TRANSVERSE ELECTRON BEAM SIZE MEASUREMENTS USING THE LLOYD'S MIRROR SCHEME OF SYNCHROTRON LIGHT INTERFERENCE

O. V. Chubar, Russian Research Center KURCHATOV INSTITUTE, Moscow, 123182, RUSSIA

Method for horizontal and vertical electron beam size measurements using the Lloyd's Mirror with monochromatic synchrotron light is presented. The use of the interference scheme with synchrotron light may result in fringes of the light intensity distribution, the way it takes place in the case of point light sources. Dimensions of the interference pattern and the fringes contrast are found to essentially depend on transverse size of the emitting electron beam. The resulting light intensity distributions can be calculated analytically. This allows one to determine the transverse size of electron beam immediately from the results of the intensity distribution measurements. Analytical expressions for the intensity distributions of synchrotron light in the Lloyd's Mirror interference scheme are given. Beam size measurement results on the Siberia-1 storage ring by the method concerned are presented.

I. INTRODUCTION

Interference of light is known to be very powerful research instrument in modern physics. In a wide range of experiments and measurements, interferometry normally provides very high precision unattainable with other methods. This paper describes an attempt to use the high information potential of the interference patterns to determine transverse size of electron beam in a storage ring.

It is well-known that "quality" of interference pattern (for example, its visibility or contrast) in most interference schemes essentially depends on characteristics of emitting light source [1]. For a finite-size source, the smaller is the source size, the better is the visibility of the interference pattern. Just this simple feature, with electron beam as a source emitting the synchrotron radiation (SR), is used in the method to be described.

In practice, precise calculation of interference patterns in view of the emitting beam and interference scheme parameters is needed to determine actual values of the beam parameters. Advantageously, SR intensity distributions in the patterns produced with simple interference schemes can be calculated to a very high accuracy. With that, the procedure of determining the beam transverse sizes consists of measuring the intensity distributions concerned and fitting the measured and calculated distributions by varying "guess values" of the actual beam transverse sizes.

Formally, the method under discussion is similar to the edge radiation (ER) based method [2] - [3]: in both cases interference effects are used and the procedures of determining beam parameters consist of very similar steps. Yet there is essential difference between the two methods. In the case of ER, the interference of light appears due to specific emission conditions of the electron beam. Meanwhile, in the method under discussion, standard bending magnet SR is used, and the interference is a result of applying specific scheme for propagation and detection of the light. An important consequence of the difference is discussed in the following chapter among other aspects.

To measure horizontal and vertical size of electron beam, the simplest "Lloyd's Mirror" interference scheme [1] was chosen. We used two modifications of the scheme (see Figures 1 and 2), the one with vertical mirror located at small distance h_x from optical axis as shown in Figure 1-a (to determine horizontal size of electron beam), and one with horizontal mirror shifted by small distance h_z from median plane as shown in Figure 1-b (to determine vertical size).

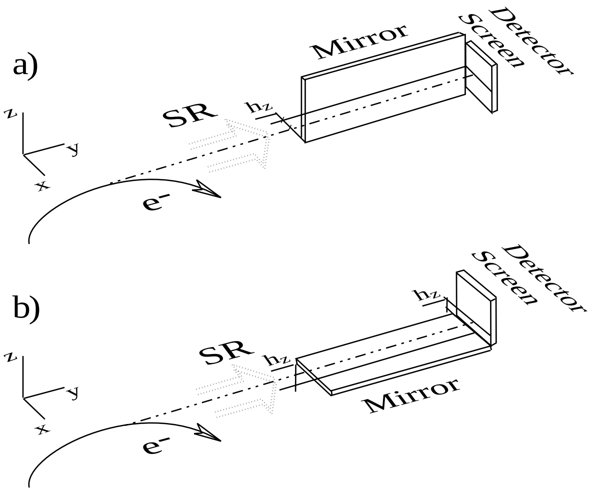

Figure 1. Lloyd's Mirror schemes of synchrotron light interference:
a) vertical mirror (to determine horizontal beam size);
b) horizontal mirror (to determine vertical beam size).

II. ANALYTICAL DESCRIPTION OF SYNCHROTRON RADIATION INTERFERENCE IN LLOYD'S MIRROR SCHEME

To describe the SR intensity distribution to appear at detector screen in the Lloyd's Mirror interference scheme, one should take into account synchrotron light coming to the screen from electron beam directly, as well as the reflected light. Also, one should keep in mind that the radiation emitted by single electron is coherent, whereas the light by different particles of the beam is dominantly incoherent (as in most cases in electron storage rings).

The following expressions can be obtained for spectral angular distribution of synchrotron light emitted by the total electron beam in the Lloyd's Mirror schemes (in terms of photon flux per unit solid angle per unit relative spectral interval), for the vertical mirror:

$$\frac{dN_1}{dt d\Omega (d\lambda/\lambda)} = \frac{8\alpha I}{e} \cdot p \cdot \left[\mathrm{Ai}'^2(\mathrm{Z}) + p\zeta^{\,2}\, \mathrm{Ai}^2(\mathrm{Z}) \right] \times$$
$$\times \left[1 - \cos\left(4\pi h_x \xi/\lambda\right) \cdot \exp\left(-8\pi^2 \sigma_x{}^2 \xi^{\,2} / \lambda^2 \right) \right]; \qquad (1)$$

and for the horizontal one:

$$\frac{dN_2}{dt d\Omega (d\lambda/\lambda)} = \frac{4\alpha I}{e} \cdot p \cdot \Big\{ \mathrm{Ai}'^2(\mathrm{Z}_+) + \mathrm{Ai}'^2(\mathrm{Z}_-) + p\zeta_+{}^2\, \mathrm{Ai}^2(\mathrm{Z}_+) +$$
$$+ p\zeta_-{}^2\, \mathrm{Ai}^2(\mathrm{Z}_-) - 2\left[\mathrm{Ai}'(\mathrm{Z}_+)\, \mathrm{Ai}'(\mathrm{Z}_-) - p\zeta_+\zeta_-\, \mathrm{Ai}(\mathrm{Z}_+)\, \mathrm{Ai}(\mathrm{Z}_-) \right] \times$$
$$\times \cos\left(4\pi h_z \zeta/\lambda\right) \cdot \exp\left(-8\pi^2 \sigma_z{}^2 \zeta^{\,2} / \lambda^2 \right) \Big\}, \qquad (2)$$

where
$\mathrm{Z} = p\left(\gamma^{-2} + \zeta^{\,2}\right);\ \mathrm{Z}_\pm = p\left(\gamma^{-2} + \zeta_\pm{}^2\right);\ p = (\pi\rho/\lambda)^{2/3};$
$\xi = x^*/y^*;\ \zeta = z^*/y^*;\ \zeta_\pm = \left(z^* \pm h_z\right)/y^*;$ Ai and Ai ′ are the Airy function and its derivative respectively, α is the fine-structure constant, e the charge of electron, I the electron current, γ the reduced energy of electrons, λ the radiation wavelength, ρ bending magnet radius; σ_x is horizontal size of the electron beam, σ_z its vertical size; y^* is distance from "radiation point" to detector screen, x^* and z^* are horizontal and vertical coordinates of the observation point (belonging to the detector screen).

Eqs. (1) and (2) are valid with the constraints
$$\gamma >> 1;\ |\xi| << 1;\ |\xi| << y^*/\rho;\ |\zeta| << 1;\ \lambda > \lambda_c;$$
$$\sigma_{z'} << (\lambda/\rho)^{1/3};\ \sigma_{x'} << 1;\ \sigma_z << y^* (\lambda/\rho)^{1/3};$$
$$\sigma_z << h_z << y^*;\ \sigma_x << h_x << y^*,$$

where $\sigma_{x'}$ and $\sigma_{z'}$ are horizontal and vertical angular divergences of the electron beam, λ_c the critical wavelength of synchrotron radiation.

The well-known Fresnel formulae for the reflected wave [1] were taken into account when deriving Eqs. (1) - (2), and the mirrors' reflectivity was assumed ≈ 100 % (the latter being quite realistic assumption for the grazing incidence taking place in the case under consideration).

As one can see from Eqs. (1) and (2), the angular distribution of monochromatic SR in the case of vertical mirror depends on the horizontal beam size σ_x, and in the case of horizontal mirror the distribution depends on the vertical size σ_z. In both cases the finite size of the beam suppresses visibility of the interference fringes (or fringes' contrast). With that, the fringes' width depends on distance from the mirror plane to the beam (h_x, h_z), but not on σ_x, σ_z. The latter is beneficial for the beam size measurements: for a reasonable range of the horizontal and vertical beam size values, there is a possibility to set such values of h_x and h_z that make the beam size measurements realizable in practice. Besides, there is no need in precise geodetic installation of the mirrors before the measurements; the actual values of h_x and h_z can be determined together with σ_x and σ_z, as a result of fitting the expressions (1) and (2) to measurement results.

It is worth noting that, according to Eqs. (1) and (2), the SR intensity distribution in the Lloyd's Mirror interference schemes does not depend on angular divergence of the electron beam (if we neglect the standard SR dependence on the vertical beam divergence, being very weak for long-wave SR, see conditions after Eqs. (1), (2)). With that, the distribution dependence on the beam transverse size takes place at any distance from the "radiation point" (assuming the mirror to be sufficiently large). For reference, as distinct from the case under consideration, intensity distribution in interference patterns of the edge radiation (as well as the distributions of undulator radiation) is known to be very sensitive to the electron beam divergence [2]. Therewith, the distribution dependence on the beam transverse size disappears at large distance ($y^* \gg \sigma_{x,z} / \sigma_{x',z'}$) in this case. The situation is explained by different origination of the interference in the two cases.

II. DETERMINING TRANSVERSE SIZE OF ELECTRON BEAM

An experimental system for measuring the SR intensity distributions in the Lloyd's Mirror interference schemes was installed on Siberia-1 450 MeV electron storage ring (small ring of Kurchatov SR Source, Moscow) in order to determine transverse size of the electron beam (see Figure 2).

The wavelength of the radiation passing through the interference filter was 560 nm at 4 nm bandwidth. Glass plates of 10 cm × 10 cm size with high-quality surfaces were used as the horizontal and vertical mirrors. The mirrors' reflectivity at the incidence angles used was better then 97 %.

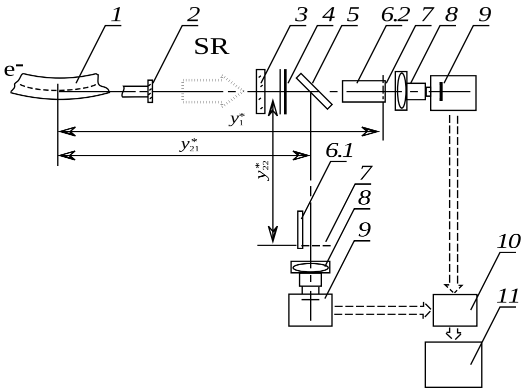

Figure 2. The system for measuring SR intensity distributions in Lloyd's Mirror interference schemes on the Siberia-1 ring. 1- bending magnet; 2- extraction window; 3- neutral filters; 4- interference filter; 5- semitransparent mirror; 6.1- vertical mirror; 6.2- horizontal mirror; 7- object plane; 8- lens; 9- CCD-matrix camera; 10- interface; 11- computer.

The system allowed one to simultaneously measure the SR intensity distributions in the two "object planes" intersecting the horizontal and vertical mirror as shown in Figure 2. Lenses were used to fit the dimensions of the intensity distributions of interest (i.e., interference patterns) to photo-sensitive windows of the CCD matrices. Optical magnification was 4.25 in each channel. Dynamic range of the cameras used was approximately 50. The distances from "radiation point" to object planes were $y^*_1 = 202$ cm and $y^*_2 = y^*_{21} + y^*_{22} = 415$ cm. We tried to use larger distance y^* for measurements with vertical mirror in order to make the corresponding interference pattern larger without using large optical magnification: since the horizontal size of the beam was relatively large, the size of the interference pattern in this case appeared too small at $y^*_2 = y^*_1$.

The intensity distribution registered by the first camera with the horizontal mirror is shown in Figure 3 as a half-tone picture (in order to suppress noise, the distribution was partially averaged in horizontal direction). The interference pattern registered agrees qualitatively with the one predicted by Eq. (2).

The fitting of the measured and calculated intensity distributions over $\sigma_{x,z}$ and $h_{x,z}$ is illustrated by Figure 4. According to the Figures 4-a and 4-b, the calculated best-fits are in good agreement with the measurement results. The values of the beam sizes corresponding to the best-fits are: $\sigma_x = 1.41 \pm 0.08$ mm (at $h_x = 8.8$ mm) and $\sigma_z = 0.34 \pm 0.02$ mm (at $h_z = 9.1$ mm). We see that $\sigma_x \ll h_x$ and $\sigma_z \ll h_z$, being in agreement with the constraints for Eqs. (1) and (2).

The measurements concerned were performed when the Siberia-1 operated as an injector for the Siberia-2 ring at the energy of $E = 350$ MeV. In agreement with data from other detectors, the vertical beam size appeared to be larger, and the horizontal size little bit smaller in this mode then in the mode when the Siberia-1 operated as light source at $E = 450$ MeV [3].

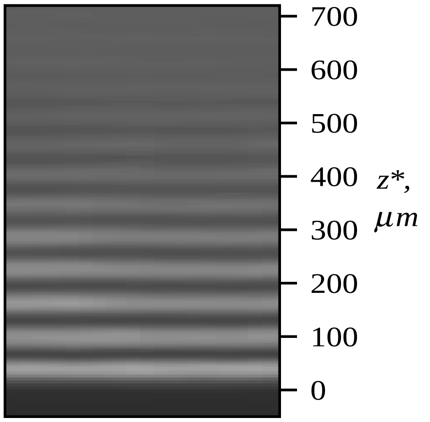

Figure 3. Half-tone representation of the intensity distribution registered by CCD-camera in the case of horizontal mirror. In the case of vertical mirror, the interference pattern was very similar, with the fringes being parallel to the mirror plane.

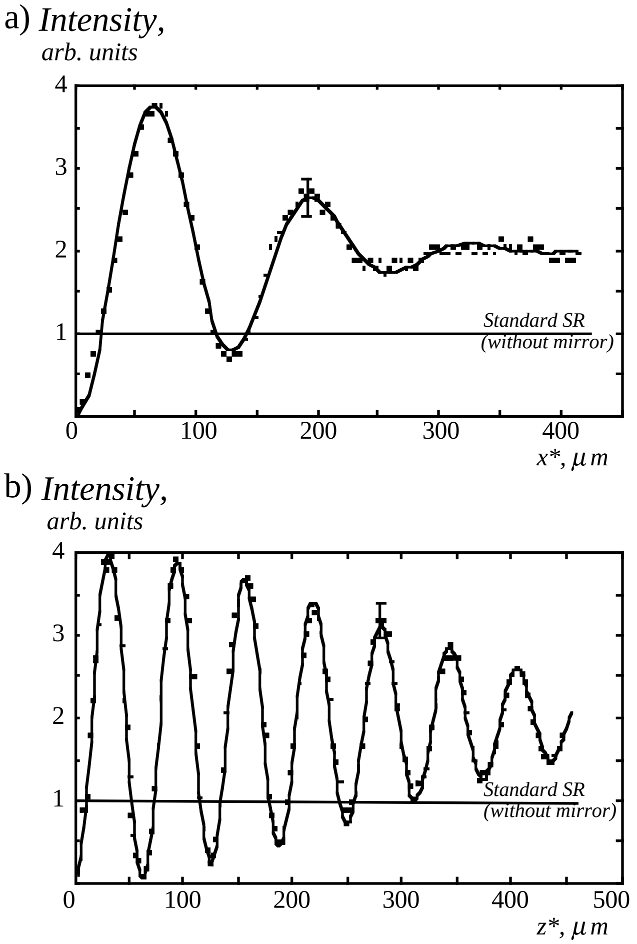

Figure 4. Measured SR intensity distributions for the channel with the vertical (a) and the horizontal (b) mirror (points), and the corresponding best-fits (solid lines). The fitting gives one: $\sigma_x = 1.41 \pm 0.08$ mm and $\sigma_z = 0.34 \pm 0.02$ mm.

IV. ACKNOWLEDGEMENTS

I would like to thank Prof. G.N.Kulipanov (Budker Inst. of Nuclear Physics, Novosibirsk) for very important notes he made on the subject of this work, and the joint team of accelerator physicists from the INP and Kurchatov SR Source, who helped to perform measurements even during the period of very intensive commissioning works on the Siberia-2 ring.

V. REFERENCES

[1] See, for example, M.Born and E.Wolf, Principles of Optics, 6th ed., Pergamon, Oxford (1980).
[2] O.Chubar, E.Masunov, "Electron Beam Diagnostics by Means of Edge Radiation", Proceedings of the PAC93, vol.3, p. 2474.
[3] O.Chubar, "Determining Electron Beam Parameters from Edge Radiation Measurement Results on Siberia-1 Storage Ring", these proceedings.