THE DISTORTION OF THE ACCELERATING FIELD DISTRIBUTION IN COMPENSATED STRUCTURES DUE TO STEADY-STATE BEAM LOADING

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Abstract

Using general electrodynamic approach and particular properties of compensated accelerating structures the effect of distortions in accelerating field distribution due to RF power losses in surface and for beam acceleration is considered. Consideration shows, that for proton linacs of 'meson facility' type (impulsive beam current $I_b \simeq 50\ mA$, acceleration rate $U_a \sim 2 \div 6\ MV/m$) this effect is not important for all accelerating structures known. When beam current $I_b \sim 100 \div 300\ mA$ and $U_a \sim 1\ MV/m$ (like linacs for accelerator-driven transmutation technologies), in the structures with small group velocity ($\beta_g \approx 0.04$) significant distortions will arise and efficiency of the structure for beam acceleration will drop. For structures with large group velocity ($\beta_g \approx 0.4$) such effects will arise at beam current in several amperes.

I. INTRODUCTION

The compensated accelerating structures are now widely used for acceleration of charged particles for high energies (relative velocities $\beta_p >$ 0.4). Remember, that the 'compensated' is named structure in which at operating frequency coincide frequencies of two modes with differing parity of field distribution with respect to symmetry plane (accelerating and coupling modes) [1]. Examples of compensated structures are well known Side-Coupled Structure (SCS), Annular-Coupled Structure (ACS), On-axis Coupled Structure (OCS), Disk And Washer structure (DAW) and so on. Particularity of compensated structures is ability of propagation of traveling wave 0 or π type with group velocity differing from zero. These structures are used in proton linear accelerators of 'meson facility' type Under consideration now are projects of accelerators. with beam current in several hundreds mA. In this case RF power needed for beam acceleration in several times exceeds RF losses in the structure. It leads to the additional distortion of accelerating field distribution along the structure in steady-state regime.

II. THE FIELD DISTRIBUTION

Below we shall consider (without loss of generality) the accelerating structures with operating π mode. The electromagnetic field in the cavity is a result of adding of two traveling waves - forward wave, from RF source, and backward wave, reflected from the end of the accelerating cavity. The losses of the RF power in really conductive surface and for beam acceleration lead to the attenuation of propagating waves. Suppose that beam bunches are in all accelerating gaps, each bunch we consider as a solid and process is steady-state. Under these assumptions all periods of the structure are equivalent and the field distribution in the structure must satisfy to the Floquet theorem. Suppose, the accelerating mode and the coupling one have the field distributions, described by:

$$H_a = H_a(\varphi, r, z), \;\; 0 \leq z \leq d, \qquad (1)$$
$$H_c = H_c(\varphi, r, z), \;\; 0 \leq z \leq d,$$

where d - is the length of the period of the structure.

Traveling waves of 0 or π type can propagate if there is no stop-band at the dispersion diagram. In practice it is sufficient to have overlapped resonant curves of accelerating and coupling modes. In this case in positive direction along z axis propagates forward wave with the field distribution in n-th period $nd \leq z' \leq d + nd$:

$$H_+^{"}(\varphi, r, z') = [H_a(\varphi, r, z) - \iota H_c(\varphi, r, z)]e^{\iota n\theta}, \qquad (2)$$

with normalization [1,2]:

$$\int_v H_a^2 dV = \int_v H_c^2 dV, \qquad (3)$$

where θ - is phase shift per period, v is the volume of the one period of the structure. Propagating in negative direction backward wave have to be described with complex conjugated to (2) expression. With all RF losses θ must be complex value $\theta = \pi$-$\iota\alpha$, where α have to be founded as the solution of our problem. For the field distributions of forward H_+, and backward H_- waves, we can write:

$$H_+ = A_+ e^{-\alpha z'} H_+^{"}(\varphi, r, z), H_- = A_- e^{\alpha z'} H_+^{"}(\varphi, r, z)^*, \qquad (4)$$

where A_+ and A_- - amplitudes. The reflection of the forward wave at the end plate of the cavity ($z = Nd$) provides the relationship for amplitudes: $A_- = A_+ e^{-2N\alpha d}$. For the magnetic field of the standing wave in the n-th period, taking into account (2) -(4), we get:

$$H_\tau = A_+ e^{-\alpha z'} cos(n\pi)(H_a(1 + e^u) - \iota H_c(1 - e^u)), \qquad (5)$$
$$u = 2\alpha(z' - Nd).$$

III. THE RF POWER BALANCE

We can write the RF power balance in the plane $z = 0$: difference in the RF powers, carried by forward P_+ and backward P_- waves is equal to the RF power losses in surface P_s and the RF power for beam acceleration P_b, and using well known relations for traveling wave power flux transform it to:

$$P_s + P_b = \frac{c\beta_g(W_+ - W_-)}{d} = \frac{c\beta_g W_a(1-e^{-4N\alpha d})}{2d}. \qquad (6)$$

where W_+ and W_- are energies, stored by forward and backward waves in the period, β_g - relative group velocity of waves, N - number of the structure period from the RF input point to the end of the cavity, W_a - energy, stored by standing wave accelerating mode in the period. This paper we shall use set of 'ideal' notions. For example, W_a is the energy, stored by operating - accelerating mode in the period of the structure if forward wave has the amplitude A_+.

In calculations of RF losses in the surface $P_s = \frac{R_s}{2}\int H_\tau^2 dS$ we have to know the field distribution including changing due to attenuation. In general form it is impossible, because in cavity with complex real form, field distributions $H_a(\varphi, r, z)$ and $H_c(\varphi, r, z)$ are not described analytically. To calculate P_s, we represent the integral over all surface as a sum of integrals over all periods, and each integral over period represent as a product of integrals from 'ideal' field distribution $H_a(\varphi, r, z)$ and from attenuation.

$$P_s = \frac{N}{8}(P_a(\frac{1-e^{-4N\alpha d}}{N\alpha d} + 4e^{-4N\alpha d}) + P_c(\frac{1-e^{-4N\alpha d}}{N\alpha d} - 4e^{-4N\alpha d})), \qquad (7)$$

where P_a and P_c are RF power losses in surface for accelerating mode and coupling one (like W_a). The RF power for beam acceleration P_b, taking into account E_z distribution like H_τ (3), is:

$$P_b = I_b \int E_z e^{\frac{\imath k z'}{\beta_p}} dz' \approx \frac{I_b U_a N(1-e^{-2N\alpha d})}{2N\alpha d}, \qquad (8)$$

where k is wave value, I_b - beam current, E_a and T_a are tension and transit time factor for accelerating mode, $U_a = E_a T_a$ - designed energy gain per period (like W_a). One can neglect contribution of the coupling mode to beam acceleration, because its field tension E_c is several orders less in comparison with E_a for accelerating mode.

Introducing (8) and (8) into (6), with simple transformation, one get for α determination:

$$\frac{\beta_g Q_a(1-e^{-4N\alpha d})}{2\pi\beta_p} = \frac{I_b U_a N(1+e^{-2N\alpha d})}{2N\alpha d P_a} + \frac{N}{8}(\frac{(1-e^{-4N\alpha d})(Q_a+Q_c)}{N\alpha d Q_c} + 4\frac{(Q_c-Q_a)e^{-2N\alpha d}}{Q_c}), \qquad (9)$$

where Q_a and Q_c - are quality factors of the accelerating mode and the coupling one.

IV. DISCUSSION

In the case of small total attenuation ($N\alpha d \ll 1$), transforming (10) in series with $N\alpha d$ power, one find:

$$N\alpha d \approx \frac{1+\frac{I_b U_a}{P_a}}{\frac{2\beta_g Q_a}{N\pi\beta_p} + 2 + \frac{I_b U_a}{P_a}} \approx (1+\frac{I_b U_a}{P_a})\frac{N\pi\beta_p}{2\beta_g Q_a}. \qquad (10)$$

In the case $N\alpha d \ll 1$ attenuation coefficient α don't depends on number of periods in the cavity and its increasing due to beam loading is proportional to the ratio of RF power for beam acceleration to RF losses in the surface, or, to the effective increasing of total RF losses in the period. The coupling mode excites in first order of αd (4) and increasing in total RF losses due to this mode excitation is in second order. In this case the radiotechnical parameters of the structure for acceleration are practically the same, as designed. Quality factor don't changes and there is no decreasing in efficiency of acceleration.

Let's define accelerating field tilt as:

$$\frac{\Delta E}{E} = \frac{E_0 - E_N}{E_0} = \frac{(1-e^{-N\alpha d})^2}{1+e^{-2N\alpha d}} \approx (N\alpha d)^2 \qquad (11)$$

Operating regimes of proton linacs of 'meson facility' type satisfy to the condition:

$$\frac{I_b U_a}{P_a} = \frac{I_b Z_a cos^2\Phi}{U_a} < 1. \qquad (12)$$

where Φ is synchronous phase. Comparing two accelerating structures - SCS ($\beta_g/\beta_p \approx 0.04$) and DAW structure ($\beta_g/\beta_p \approx 0.4$), assuming these structures to be equivalent with shunt impedance and quality factor ($Z_a = 29\ MOhm/m, Q_a = 1.8 * 10^4$), with 40 periods from RF input to the end of the cavity, $U_a = 2\ MV/m, I_b = 50\ ma$, one will have tilt 0.58% for SCS structure and 0.0054% for DAW one. The RF power flux, carried through the structure by forward wave at accelerating rate given U_a, is:

$$P_+ = \frac{c\beta_g U_a^2 Q_a}{2\pi\beta_p Z_e cos^2\Phi}, \qquad (13)$$

and for typical regime of the 'meson facility' linacs P_+ is several hundreds MWt, exceeding in several orders the RF losses in surface P_s and for beam acceleration P_b. This case all RF losses are small perturbation with respect to the RF power flux, carrying through the structure.

Now under consideration are projects of continuous mode linear accelerators with beam current several hundreds of mA - [3,4]. Beam loading in such accelerators in orders exceeds one for 'meson facility' linacs. Attenuation αd becomes larger then 10^{-2}. All assumptions, made in derivation (10) remain valid, but, to find attenuation one must solve equation (10), because estimation (10) gives lowered value for αd. This case αd depends on both beam loading and number of periods in the cavity.

Suppose in the cavity, containing $2N$ periods (RF input is placed in the middle), designed energy gain per period is U_a. From calculations or signal RF level measurements we know quality factor Q_a and effective Z_a. Using this data, we can estimate RF power needed to have designed total energy gain with acceleration of large current. For the structure with ($\beta_g/\beta_p \approx$

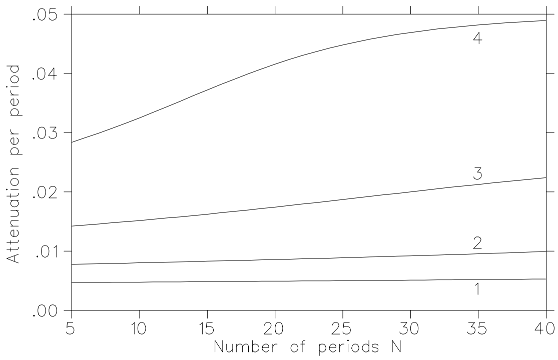

Figure 1. Dependencies of the total attenuation $N\alpha d$ vs number of periods in the cavity N with different beam current $I_b, 1-125\ mA, 2-250\ mA, 3-500\ mA, 4-1.0\ A.$

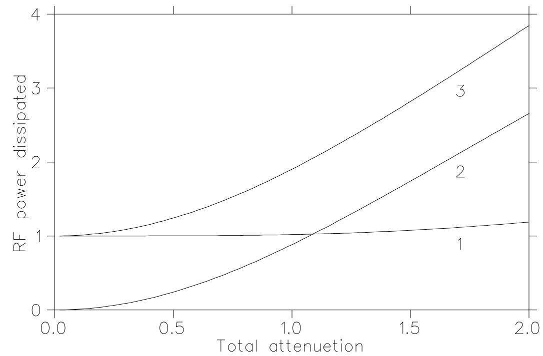

Figure 2. Dependencies of normalized RF power losses for operating mode $P^{"}_{a'} = \frac{P_{a'}}{P_a}$ and for coupling one $P^{"}_{c'} = \frac{P_{c'}}{P_a}$ vs total attenuation $N\alpha d$. $P_{o'} = P^{"}_{a'} + P^{"}_{c'}$.

0.04) and parameters, mentioned above, $Q_c = Q_a/3$, and middle accelerating rate 1.5 MV/m plots of the dependencies of the total attenuation along the cavity are shown at Fig.1 for different beam currents.

If the total attenuation becomes enough large, $N\alpha d \geq 0.2$, the dependence of the field tilt differs from square and the value of the tilt $\frac{\Delta E}{E}$ reaches significant value. At least, phase oscillation of particles inside separatrix will occur. To provide RF power flux needed along the cavity, coupling mode in periods near RF input point must be strongly exited (6). For all structures known the quality factor for coupling mode is in several times less than for accelerating one and the excitation of coupling mode leads to significant additional losses near RF input point. Total RF power dissipated increases not so fast, but this increasing presents. It means, that effective shunt impedance of the structure decreases. Relation of the total RF power $P_{a'}$, dissipated in the cavity by accelerating mode and $P_{c'}$ for coupling one to have the designed value of total acceleration $2NU_a$ depends on total attenuation $N\alpha d$ along the cavity, which is proportional to the number of periods:

$$P_{a'} = P_a N\alpha d \frac{1 - e^{-4N\alpha d} + 4N\alpha d e^{-2N\alpha d}}{2(1 - e^{-2N\alpha d})^2}, \qquad (14)$$

$$P_{c'} = P_a Q_a N\alpha d \frac{1 - e^{-4N\alpha d} - 4N\alpha d e^{-2N\alpha d}}{Q_c 2(1 - e^{-2N\alpha d})^2}.$$

Fast rise $P_{c'}$ with $N\alpha d$ explains dependence of αd vs number of periods in the cavity.

Analyzing relationships given and plots presented one see, that for beam current 500 mA and more in structures with small group velocities arise unacceptable effects. Practically the structure closes. Acceleration take place in several periods near RF input points and another parts are not used effectively. The ratio of RF losses between the first (near input point) and the last periods rises very fast with the beam current increasing. For acceleration of beam with current in several hundred mA, structures with small β_g have to be tuned specially. Using analogy with disk-loaded waveguide for electron linac, it is preferable to have constant gradient along the cavity with current, differing from constant impedance sections of cavities, which are now used in 'meson facility' type linacs. Moreover, number of periods of the structure between RF input points have to be not large. For structures with high group velocity this problems are absent.

V. SUMMARY

In this paper, using electrodynamic approach and general properties of compensated accelerating structures, the effect of distortions in accelerating field distribution due to RF losses in surface and for beam acceleration is considered. Basing on the RF power balance, simple analytical expressions for evaluation of attenuation constant and estimation of RF parameters of the structure are obtained.

Consideration shows, that for proton linacs of 'meson facility' type this effect is not important for all accelerating structures known. When beam current is several hundred mA, in the structure with small group velocity ($\beta_g/\beta_p \approx 0.04$) significant distortions will arise and efficiency of the structure for beam acceleration will drop. It will need special care in the design of the structure.

REFERENCES

1. Linear accelerators /Ed. P.M.Lapostolle and A.L. Septier.- Ams.: North-Holland Publ. Co., 1970.

2. M. Bell, Particle Accelerators, v.8, n. 2, p. 71, 1978

3. G.P. Lawrence, Proc. of the 1994 EPAC, v.1, p.236, 1995

4. I.M. Kapchinskiy et al. Proc. of the 1993 IEEE PAC, v.3, p.1675, 1994