THE INTRINSIC UPPER LIMIT TO THE BEAM ENERGY OF AN ELECTRON-POSITRON CIRCULAR COLLIDER

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Abstract

In this paper we predict that there exists an intrinsic upper limit to the beam energy of an e^+e^- circular collider due to the beam-beam effects. The maximum beam energy is given by $E_{0,max} = 85.17\sqrt{N_{IP}}$(GeV), where N_{IP} is the number of the interaction points in the ring. It is concluded that LEP is the only machine on this planet which has the potential to test the validity of this theoretical prediction by reducing its interaction points to one.

1 INTRODUCTION

Since the first report on the interactions of 200MeV+200MeV e^+e^- beams in the single storage ring device, named AdA, was appeared in 1964 [1], the energy of the interacting electron and positron beams has increased by a factor of 300 in about 30 years. Due to synchrotron radiation the cost of an e^+e^- circular collider is increasing with energy squared [2]. It is taken for granted that e^+e^- collisions at energies beyond LEP II [3] can only be realized by using a linear collider. The questions one may ask, however, are that disregarding the problem of cost whether there exists an intrinsic upper limit for the beam energy of an e^+e^- circular collider, and if it exists, which value it takes. In this paper we try to answer these questions.

2 THEORY

Let's consider a storage ring of single beam (without interaction points and the beam-beam effects of course). Apparently, in this case we could not see any physical reason which limits the possible beam energy. As far as an e^+e^- storage ring collider is concerned the situation is quite different, and obviously, the difference is due to the additional interaction points and the beam-beam effects. To answer our questions it is worthwhile to start with the discussion on the beam-beam effects and the maximum beam-beam tune shift. The luminosity of a circular collider can be expressed as

$$\mathcal{L} = \frac{I_{beam}\gamma\xi_y}{2er_e\beta_y^*}\left(1 + \frac{\sigma_y^*}{\sigma_x^*}\right) \quad (1)$$

where r_e is the electron radius, β_y^* is the beta function value at the interaction point, γ is the normalized particle energy, σ_x^* and σ_y^* are the bunch transverse dimensions after the pinch effect, respectively, I_{beam} is the circulating current of one beam, and

$$\xi_y = \frac{N_e r_e \beta_y^*}{2\pi\gamma\sigma_y^*(\sigma_x^* + \sigma_y^*)} \quad (2)$$

is the vertical beam-beam tune shift. At each interaction point particles in a bunch will be deflected transversely by the counter-rotating bunch. According to the linear theory of beam-beam dynamics [4], one knows that for two equal charge Gaussian bunches after each collision, the average beam-beam kicks of each particle in the horizontal and the vertical planes are expressed as follows

$$\delta x' = -\frac{2N_e r_e x}{\gamma\sigma_{x,+}^*(\sigma_{x,+}^* + \sigma_{y,+}^*)} \quad (3)$$

$$\delta y' = -\frac{2N_e r_e y}{\gamma\sigma_{y,+}^*(\sigma_{x,+}^* + \sigma_{y,+}^*)} \quad (4)$$

where $\sigma_{x,+}^*$ and $\sigma_{y,+}^*$ are the bunch transverse dimensions just before the interaction point. In fact, these kicks are *random* and they will move the zero-current equilibrium transverse beam sizes to the new values which depend on the bunch current [5]. Assuming that there are N_{IP} interaction points in the machine, and taking into account of these additional independent random kicks, one gets the new equilibrium horizontal and vertical emittances expressed as [5]:

$$\epsilon_x = \epsilon_{x0}\left(1 - \frac{(e^2 N_e \mathcal{K}_{IP,BB,x})^2 N_{IP}\tau_x}{4T_0 E_0^2}\right)^{-1} \quad (5)$$

and

$$\epsilon_y = \epsilon_{y0}\left(1 - \frac{(e^2 N_e \mathcal{K}_{IP,BB,y})^2 N_{IP}\tau_y}{4T_0 E_0^2}\right)^{-1} \quad (6)$$

where T_0 is the revolution period, E_0 is the beam energy, τ_x, τ_y are the horizontal and vertical damping time, respectively, ϵ_{x0}, ϵ_{y0} are the horizontal and vertical natural emittance, respectively, and

$$\mathcal{K}_{IP,BB,x} = \frac{\beta_x^*}{2\pi\epsilon_0\sigma_{x,+}^*(\sigma_{x+}^* + \sigma_{y,+}^*)} \quad (7)$$

$$\mathcal{K}_{IP,BB,y} = \frac{\beta_y^*}{2\pi\epsilon_0\sigma_{y,+}^*(\sigma_{x,+}^* + \sigma_{y,+}^*)} \quad (8)$$

where ϵ_0 is the permittivity of vacuum. For an isomagnetic ring, one gets

$$\epsilon_x = \epsilon_{x0}\left(1 - \frac{3\epsilon_0 R(eN_e\mathcal{K}_{IP,BB,x})^2 N_{IP}}{2m_0 c^2\gamma^5 J_x}\right)^{-1} \quad (9)$$

and

$$\epsilon_y = \epsilon_{y0}\left(1 - \frac{3\epsilon_0 R(eN_e\mathcal{K}_{IP,BB,y})^2 N_{IP}}{2m_0 c^2\gamma^5}\right)^{-1} \quad (10)$$

where R is the local bending radius. For a flat bunch ($\sigma_{y,+}^* \ll \sigma_{x,+}^*$, in the following we restrict ourselves to this case), from eq. 10 one knows that

$$\sigma_{x,+}^* + \sigma_{y,+}^* > \left(\frac{3RN_{IP}(eN_e\beta_y^*)^2}{8\pi^2\epsilon_0 m_0 c^2 \gamma^5}\right)^{1/2} \qquad (11)$$

Defining

$$H = \frac{\sigma_{x,+}^* + \sigma_{y,+}^*}{\sigma_x^* \sigma_y^*} \qquad (12)$$

where H is a measure of the *pinch* effect, and keeping in mind the physics of beam-beam effect at the interaction point, one can write

$$H = \frac{H_0\sqrt{N_{IP}}}{\gamma} \qquad (13)$$

where H_0 is a constant and its value will be determined later. H is an important quantity and we will go back to it later. Combining eqs. 2, 11 and 13 one gets finally [5] (isomagnetic case)

$$\xi_y \le \xi_{y,max} = H_0\sqrt{\frac{\gamma r_e}{6\pi R}} \qquad (14)$$

or, for general cases

$$\xi_y \le \xi_{y,max} = \frac{H_0}{2\pi\gamma}\sqrt{\frac{T_0}{\tau_y}} \qquad (15)$$

We have therefore found the analytical expression for $\xi_{y,max}$ and explained the well-known phenomenon in circular colliders that $\xi_y \le \xi_{y,max}$. The experimentally reached maximum H_0 is found to be about $\frac{1}{6} \times 10^6$. The validity of eqs. 14 and 15 has been demonstrated in ref. 5. On the road to arrive at eqs. 14 and 15 one has defined, expressed and used the pinch effect factor, H (see eq. 13). Now we go back to it and bear in mind that this quantity is one of the base stones of the analytical expressions for the maximum beam-beam tune shift which agree well with the experimental results. The reason why we call H a measure of the *pinch* effect lies in the fact that one has always (for the existing operational e^+e^- circular colliders) $H > 1$. The beam-beam effects not only increase the beam transverse emittances, as shown in eqs. 9 and 10, hence the transverse beam sizes, but also contribute to the maintenance of the beam transverse stabilities. One can imagine that one interaction point acts as a locally fastened belt on the beam "waist". At this point it is natural for one to raise his hand and ask the following question: *what happens if $H \le 1$?* It is clear that by definition when $H = 1$ the pinch effect disappears, and the colliding beams in the collider act as a single beam in a storage ring (but with larger transverse equilibrium emittances due to the random beam-beam excitations). As for the case when $H < 1$ the physical intuition suggests us that this situation be unstable
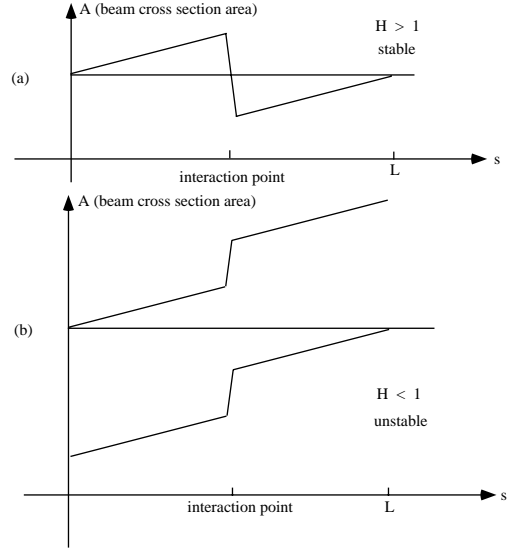

Figure 1: Schematical showing the difference between $H > 1$ and $H < 1$, where there is only one ineraction point in the ring for simplicity, and L is the perimeter of the ring.

since the interaction point explodes the beam as shown in Fig. 1. As bold as one can be we predict that

$$\gamma_{max} = H_0\sqrt{N_{IP}} \qquad (16)$$

which corresponds to $H = 1$, as the intrinsic upper limit to the beam energy of an e^+e^- circular collider. For $N_{IP} = 1$ one gets from eq. 16 that $\gamma_{max} = H_0 = \frac{1}{6} \times 10^6$ or $E_0 = 85.17$GeV. Apparently, LEP has not met the limit due to its multi-interaction points ($E_{0,max}$(GeV)$= 85.17\sqrt{N_{IP}}$). It is interesting to predict that LEP II [6] (90GeV+90GeV) can not be realized if there is only one interaction point. In this planet LEP is the largest e^+e^- circular collider and the only machine having the potential to test this theoretical prediction by reducing the number of its interaction points to one.

From the point of view of the author the choice of an e^+e^- linear collider as the replacement to the circular one at energies beyond LEP II has its physical reason behind.

3 CONCLUSION

In this paper we predict an intrinsic upper limit to the beam energy in an e^+e^- circular collider due to beam-beam instabilities. The maximum energy is expressed as $E_{0,max} = 85.17\sqrt{N_{IP}}$(GeV). It is obvious that LEP is the only machine on this planet to have the potential to test this theoretical prediction by reducing the number of its interaction points to one.

4 REFERENCES

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