THE REGIMES OF POLARIZATION IN A HIGH ENERGY e^+e^- STORAGE RING

R.W. Assmann, CERN, SL Division, Geneva, Switzerland

Abstract

Several regimes of polarization must be considered for high-energy e^+e^- storage rings. Based on a theoretical paper by Derbenev, Kontratenko and Skrinsky from 1979 we describe the different cases. Particularly, it is shown that from a certain high energy onwards the polarization degree is expected to increase with energy. This is in sharp contrast to the usually considered regime where the expected polarization level decreases for higher beam energies. The theory of Derbenev, Kontratenko and Skrinsky is applied to the LEP storage ring with its energy range from the Z at 91 GeV to the W at 200 GeV. Though the theoretical expectations for beam polarization at the highest beam energy remain low, it is shown that the depolarization can move into a new regime for LEP above 60 GeV. The high energy LEP is the first storage ring that operates in this new and experimentally unknown regime of beam spin dynamics.

1 INTRODUCTION

Radiative polarization of the particle beams provides the most accurate tool to measure the beam energy in LEP [1,2]. In order to determine the W mass with the required accuracy it is important to establish polarized beams for the highest possible energies. Sufficient radiative spin-polarization in LEP was measured up to 60.6 GeV [3]. In order to examine the possibilities for establishing polarization at even higher energies, the theory of radiative spin polarization in e^+e^- storage rings must be analyzed carefully. This paper reviews the relevant theory and applies it to LEP parameters.

2 THEORY OF POLARIZATION AT ULTRA-HIGH ENERGIES

The particle beams in LEP spontaneously polarize due to the Sokolov-Ternov effect [4]. A polarization build-up time τ_p and an ideal final polarization degree of 92.4% characterize the process. The polarization rate λ is the inverse of the build-up time τ_p and is here used in units of the LEP revolution frequency. It is a steep function of the spin tune ν. The spin tune is the precession frequency of the spin vectors and can be expressed via the beam energy E: $\nu = a\,\gamma = E / 440.6486$ MeV. Unavoidable imperfections in the vertical orbit cause depolarization. It turns out that synchrotron radiation drives both polarizing and depolarizing processes. The depolarization is characterized by a depolarization time τ_d and the asymptotic degree of polarization is reduced to:

$$P = \frac{92.4\%}{1 + \tau_p / \tau_d} \qquad (1)$$

Polarization theories aim at estimating the depolarization term τ_d. Here, we follow the basic theory by Derbenev, Kondratenko and Skrinsky from their summary paper in 1979 [5].

2.1. Basic Quantities

A few basic beam and machine parameters determine the behavior of polarization:
1) The *spin tune* ν describes the energy dependence of polarization. 2) The *polarizing rate* λ determines the speed of polarization buildup. 3) The *synchrotron tune* Q_s gives the distance between synchrotron sidebands of spin resonances. 4) The *spin tune spread* σ_ν causes a smearing out of spin precession frequencies so that they eventually overlap Q_s sideband resonances.
The particles in LEP traverse the ring about 11000 times per second. Let's assume the average spin tune ν_0 is not on any resonance. However, particles perform synchrotron oscillations around the average spin tune: $\nu = \nu_0 + \delta\nu$. Depending on the spin tune spread some particles might be on a spin resonance, for example $\nu = k \pm n \cdot Q_s$. During a large number of subsequent turns the particles will periodically cross the spin resonance. In order to evaluate the depolarizing effect on the ensemble polarization, it must be determined whether subsequent passings of a spin resonance are correlated or not. As shown by Derbenev, Kondratenko and Skrinsky, the criterion for correlated passings is:

$$\alpha = \frac{\nu^2 \lambda}{Q_s^3} < 1 \qquad (2)$$

If subsequent passings are correlated then spin rotations can average out to some extent and their effect is less severe.

2.2. Correlated Spin Resonance Passings

The following theory applies if the correlation criterion in Equation 2 is true. Polarization can be described by:

$$\frac{\tau_p}{\tau_d} = \frac{11}{18} \nu^2 \sum_{k,m} \frac{|w_k|^2 \left\langle T_m^2(\Delta / Q_s) \right\rangle}{\left[(k - \nu - mQ_s)^2 - Q_s^2 \right]^2} \qquad (3)$$

Here, w_k is the complex strength of the spin resonance at integer k, ν is the spin tune averaged over the ensemble and m an integer giving the order of the synchrotron sideband resonance. The equation contains a Bessel function term T_m. Assuming a Gaussian distribution over squared amplitudes Δ of synchrotron oscillations one obtains:

$$\langle T_m^2 \rangle = I_m\left(\frac{\sigma_\nu^2}{2Q_s^2}\right) \cdot \exp\left(-\frac{\sigma_\nu^2}{2Q_s^2}\right) \qquad (4)$$

The I_m are the modified Bessel functions. The spin tune spread is of central importance for the strength of the T_m term. The above equations are valid in the approximation of high energy. Note that betatron spin resonances with the transverse tunes Q_x and Q_y do not appear. For high energy lepton storage rings they are much weaker than synchrotron resonances and are therefore neglected here.
Two regimes are distinguished in the regime of correlated spin resonance passings. If the spin tune spread is much smaller than the synchrotron tune then higher order synchrotron sidebands are not important and only the linear spin resonances ($k \pm Q_s$) affect the achievable polarization degree. In the following this is called the "linear" theory. If the spin tune spread becomes larger than the synchrotron tune then the higher order synchrotron sidebands limit the achievable polarization degree. This is referred to as "higher-order theory".

2.2. Uncorrelated Spin Resonance Passings

A different situation is encountered if subsequent passings of spin resonances are uncorrelated. They are uncorrelated if the criterion from Equation 2 is not true and in addition $\sigma_\nu >> \nu_\gamma$. In this case passings of synchrotron resonances are completely uncorrelated. For LEP uncorrelated passings are always completely uncorrelated. With $\sigma_\nu << 1$ the polarization can be calculated from:

$$\frac{\tau_p}{\tau_d} = \frac{11\pi^4}{54} \cdot \nu^2 \cdot \left|w_{[\nu]}\right|^2 \cdot \left[1 + \frac{108\exp(-2\sigma_\nu)^{-2}}{11\pi^3\sqrt{\pi}\,\nu^2\lambda}\right] \qquad (5)$$

In the case of $\sigma_\nu >> 1$ Derbenev, Kondratenko and Skrinsky have obtained a very simple result for the expected depolarization:

$$\frac{\tau_p}{\tau_d} = \frac{\pi\left|w_k\right|^2}{\lambda} \qquad (6)$$

Polarization does not show any resonant dependence on beam energy in this regime, but exhibits an increase with energy, as the polarizing rate λ becomes very large for highest energies. In this regime the spin tune spread σ_ν is very large and particles constantly sweep over spin resonances. As the polarization rate increases, depolarization does not increase as rapidly any more.

The theory by Derbenev, Kondratenko and Skrinsky does not include the LEP energy sawtooth that causes rapid energy changes of ± 500 MeV with 100 GeV average beam energy. The single particles constantly cross the integer and linear spin resonances. The crossings are very fast, about 40 times faster than the synchrotron oscillation. Therefore the associated crossings of spin resonances might be fully correlated and cause little harm. However, the consequences of the sawtooth on the spin motion in LEP are not clear and require further study.

3 LEP PARAMETERS

The polarizing rate λ (in units of the revolution frequency) for LEP is:

$$\lambda = \frac{1}{\tau_p} = 3.9 \times 10^{-19} \cdot \left(\frac{E}{0.44065\ \mathrm{GeV}}\right)^5 \qquad (7)$$

The resonance strength w_k is calculated from 57% polarization at 44.7 GeV [6][1]:

$$\left|w_k\right|^2 = 1.94 \times 10^{-10} \cdot \left(\frac{E}{0.44065\ \mathrm{GeV}}\right)^2 \qquad (8)$$

The spin tune spread σ_ν is proportional to the square of the beam energy E:

$$\sigma_\nu = 6.76 \times 10^{-6} \cdot \left(\frac{E}{0.44065\ \mathrm{GeV}}\right)^2 \qquad (9)$$

4 PREDICTIONS FOR LEP

The numerical evaluation of the correlation criterion is shown in Figure 1. It is seen that the LEP1 working point is clearly in the correlated regime as is the working point at 60.6 GeV with Q_s = 1/11. With high energy and small Q_s LEP moves into the uncorrelated regime of spin polarization.

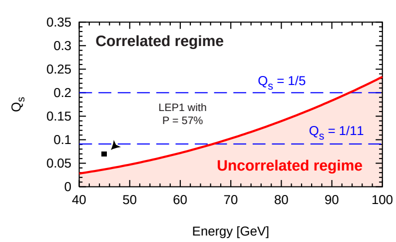

Figure 1: Evaluation of correlation criteria as a function of beam energy and Q_s. LEP can stay in the correlated regime by increasing the value of the synchrotron tune Q_s.

The polarization optics (60°/60°) allows a high Q_s of above 0.25 until its aperture limit at around 75 GeV (see Figure 2). With that Q_s LEP remains in the correlated

[1] The linear polarization then decreases with the fourth power of energy. This result by Derbenev, Kontratenko and Skrinsky is not in agreement with other predictions. Their result might be overly pessimistic.

regime of spin resonance passings and the higher-order polarization theory can be used to predict the achievable polarization degree (Equations 3 and 4). Note that this theory correctly predicts the decrease of polarization with energy that is observed in LEP (see Figure 3). The predicted polarization degree has been evaluated as a function of beam energy and for different values of the synchrotron tune Q_s. In order to achieve the maximum distance to all spin resonances, Q_s is chosen to be equal to one over an odd integer. For illustration we consider Q_s values of 1/13 (LEP1 value), 1/5 and 1/3. The predicted dependence of polarization on energy is shown in Figure 4. High Q_s values significantly improve the chances of spin polarization for higher beam energies.

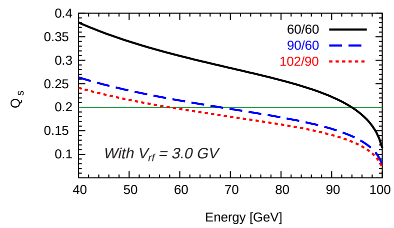

Figure 2: Maximum achievable Q_s as a function of energy and for different optics (with 3 GV RF voltage).

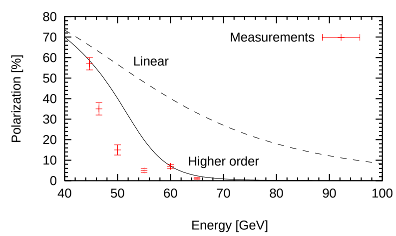

Figure 3: Measured maximum polarization degrees in LEP compared with higher-order correlated polarization theory (solid) and linear theory (dashed).

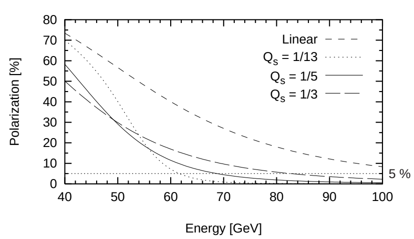

Figure 4: Predicted transverse spin polarization in LEP as a function of beam energy for different values of Q_s.

The expected polarization is shown in Figure 5 for both the correlated and uncorrelated regime. For the highest LEP energies at 90-100 GeV the uncorrelated regime must be considered. We evaluate this regime under the assumption of a large spin tune spread ($\sigma_\nu >> 1$). Including the energy sawtooth LEP will just enter into this regime at the highest beam energies. It is seen that the polarization prediction in this regime increases with the beam energy. At about 100 GeV a polarization degree of roughly 1% is expected. This is not sufficient for energy calibration. However, in view of the uncertainties in this energy regime, an experimental test is clearly warranted. At the end of the 1998 run it was tried to measure polarization at 90 GeV. This attempt was unsuccessful due to problems in the detector shielding at high energies.

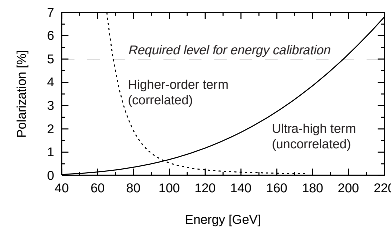

Figure 5: Expected polarization in LEP for ultra-high energies. The higher-order theory in the correlated regime (Q_s = 1/5) and the "ultra-high term" (Equation 6) in the uncorrelated regime are shown. LEP is expected to enter the uncorrelated regime at around 80 GeV.

5 CONCLUSIONS

Transverse beam polarization and accurate energy calibration has been extended to 60.6 GeV in 1998. The additional range in energy calibration helps to reduce the extrapolation error for physics energies and the W mass.

The application of the polarization theory to LEP parameters shows that a 5% polarization degree can be expected up to about 70 GeV with the polarization optics and high Q_s. This extension of polarization range will be studied at the end of regular energy calibrations in 1999. If polarization is found at up to 70 GeV, it can be used for energy calibration.

The prospects for polarization in LEP at around 100 GeV are very uncertain. However, it cannot be excluded that polarization of a few percent is possible in the so-called uncorrelated regime. A dedicated experiment in 1999 will try a to explore this regime of beam polarization.

6 REFERENCES

[1] L. Arnaudon et al, "Accurate Determination of the LEP Beam Energy by Resonant Spin Depolarization", Z. Phys. C66(1995)45.

[2] R. Assmann et al, "Calibration of Centre-of-Mass Energies at LEP1 for Precise Measurements of Z Properties", CERN-SL/98-12.

[3] R. Assmann et al, "Observation of Radiative Spin-Polarization in LEP at 60 GeV", these proceedings.

[4] A.A. Sokolov and I.M. Ternov, "On Polarization and Spin Effects in the Theory of Synchrotron Radiation", Sov. Phys. Dokl. 8(1964)1203.

[5] Ya.S. Derbenev, A.M. Kondratenko and A.N. Skrinsky, "Radiative Polarization at Ultra-High Energies". Part. Acc. 1979, Vol. 9, pp. 247-266.

[6] R. Assmann et al, "Polarization Studies at LEP in 1993", CERN-SL/94-08 (AP).