On the Higher Order Mode Coupler Design for Damped Accelerating Structures

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Abstract

In this paper we will discuss mainly damped structures and class them into three types. The procedures to make a higher order mode (HOM) coupler design have been given, and some useful analytical formulae, which can be used to determine the coupling apertures' dimensions and the equivalent loaded quality factors, have been derived. It is shown that few couplers to dampe a detuned structure is not at all efficient.

I. Introduction

For the linear colliders working in the multibunch mode the long range wakefields have to be properly controlled by using detuned, damped or damped detuned structures depending on the concrete machine design parameters. In this paper we try to class damped structures into three types and to make a detail discussion about how to determine the coupling aperture dimension of a HOM coupler and how to estimate its effect on the damped HOM. In section 2, the first type damped structure is discussed, where each cavity is damped by HOM couplers. In section 3, the second type damped structure is defined as a section of constant impedance structure is damped by a HOM coupler. The essential difference between the first and the second type damped structures is demonstrated. To estimate the effect of this HOM coupler an equivalent quality factor has been introduced and formularized. Finally, in section 4, damped detuned structures are discussed and it is shown that few couplers to dampe a detuned structure is not at all efficient.

II. The First Type of Damped Structure

We consider a single pill-box resonant rf cavity resonating at a resonant mode, for example the TM$_{110}$ mode. The quality factor corresponding to this mode is Q_{110} If this cavity is loaded with some waveguides, the loaded quality factor $Q_{L,110}$ is defined as

$$Q_{L,110} = \frac{Q_{0,110}}{1+\beta_{N,110}} \tag{1}$$

where $\beta_{N,110}$ is called the coupling coefficient between waveguides and the resonant cavity corresponding to the TM$_{110}$ mode. Where the subscript N denotes the number of the waveguides. If the cavity is loaded with four waveguides distributed uniformly around the cavity's cylindrical surface, from the general expression derived in refs. [1] and [2] one finds that $\beta_{4,110}$ is TM$_{110}$ mode polarization independent and can be expressed analytically

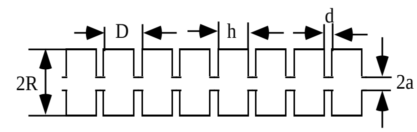

Figure. 1. Constant impedance accelerating structure

as

$$\beta_{4,110}(l) = \frac{\pi Z_0 k k_{10} l^6 e^{-2\alpha_c t} J_1'(u_{11})^2}{144(ln(\frac{4l}{w})-1)^2 ABRR_{s,110}(R+h)J_2^2(u_{11})} \tag{2}$$

where $k = 2\pi/\lambda_{110}$, $k_{10} = k(1-(\lambda_{110}/2A)^2)^{1/2}$, $\alpha_c = (2\pi/\lambda_{110})((\lambda_{110}/2l)^2-1)^{1/2}$, $R_{s,110} = (\pi c\mu_0/\sigma\lambda_{110})^{1/2}$, μ_0 is the magnetic permeability and σ is the electric conductivity, A and B are the width and the height of HOM waveguides, l and w are the width and the height of the four rectangular coupling slots with l parallel to the magnetic field, and t is the wall thickness between cavity inner surface and waveguide. Now, if many of this kind of waveguide loaded cavities are coupled together, one obtains the first kind of damped accelerating structure under the condition that the fundamental mode is not damped. For the TM$_{11}$ mode passband one can say that the loaded Q is almost $Q_{L,110}$. If $K_h Q_{L,110} < 2$ there will be no coupling between cavities for the TM$_{11}$ mode [3], where K_h is the coupling coefficient for the TM$_{11}$ mode passband.

III. The Second Type of Damped Structure

Now we consider the second kind of damped structure which is essentially different from the first. Given a constant impedance disk-loaded structure of length L as shown in Fig. 1, one can calculate the passband of the TM$_{11}$ mode (which is the most dangerous mode for multibunch operation). We define $\theta_{1,s}$ as the phase shift of the TM$_{11}$ mode passband at which the phase velocity equals to the velocity of light. Assuming now a charged particle is injected into the structure off axis at a velocity close to that of light, one knows that this particle will loss its energy to the TM$_{11}$ mode mainly at synchronous phase shift $\theta_{1,s}$ and generates behind it so-called wakefield $W_{\perp,11}(s)$, where s is the distance between this exciting particle and a following test particle (of course it will generate wakefields oscillating at other

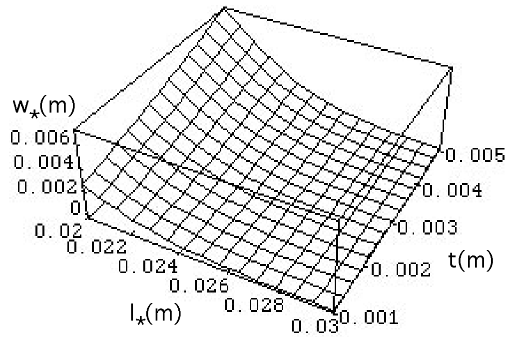

Figure. 2. $\beta_{4,11} = 1$ surface with $v_g(\theta_{1,s})/c = 0.001$

frequencies also). If $0 < \theta_{1,s} < \pi$ the energy of the wakefield $W_{\perp,11}$, which is distributed uniformly along the structure length L just after the passage of the exciting electron, will tend to move upstream with the group velocity of $v_g(\theta_{1,s})$. Assuming that four distributed waveguides, for example, are connected to the first cavity at the beginning of the structure, in order to absorb the energy of $W_{\perp,11}$ from the structure via the HOM waveguides without any reflection, one has to match this HOM coupler to the TM_{11} *travelling wave mode* working at $\theta_{1,s}$ by carefully choosing the slot dimensions on the coupler wall (just like matching the fundamental mode input and output couplers of a constant impedance travelling wave accelerating structure). The coupling slots' width l_* and height w_* have to satisfy the following relation [2][4]:

$$\beta_{4,11}(l_*) = \frac{\pi Z_0 k k_{10} l_*^6 \exp(-2\alpha_c^* t) J_1'(u_{11})^2}{144(ln(\frac{4l_*}{w_*}) - 1)^2 ABRR_{s,110}(R+h) J_2^2(u_{11})} \times$$

$$\frac{1}{(1 + \frac{Z_0 R}{2R_{s,110}(R+h)}(\frac{v_g(\theta_{1,s})}{c}))} = 1 \qquad (3)$$

where $\alpha_c^* = (2\pi/\lambda_{110})((\lambda_{110}/2l_*)^2 - 1)^{1/2}$ and c is the velocity of light. Comparing eq. 2 with eq. 3, one finds out the fundamental difference between the two kinds of damped structure, that is, the coupling slots of the HOM coupler of the second type damped structure should not be too large or too small but satisfy matching condition shown in eq. 3, however, for the first type damped structure this limitation doesn't exist. Here we will use eq. 3 to find the dimensions of HOM coupling apertures of a 3GHz accelerating structure with $a = 1.5$cm, $D = 3.33$cm, $R = 4.1$cm, $h = 2.83$cm, $A = 4.4$cm and $B = 2.$cm. Fig. 2 shows the relation between w_* and l_* for different wall thickness at $v_g(\theta_{1,s})/c = 0.001$.

Once the HOM coupler is matched one knows the deflecting force of the transverse wakefield $W_{\perp,11}$ on the test particle of charge q which is behind the exciting particle by a distance of s:

$$F_{\perp,11} = qW_{\perp,11} e^{-\frac{\omega(\theta_{1,s})}{2Q_{0,11}}(\frac{s}{c})} D(s) \qquad (4)$$

where

$$D(s) = L - v_g(\theta_{1,s})\frac{s}{c}, (s \leq cL/V_g(\theta_{1,s}))$$

$$D(s) = 0, (s > cL/v_g(\theta_{1,s})) \qquad (5)$$

where $\omega(\theta_{1,s})$ is the angular frequency of the travelling TM_{11} mode at the phase shift of $\theta_{1,s}$, $Q_{0,11}$ is the quality factor of the structure working at the angular frequency $\omega(\theta_{1,s})$. It is obvious that when $v_g(\theta_{1,s}) = 0$ there is no damping effect (only the coupler cavity is damped). For those who are used to think in terms of loaded quality factor to judge the damping effect of this HOM coupler, we will use an exponential function $Le^{-\frac{\omega(\theta_{1,s})}{2Q_{e,11}}(\frac{s}{c})}$ to simulate function $D(s)$ in eq. 5 by choosing $Q_{e,11}$ as

$$Q_{e,11} = \frac{\omega(\theta_{1,s})L(1 - e^{-1})}{2v_g(\theta_{1,s})} \qquad (6)$$

in the way that both functions drop to the the same value Le^{-1} at the same s. Eq. 4 can be expressed as

$$F_{\perp,11} = qW_{\perp,11} Le^{-\frac{\omega(\theta_{1,s})}{2Q_{L,11}}(\frac{s}{c})} \qquad (7)$$

where $Q_{L,11}$ is called the equivalent loaded quality factor which is

$$Q_{L,11} = \frac{Q_{0,11}}{1 + \frac{Q_{0,11}}{Q_{e,11}}} = \frac{Q_{0,11}}{1 + \beta_{eq}} \qquad (8)$$

where $\beta_{\mathbf{eq}}$ is called equivalent coupling coefficient which has nothing to do with the coupling coefficients expressed in eq. 2 and eq. 3. If $Q_{0,11} \gg Q_{e,11}$, $Q_{L,11} \approx Q_{e,11}$. From eq. 6 it is very important to note that there are two ways to decrease $Q_{L,11}$, either to reduce L or to increase $v_g(\theta_{1,s})$. The total number N_c of the HOM couplers which should be used to reach a desired $Q_{L,11}^*$ for a constant impedance structure of length L can be calculated from the following expression:

$$N_c = \frac{L\omega(\theta_{1,s})(1 - e^{-1})(Q_{0,11} - Q_{L,11}^*)}{2cQ_{0,11}Q_{L,11}^*(\frac{v_g(\theta_{1,s})}{c})} \qquad (9)$$

The second kind damped structure is used, for example, in TESLA linear collider project.

IV. The Third Type of Damped Structure

The third type of damped structure is to damp a detuned structure by few HOM waveguides. We will first discuss it without adding HOM coupler and assuming that cavities' dimensions are different from each other adiabatically. The TM_{11} modes dispersion curves of this detuned structure are shown in Fig. 3. When a particle traverse the structure off axis it will deposit part of its energy on TM_{11} mode oscillating at the frequencies mainly from f_1 to f_n as shown in Fig. 3, where the subscript n is the total number of different cavities in this detuned structure of length L. The deflecting force felt by a test particle can be expressed as

$$F_{\perp,11} = \sum_{i=1}^{n} 2qK_{\perp,11}\frac{L}{n}\sin(\omega(\theta_{1,s,i})\frac{s}{c})\exp(-\frac{\omega(\theta_{1,s,i})}{2Q_{0,11,i}}(\frac{s}{c})) \qquad (10)$$

with

$$K_{\perp,11} = \left(\frac{k_{1,i}c}{a_i^2\omega(\theta_{1,s,i})}\right) \qquad (11)$$

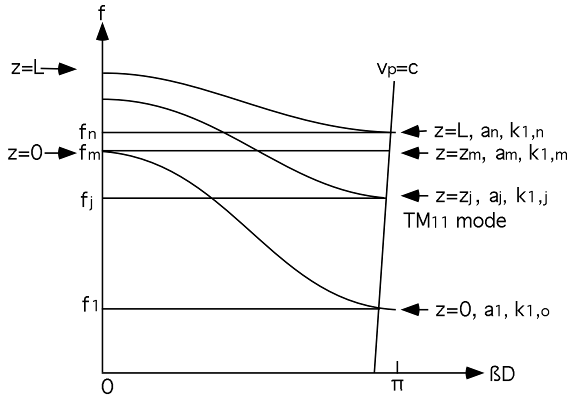

Figure. 3. Dispersion curve of a detuned structure

where $k_{1,i}$ is the TM$_{11}$ mode loss factor corresponding to the ith cavity, and it can be calculated analytically [2]

$$k_{1,i} = \frac{a_i^2 u_{11}^2 D \eta_{\theta_{1,s,i}}^2}{4\pi\epsilon_0 h R_i^4 J_2^2(u_{11})} \left(\frac{2R_i}{a_i u_{11}} J_1\left(\frac{u_{11}}{R_i} a_i\right)\right)^2 \qquad (12)$$

$$\eta_{\theta_{1,s,i}} = \frac{2}{\theta_{1,s,i}} \sin\left(\frac{\theta_{1,s,i} h}{2D}\right) \qquad (13)$$

If we consider an uniformly detuned structure with its iris radius adiabatically decreasing from a_1 at the beginning to a_n at the end, the deposited energies can move inside the structure instead of oscillating locally. The motions of the TM$_{11}$ modes' energies inside the structure can be classified into three different ways. For the modes deposited at the beginning of the structure they will oscillate locally since they could not propagate downstream. For the modes generated at the end of the structure (from f_m to f_n, $v_g(\theta_{1,s,i}) < 0, i = m, \cdots, n$), however, the energies will propagate upstream and be trapped finally somewhere in the downstream cavities where the group velocities corresponding to these frequencies are equal to zero. Finally, the energies of the modes between f_1 and f_m will propagate upstream until they are reflected by the first cavity. The reflected energies will move downstream and finally be trapped in the cavities where the group velocities are zero.

To damp the TM$_{11}$ modes in a detuned structure via a few HOM couplers is the main idea of the so-called damped detuned structure currently adopted in the S-Band linear collider main linac design [5]. To demonstrate the behaviour of a HOM coupler let's assume that at the beginning of a detuned structure a HOM coupler (four waveguides) is installed and is matched to the ith TM$_{11}$ mode. The equivalent loaded quality factor corresponding to the ith mode can be expressed as

$$Q_{L,11,i} = \frac{Q_{0,11,i}}{1 + \frac{Q_{0,11,i} 2 < v_g(\theta_{1,s,i}) >}{\omega(\theta_{1,s,i}) z_i (1-e^{-1})}} \qquad (14)$$

where $< v_g(\theta_{1,s,i}) >$ is the average group velocity of the ith mode travelling from $z = z_i$ to $z \approx 0$ where the HOM coupler is located. The damping effects of this HOM coupler on the other modes, however, are less efficient and different from one to another due to mismatching. The reflected HOM energies by this mismatched HOM coupler will be trapped somewhere in the structure. It seems that the first type damping scheme is the most suitable choice for a damped detuned structure (SLAC damped detuned structure is a good example [6]).

V. Acknowledgements

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References

[1] J. Gao, "Analytical formula for the coupling coefficient β of a cavity-waveguide coupling system", *Nucl. Instr. and Meth.*, **A309** (1991) p. 5.

[2] J. Gao, "Analytical approach and scaling laws in the design of disk-loaded travelling wave accelerating structures", *Particle Accelerators*, **Vol. 43(4)** (1994) pp. 235-257.

[3] J. Gao, "The criterion for the coupling states between cavities with losses", *Nucl. Instr. and Meth.*, **A352** (1995) pp. 661-662.

[4] J. Gao, "Analytical formulae for the coupling coefficient β between a waveguide and a travelling wave structure", *Nucl. Instr. and Meth.*, **A330** (1993) p. 306.

[5] N. Holtkamp, "S-band test accelerator", Proceedings of the LC93, Oct. 13-21 (1993) Stanford, SLAC-436, 1994.

[6] N. Kroll, K. Thompson, K. Bane, R. Gluckstern, K. Ko, R. Miller, and R. Ruth, "Higher order mode damping for a detuned structure", SLAC-PUB-6624, Aug. 1994.