Electromagnetic Field Vector Components Precise Measurements in Accelerating Structures

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Abstract

Precise method for resonator electric or magnetic vector components values and their space positions measurements, based on application of photosemiconductor plate with different configuration lighted images, formed by projections forming and measuring optical system of amplitude modulated light radiation, is presented. The optical system for $433\,\mathrm{MHz}$ RFQ accelerating structure is realized by means of serial producted microalignment telescopes; the method allows to descriminate the field axis fluctuations on micron level and provides several percents and tenths of percents precision for accelerating efficiency and modulation period measurements respectively.

I. INTRODUCTION

An electromagnetic field distribution measurements in accelerator resonant structures are usually carried out by perturbation method, the data processing gives the field vector modulus as averaged volumatric value on the perturbation object. The measurements accuracy is limited in principle by the object carrying system distortions and unperturbated resonant frequency reading inadequacy. Operative frequency rise as well as applied fields complication will cause inadmissible growth of these inaccuracies. Electro-optical principle is proposed to exclude these errors and to realize different vector components measurements of electric or magnetic fields distributions, the method and system development for RFQ structure is considered.

II. ELECTRO-OPTICAL PRINCIPLE

Perturbation object for the method is designed as a high resistivity photosemiconductor flat plate, that can be installed inside the resonator on a thin filament as before. Light radiation of amplitude modulated source passes through a controller of the light beam spatial position and lights up desired configuration region on the plate surface in required position. The resonant frequency difference between the readings in unlighting amplitude modulation half-cycle and in the next lighting one will determine the field in perturbated region. Electric field vector components measurements can be carried out by thin strip light configurations, oriented along the components; for the plate normal magnetic field component a closed-loop configuration is sutable; average properties can be determined by a spot of accordent space. A normal to electric field lighting minimizes the dark and light readings difference (that vanish for infinitesimal thickness of the normal), in quasistatic field it is equipotentials detection without quantity measurements.

Carrying system distortions are entirely excluded, because the perturbation is formed by exact strightforward light beam. The second mentioned error is excluded almost completely by the dark reading in any measurement point, because high-spead (acusto-optical, e.g.) devices allow to decrease amplitude modulation cycle up to the doubled transient duration. For all this, the precise measurement problem is reduced to implementation of correlated with the resonant mode light configurations.

III. RFQ MEASUREMENTS

The field symmetry axis can be detected by equal thin strips lighting on the round plate (Π), fig.1. By (1) and (2) couple removing in OY direction the equal dark-light frequency differences for each strip can be achieved, i.e. E_y components in (1) and (2) regions are equal, the axis coordinate y_q is geometrical center of the couple; similar OX removing of (3),(4) gives x_q coordinate. Vanes curvature is determined by the strips turn refer $O_q x$ and $O_q y$ untill maximum (but a.m. equal) dark-light differences for $(1'), (2'); (3'), (4')$ will be obtained – fig.1 presents symmetrical ψ bend; for a single element distortion the geometrical centers will not form straight lines under OX displacing of (1),(2) couple, (3),(4) – in OY direction. So, that kind positions research yields the field axis coordinates and symmetry destorting causes all information.

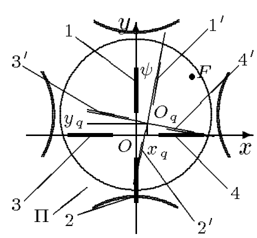

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Figure 1: Lightings for RFQ measurements.

Obtainable precision analysis is conducted at known [1] electric field in the bore with modulation period l_m, accelerating efficiency θ, inner radius mean r_0:

$$\begin{aligned} E_x &= Ux(\Lambda - 1/r_0^2), \qquad \Lambda = 2\theta\kappa \sin \kappa z I_1(\kappa r)/\kappa r, \\ E_y &= Uy(\Lambda + 1/r_0^2), \qquad r = \sqrt{x^2 + y^2}, \\ E_z &= (U/\pi) 2\theta\kappa \cos \kappa z \cdot I_0(\kappa r); \quad \kappa = \pi/l_m. \end{aligned} \tag{1}$$

Spatial selectivity is defined by frequency deviations ratio of interfluent and separated components, e.g. l length, $a_s \times a_s$ cross-section strip (4) E_x selectivity in (1) field according to [App.] is $\Pi_{\perp} = (\delta f_{E_z}/\delta f_{E_x}) \le 4 \cdot 48 (a_s/2l)^2 (\ln 2l/a_s - 1)(r_0\theta/l_m)^2$ – the cylinder curcle perimeter is equal to square one. In the structure with $\theta \in [0.003; 0.5]$, $l_m \in [4; 20]\,\mathrm{mm}$, $\mathrm{r}_0 = 3.5\,\mathrm{mm}$ and operating frequency $f_n = (k_n/2\pi\sqrt{\varepsilon_0\mu_0}) = 433$ MHz for $0.1 \times 0.1 \times 2.5\,\mathrm{mm}$ strip the averaged selectivity is $\Pi_{\perp} < 2 \cdot 10^{-4}$, that is greatly less than instrumental resolution. The cavity analysis in the form of coupled shortcircuated sector radial waveguide sections, loaded by end capacitances, gives according to [App. (11)] the deviation

$$\mid \frac{\delta f_{Ex}}{f_n} \mid = M \left((\frac{x_0}{r_0})^2 + (s_1 - \frac{1}{4})(\frac{l}{r_0})^2 \right), \qquad (2)$$

$M \cong 5.52 \cdot 10^{-6}$. An error in desired equality of deviations at RF phase measurements is determined by minimal phase count discrete $d\varphi_m$, and (2) result yields the strip displacement resolution $dx_0 \cong 2r_0(\pi M Q)^{-1} d\varphi_m$, Q - quality factor. For $d\varphi_m = 2 \cdot 10^{-5}, Q = 5 \cdot 10^3$ it is $dx_0 \simeq 1.6\,\mathrm{mkm}$, that cause the development of precise optical system for the images forming inside small aperture, lengthy ($L = 1445\,\mathrm{mm}$) bore without vanes lighting – the plate excitation by reflection phone is inadmissible. Autoreflective image forming in convergent rays of telescopic objective can be effected by micro-alignment telescope only by placing the light source beyond the graticule on the eyepiece side.

The system fig.2 comprises (4) and (5) micro-alignment telescopes [2]. Collimated image former (1) containes transparency (2) with adjustable in independent square directions transparent region; projective telescope (4) forms the image along its datum optical axis with displacements possibility by optical micrometers.

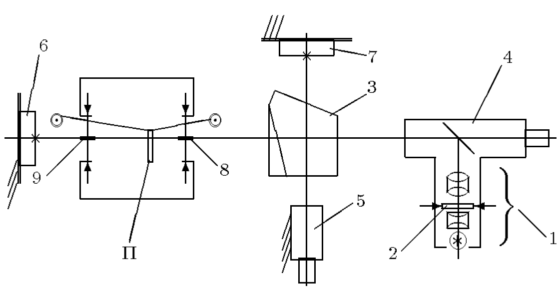

Figure 2: Images forming and measuring system.

Another telescope (5) is interconnected by pentaprism with wedge (3) for the image dimensions measurings on the plate (Π). Alignment on the base datum targets (6),(7) sets optical axes of the former and both telescopes in coincidence; the resonator is fixed on the datum axis by (4) viewfinding on removable transparent targets (8),(9); and datum line of sight can be ascertaned always by (5) viewfinding on (7) target after (3) removing. Transparency (2) is equipped by rotary device with $30''$ count accuracy. The system allows to form rectangular images with variable $0.05 \ldots 3\,\mathrm{mm}$ sides, total error of the image coordinates is $\pm(3 + 2D)\,\mathrm{mkm}$, D - image distance, M, but the systematic error can be excluded, [2]. So, the strip position inaccuracy is $3.5\,\mathrm{mkm}$ and the field axis coordinates real error $\approx 4\,\mathrm{mkm}$. The structure fine tuning properties θ,l_m are defined by the minimum dark-light deviation position detection under radial strip lighting refer to determined axis. According to [App.], this position corresponds to minimal integral value in (6) equation at the current distribution (7) for (1) field. Direct variation yields the minimum deviation angle α between Ox and the strip

$$\alpha = \frac{\arccos{(G\theta \sin \kappa z)}}{2}\ ; \ \ G = \frac{2(\kappa r_0)^2 \int_0^l I_1(\kappa x) F(x)\, dx}{\pi \int_0^l \kappa x F(x)\, dx},$$

$F(x) = kx - kl \sin kx {\cdot} \csc kl$, and it is enough to measure α under the plate removing along the structure for α^{max}, α^{min} values and $\alpha = \frac{\pi}{4}$ positions determination, because

$$\theta = (\cos 2\alpha^{min} - \cos 2\alpha^{max})(2G)^{-1}. \qquad (3)$$

Two counts phase method of α mesurment with rms error $(\delta\tilde{\varphi}/\varphi) = 2 \cdot 10^{-4}$ in (1) field gives inaccuracy $(\delta\tilde{\alpha_m}/\alpha) \le 1.7 \cdot 10^{-6}$, but the phase dependence is not symmetric refer α (except $\alpha = \pi/4$), and systematic innacuracy $(\delta\alpha_s/\alpha) \le 4.4 \cdot 10^{-5}$. Spatial selectivity is ample as before, sensitivity decreace is $2.5 \cdot 10^{-3}$ only. The angle reading rms error of the optical system is $1.4 \cdot 10^{-4}$, that determines real precision: for longitudinal coordinate count error $\simeq 5 \cdot 10^{-3}$ the modulation period innacuracy is $(\delta\tilde{l_m}/l_m) \le 7.1 \cdot 10^{-3}$, and (3) result gives accelerating efficiency with $(\delta\tilde{\theta}/\theta) \le 1.3 \cdot 10^{-2}$ error.

Asymmetric positioning of R_d radius, T thickness, ε dielectric constant plate will distort investigating field (on fig.1: F is fastening point, $\sqrt{2}b$ – sag to 3 quadrant). But only transverse field components inside the plate are used, l_m accuracy under $\alpha = \pi/4$ position determination in 1 and 3 quadrants is not changed, because $\delta\alpha_m$ rise even to two orders due to sensitivity lowering will not exeed determinative value. The θ error electrostatic estimating for $R_d = l, \varepsilon = 10, T = 0.1\,\mathrm{mm}, \mathrm{b} = 0.05\,\mathrm{mm}$ gives $(\delta\theta_\varepsilon/\theta) \le 6 \cdot 10^{-3}$, the efficiency error grow up to $1.5 \cdot 10^{-2}$.

Photosemiconductor characteristics analysis by [App.] method yields optimized ratio of the active light-dark conductivity: for $\varepsilon \le 10$ the ratio is $125 \ldots 130$, and CdS or CdSe materials suit perfectly well.

IV. CONCLUSION

Electro-optical principle and its application for RFQ structures precise measurements method have been developed. Designed optical system provides several micrometers accuracy not only for the field axis coordinates, but for geometrical shapes of the accelerating bore forming elements also – autoreflection method is effected by contact bore targets or master gauges fastening on the same filament. Developed balance technique (that excludes a field quantity measurements) for the structure fine tuning could be used for precise measurements in other types cavity resonators.

APPENDIX. Thin cylinder formfactor for RF nonhomogeneous fields

Use of retarded potential $\vec{A}$ field operator singularity for axially symmetric l length a radius longitudinal current $\vec{I}(x)$ yields on its circumference in ρ, ϕ, x local coordinates

$$\vec{A} = N(\rho) \cdot \vec{I}(x) \mid_{\rho \approx a}, \qquad (4)$$

where $N(\rho) = (1/2\pi)[\ln(2/k\rho\gamma) + Ci(kl) - (\sin kl/kl)], \gamma = 0,577\ldots$ - Euler's constant, $Ci(kl)$ - integral cosine. In two-component case (4) result inaccuracy will not exceed $(a/2l)^2$

even for equal longitudinal and transverse field components, that follows, e.g., from ellipsoid depolarization tensor principal values [3]; however, the practicaly used disposing along the supposed field vector will supplement decreace of inaccuracy to $3\ldots4$ orders. Boundary conditions in external E_x^e field lead to

$$\frac{\partial I}{\partial x} = -\frac{i\omega\varepsilon_0\varphi(x)}{N(a)}; \quad \frac{\partial \varphi}{\partial x} = E_x^e - I(x)\big(i\omega\mu_0 N(a) + z_l\big), \quad (5)$$

where z_l - line active resistance of the cylinder material, φ - scalar potential of (4) field. Now a resonator ν-mode $\{\vec{E}_\nu, \vec{H}_\nu\}$ with ω_ν resonant frequency, Q_ν quality factor is excited by some source S together with the cylinder $\vec{j}$ current density, and magnetic h_ν, electric e_ν fields amplitudes equations are

$$h_\nu(\omega_\nu^2 - \omega^2) + \frac{i\omega\omega_\nu h_\nu}{Q_\nu} = S - \frac{\omega_\nu}{W_\nu}\int_v (\vec{j}, \vec{E}_\nu)dv, \quad (6)$$

$e_\nu = i\omega h_\nu/\omega_\nu$; where $W_\nu = \varepsilon_0\int_{v_{re}}\big|\vec{E}_\nu\big|^2 dv$; v_{re}, v - resonator and cylinder volumes. Thus, for any $E_{\nu x}(x)$ function the $I(x)$ distribution is defined at $E_x^e = e_\nu E_{\nu x}(x)$ substituting in (5) equations:

$$\frac{\partial^2 I(x)}{\partial x^2} + k^2\big(1 - \frac{i\omega\varepsilon_0 z_l}{k^2 N(a)}\big)I(x) = \frac{\omega^2\varepsilon_0}{N(a)\omega_\nu}E_{\nu x}(x)h_\nu, \quad (7)$$

boundary values are $I(0) = I(l) = 0$ for distant from the cavity walls cylinder. So defined $I(x)$ determines integral value in equation (6), forming the amplitude equation for only S excitation of the resonator with new resonant frequency ω_n and new quality factor Q_n, i.e. the formfactor is defined. Eigenfunctions means substantiate that the summary field of all other modes with gradient summand is described by cylinder own field (4), and only condition for exact measurements is excluding of other modes excitation by S source in the cylinder region.

Simplest homogeneous field analysis for $kl \le 0.1$ gives

$$Q_n \cong Q_\nu\frac{\omega_\nu}{\omega_n}\left(1 + \frac{\omega_\nu^2 - \omega_n^2}{\omega_\nu\omega_n}Q_\nu\frac{z_l l}{\sqrt{\frac{\mu_0}{\varepsilon_0}}N(a)}\frac{k_n l}{(k_n l)^2 + 10}\right)^{-1};$$

quality factor decrease due to finite coductivity of a thin $(N(a) > 1)$ cylinder even with $z_l l = 10\,\mathrm{Ohm}$ will be in 4 order only. Therefore, $z_l = 0$ value can be used indeed, and for any kl the method yields (in conventional Slater's form writing):

$$\frac{\omega_n^2 - \omega_\nu^2}{\omega_n^2} = -\frac{\big[E_{\nu x}(x_0)\big]^2\varepsilon_0 v}{W_\nu}K, \quad (8)$$

where x_0 - coordinate of the field reading, $K = K_0$ -formfactor:

$$K_0 = \frac{(\frac{l}{a})^2}{\pi N(a)}\frac{2\tan\frac{k_n l}{2} - k_n l}{(k_n l)^3}. \quad (9)$$

Product vK_0 for $(a/l) \ll 1, k \to 0$ coinsides with the result of electrostatic analysis [3]. However, if the cylinder approaches the cavity walls (e.g. $x = 0$ endpoint is near the wall), $I(0) = 0$ value must be interchanged by $\varphi(0) = -I(0)/i\omega C_e$, or $I(0) = \beta(\partial I/\partial x)|_0, C_e$ - the cylinder end-wall and the cavity wall capacitance, $\beta = C_e N(a)/\varepsilon_0$:

$$K_w = K_0\left(1 + \beta k_n\frac{1 - 2\tan\frac{\tau}{2}\cot\tau}{(2\tan\frac{\tau}{2} - \tau)(1 + \beta k_n\cot\tau)}\right), \quad (10)$$

where $\tau = k_n l$. The end contiguity $(\beta \to \infty)$ yields $K_w \cong 4K_0$ for $kl \le 0.1$ and than K_w decreases up to K_0 for $\beta \to 0$ with the cylinder moving off, (10) general relation corresponds to electrostatic model [4].

Practicaly interesting results of nonhomogeneous field analysis are obtained at polinomial representation, e.g. for $E_{\nu x} = A_2x^2 + A_1x + A_0$ the method gives

$$K_p = K_0\left(\Upsilon + \eta\frac{4\big(4\eta s_2 + \frac{2\xi s_3 + s_4}{1+\xi}\big) - \Upsilon(\eta + 2)}{(1+\eta)^2}\right), \quad (11)$$

where $\Upsilon = 1 + (4s_1 - 1)\big(\frac{\xi}{1+\xi}\big)^2;\ \xi = \frac{A_1 l}{2A_0},\ \eta = \frac{A_2 l^2}{4A_0(1+\xi)}$ - normalised variability,

$$s_1 = \frac{\frac{\tau^2}{3} + \tau\cot\tau - 1}{\tau(\tau - 2\tan\frac{\tau}{2})},\ s_2 = \frac{\frac{\tau^2}{5} + \tau\cot\tau - \frac{2}{3}}{\tau(\tau - 2\tan\frac{\tau}{2})} + \frac{2}{\tau^2}\big(\frac{2}{\tau^2} - 1\big),$$

$$s_3 = \frac{1}{2} - \frac{2}{\tau^2} + \frac{\cot\frac{\tau}{2}}{\tau}, \quad s_4 = 1 - \frac{4}{\tau^2} - \frac{\frac{\tau}{3}}{\tau - 2\tan\frac{\tau}{2}};$$

and $K = K_p,\ x_0 = \frac{l}{2}$ in form (8).

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