \tag{5}$$

$$\omega_\beta = \omega_{\beta o}(1 + a(y_i^2 + y_i'^2) + \xi\delta + \Delta\nu_{BNS}/\nu_\beta), \tag{6}$$

where $a(y_i^2(n) + y_i'^2(n))$ is the amplitude dependent tune shift, $\xi\delta$ is the tune spread due to chromaticity and $\Delta\nu_{BNS}/\nu_{\beta o}$ is the tune spread due to BNS damping. Here a is the nonlinearity parameter, ξ is the chromaticity and $\delta = \delta P/P$ is the relative momentum error of the particle,

The wakefield force is included by a kick:

$$\Delta y_i'(n) = -\frac{c}{\omega_\beta}\frac{N_p r_o}{N_m \gamma}\sum_J^{I<J} Y(J) W_1(z_I(n) - z_J(n)), \tag{7}$$

where $Y(J)$ is the sum of the transverse displacements of the macroparticles in the J bin, and $z_I(n) - z_J(n)$ is the longitudinal separation of the bins ($z(I)$ is the average longitudinal position of the particles in the bin I). The i^{th} particle is in bin I. Also, N_p is the number of particles in the bunch and N_m is the number of macroparticles.

We used 1000 bins for 20000 macroparticles in the simulation. The numbers of bins and macroparticles must be properly chosen, as they depend on ring parameters such as the bunch length and the radius of the beam pipe. The initial coordinates of macroparticles are chosen to have a Gaussian distribution. Since the beam intensity decreases due to the muon decay, rf voltage is also varied in time. Rf parameters used in the simulation is shown in Table-1.

Table 1: Parameters of a 50GeV collider ring and rf parameters used in the simulation.

Beam energy (E)	GeV	50
Muons per bunch (N)	10^{12}	2
Circumference (C)	m	300
Bunch length (σ_z)	cm	13
Bunch energy spread (σ_δ)	%	0.003
Slippage factor (η_1)	10^{-6}	-1
Beam pipe radius (b)	cm	3
Beam-beam parameter(ζ)	10^{-2}	1.5
RF energy (V_{rf})	MV	0.34 and 0.99
RF frequency (f_{rf})	MHz	823 and 399
RF phase-offset (ϕ_{rf})	radian	3.755 and 3.415

3 BEAM BREAKUP

Due to the very small slippage factor(η_1=-10^{-6}), the synchrotron motion is frozen in the storage time and transverse dynamics is similar to that in a linac. In the case that a beam is not centered in the beam pipe, the transverse wake field driven by the head of the bunch perturbs the tail, causing beam breakup(BBU)-like instability. A dimensionless growth parameter that characterizes the BBU strength is

$$\Upsilon(z) = N r_0 W_1(z)/2k_\beta\gamma, \tag{8}$$

where k_β is the betatron wave number, and $W_1(z) = \int_z^\infty dz' W_1(z-z')\rho(z')$ is the transverse wake function [3].

Fig.1 shows the BBU-like instability using a broad-band resonator model in the absence of betatron tune spread. The initial displacement of the beam (Δ_y) is 0.2cm. The transverse motion shows unstable although there is not much growth. The growth can be damped due to betatron tune spreads, as discussed below.

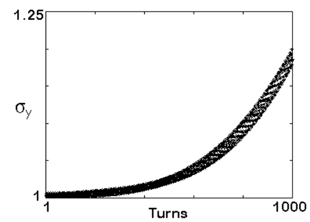

Figure 1: Blowup of the rms beam size, due to a BBU-like instability. In Figs.1-3 and 5-6, R_s=13000Ω, Δ_y=0.2cm and ν_β=28.816.

4 DAMPING OF BEAM BREAKUP BY BETATRON TUNE SPREADS

4.1 BNS damping

The BNS damping can be obtained by introducing a slightly stronger betatron focusing of bunch tail than bunch head [3]. A slightly different focusing strength for the bunch tail can eliminate the beam breakup. This happens because the additional focusing force compensate for the defocusing dipole deflection due to the wake field left behind by the bunch head. The tune spread is defined by

$$\omega_\beta = \omega_{\beta o}(1 + \frac{\Delta\nu_{BNS}}{\nu_\beta}), \tag{9}$$

$$\frac{\delta\nu_{BNS}}{\nu_\beta} = -\frac{N r_o}{2k_\beta^2\gamma\, C}\int_z^\infty W_1(z-z')\rho(z')dz', \tag{10}$$

where k_β is the betatron wave number. Fig.(2) shows that the BBU-like instability is stabilized by BNS damping. The simulation (in runs not shown) indicates that BNS damping more effective when bunch length is small compared to beam pipe radius. Fig.(3) shows the BNS tune spread which is applied in the longitudinal positions of the beam to satisfy Eq.(10). Fig.(4) separates parameter regions of beam pipe radius and shunt impedance(R_s) according to the appearance of a BBU-like instability.

4.2 Tune spread due to chromaticity

The betatron oscillation frequency of a particle in ring depends on the energy error δ of the particle. If we denote betatron frequency of an on-momentum particle as $\omega_{\beta o}$, the betatron frequency for an off-momentum particle can be written as $\omega_\beta(\delta) = \omega_{\beta o}(1 + \xi\delta)$. Fig.(5) shows that tune spread from chromaticity stabilizes the motion.

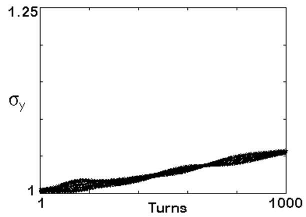

Figure 2: Rms beam size when there is a BNS tune spread.

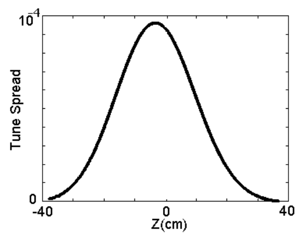

Figure 3: Maginitudes of the BNS tune variation with longitudinal position.

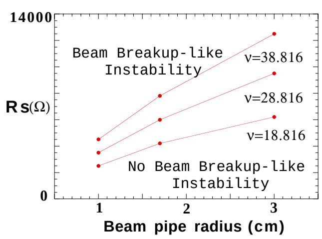

Figure 4: Parameter regions for various tunes, ν, as a function of pipe radius and shunt impedance(R_s).

4.3 *Amplitude dependent tune shift*

We consider the case when the tune spread is associated with the lattice nonlinearity. The amplitude dependent tune shift is defined by $\omega_\beta = \omega_{\beta o}(1 + aJ)$, where $\omega_{\beta o}$ is the betatron frequency at zero betatron amplitude, $J = y^2 + y'^2$ is the amplitude of the betatron oscillation of a particle. The tune spread in a beam may be caused by the amplitude dependent tune shift since the beam has a distribution in amplitude. Fig.(6) shows the stabilized motion when there is an amplitude dependent tune shift.

4.4 *Beam-beam interaction*

We use a round beam model to study beam-beam effects. The beam-beam kick is calculated by using a weak-strong model. A macroparticle is kicked at interaction point(IP) as

$$\Delta y = 0, \quad \Delta y' = \frac{8\pi\zeta\epsilon_N}{\gamma}\frac{1}{\sqrt{2}y}(1 - e^{-y^2/\sigma_y^2}), \qquad (11)$$

where $\zeta = \frac{r_o N}{4\pi\epsilon_N}$ is the beam-beam parameter and ϵ_N is the normalized emittance. A beam-beam parameter greater than ζ=0.035 damps the transverse instability.

Since a 50 GeV muon beam has very small energy spread and long bunch length, in addition to the transverse kick, the model of the beam-beam interaction should include energy change caused by the opposing bunch at the IP [4, 5]. Our simulation predicts a factor of 4.6 increase in the energy spread after 1000 turns. This is easily compensated for by rf, as will be presented in a later publication.

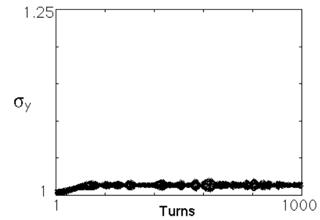

Figure 5: Rms beam size when there is tune spread due to the chromaticity, ξ=-2.

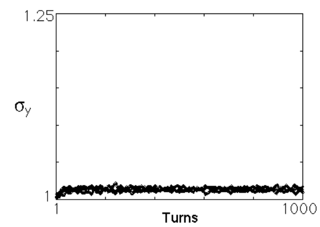

Figure 6: Rms beam size when there is the amplitude dependent tune shift, a=10^{-4}.

5 CONCLUSIONS

The transverse instability of a bunch with a small η_1 and strong wake fields is seen to be BBU-like and can be stabilized by betatron tune spreads. BNS damping, chromaticity, amplitude dependent tune shift and beam-beam parameter greater than ζ=0.035 all cause betatron tune spreads and each provide adequate Landau damping against the transverse instability for parameters of the 50GeV$\times$50GeV muon collider ring.

6 REFERENCES

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