Transverse Coupling Impedance Measurement Using Image Current *

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Abstract

Results of transverse impedance measurement using image current are reported. The results show that this technique can be used up to frequency of 3.9 GHz and high Q of 3300.

I. Introduction

The transverse coupling impedance is a measure of the interaction between the beam and deflecting modes (TM_{1n0}) in accelerator cavities. Two-wire and bead-pull techniques are usually employed for the measurement of transverse coupling impedance. The principle of the wire technique is to insert wires into cavities and beam line components to transform them into transmission lines and measure the scattering parameters. Coupling impedances (longitudinal or transverse) are extracted from S parameters [1]. For longitudinal coupling impedance measurement, a single wire is inserted in the center of cavities and beam components to form a coaxial transmission line. For transverse coupling impedance, two off-center wires are inserted to form a balanced-two-wire transmission line and a 180 degree, 3 db power splitter (hybrid) is used to excite TM_{1n0} modes. In addition to error sources in the one-wire (center wire) technique, the two-wire technique has more error sources. First, the power splitter causes phase and amplitude errors. Although amplitude differences from two output ports of power splitters are usually small, phase differences of 0-12 degrees over 0-2 GHz range are common among commercial power splitters. Second, phase and amplitude errors are introduced by geometric factors of the two wires. In practice the two wires may not be perfectly parallel to each other and the distances between the symmetric plane and the two wires may not be the same along the wires. These geometric factors can easily introduce about 5% amplitude and phase distortions. Third, unavoidable differences of electrical lengths between two wires due to non-identical connections and matching components etc. produce phase differences between the two wires. In order to avoid the aforementioned error sources and improve the two-wire technique, measurements using image current have been done. This technique uses an off-center wire and half-cylindrical structure covered with a conductive plate which serves as a mirror plane (Fig. 1.) Since the image current plays the role of the second wire, the error sources due to the differences between two wires are minimized. The error sources from the power splitter are eliminated since a power splitter is no longer needed.
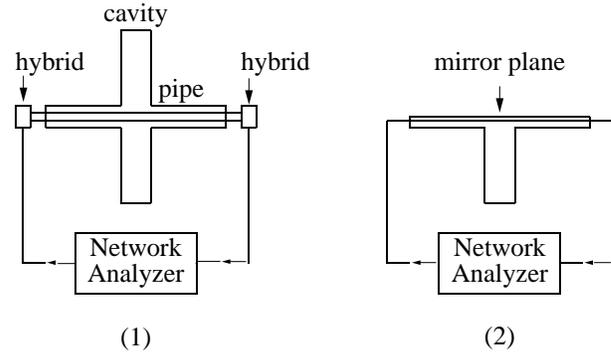
Figure 1: (1) Whole cavity with two wires and hybrids, (2) Half of (1) without hybrids.

II. Measurement Procedure

II.1. Test Apparatus

To use the image current method, an half cylindrical (pill-box) aluminum cavity with beam pipe was built. The length of the cavity was 1.575 cm. The radius of the cavity was 12.057 cm. The radius of the beam pipe was 1.740 cm. The beam pipe and other required components such as reference lines and calibration standards (will be discussed later) were made in a half-cylindrical configuration. A copper plate was mounted on the half cylindrical structure at the cut plane and covered the total length of the structure. A single wire was inserted into the pipe and cavity. The space between the wire and the copper plate could be adjusted. The radius of the wire was 0.0114 cm. To stretch the wire and connect this type of transmission line to the ordinary coaxial cables which are connected to the network analyzer, a pair of half cylindrical matching sections were made. The end of each matching section was sealed by a detachable aluminum block (end block). A pair of semi-rigid coaxial cables were inserted through the end blocks. The wire was soldered onto the inner conductors of these cables. Resistive matching was used to reduce the reflection from the mismatch between matching sections and semi-rigid coaxial cables. The drawback of resistive matching is that magnitudes of signals become much smaller. Also at high frequencies, resistive matching is not effective. In order to get reliable results, it is critical that all parts which have to be disconnected and re-connected during the process of calibration or measurement must have high repeatability. To ensure good repeatability, special measures have been taken. First, the design of detachable end blocks of the matching sections allows the wire, the resistive matching parts, the end blocks and the

coaxial cables to be moved together from the DUT (device under test) to reference lines. This feature reduces the number of times necessary to disconnect wires, resistors and connectors between matching sections and measurement cables. Second, the cavity and its attached beam pipe, the reference and calibration transmission lines can be opened by detaching the copper plate. As a result of this feature, the process of stretching wires and soldering resistors for resistive matching can be controlled more precisely than in a closed structure.

II.2. Calculation of impedance

The calculation of impedance from S parameters of a transmission line (reference line) and a cavity inserted with a wire has been discussed in several publications [1] which is outlined as follows. Since our cavity is short, a lumped impedance approximation may be used. The shunt impedance of the cavity is calculated as:

$$Z = 2Z_0 \left(\frac{S_{21\mathrm{ref}}}{S_{21\mathrm{cav}}} - 1 \right) \tag{1}$$

where $S_{21\mathrm{cav}}$ and $S_{21\mathrm{ref}}$ are the S_{21} parameters of a transmission line with and without cavity respectively while keeping the same total length. Z_0 is the characteristic impedance of the transmission line. The transverse impedance of dipole modes can be deduced from the shunt impedance as:

$$Z_\perp = \left(\frac{c}{wd} \right)^2 Z \tag{2}$$
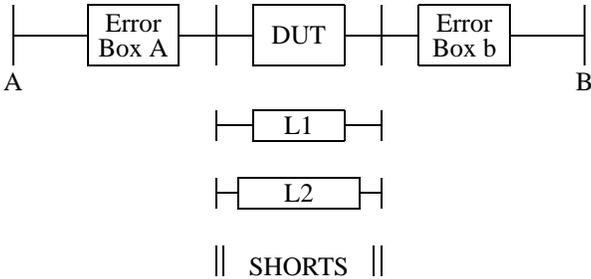
A,B: reference planes of network analyzer

Figure 2. Measurement and calibration models.

II.3. De-embedding procedure

The S parameters of a device under test (DUT) are de- embedded from the S parameters measured at the reference planes of the network analyzer as follows. Shown in Figure 2 are the models of what is measured by a network analyzer at its reference planes. The wave cascade matrix R of a two port network is defined as:

$$\begin{bmatrix} b_1 \\ a_1 \end{bmatrix} = [R] \begin{bmatrix} a_2 \\ b_2 \end{bmatrix} \tag{3}$$

where a_1 and a_2 are incident waves at ports 1 and 2, b_1 and b_2 are emergent waves at ports 1 and 2 respectively. The S parameter matrix S of a two port network is defined as:

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = [S] \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \tag{4}$$

The R matrix and S matrix are related by:

$$[R] = \frac{1}{S_{21}} \begin{bmatrix} -\Delta & S_{11} \\ -S_{22} & 1 \end{bmatrix} \tag{5}$$

From the definition of R matrix, the R matrix measured at network analyzer reference planes, $[R_m]$, can be expressed as:

$$[R_m] = [R_a] [R_{\mathrm{dut}}] [R_b] \tag{6}$$

where the $[R_a]$, $[R_b]$ and $[R_{\mathrm{dut}}]$ are the wave cascade matrcies of error boxes A, B and DUT respectively.

If $[R_a]$ and $[R_b]$ are known, $[R_{\mathrm{dut}}]$ can be obtained through (6) and the S parameters of DUT can be calculated through (5).

II.4. Calibration Procedure

To obtain $[R_a]$ and $[R_b]$, the "generalized through-reflect-line" calibration procedure is used [2][3]. This calibration algorithm requires two lengths of transmission lines and two equal reflects (shorts) of unknown reflection coefficients. In order to ensure the accuracy of the calibration, the phase difference between these two transmission lines (standards) should not be too close to 0 or 180 degrees. The procedure developed in [3] is outlined as follows. Successive measurement of line 1 and line 2 yields:

$$[R_m(L1)] = [R_a] [R_{L1}] [R_b] \tag{7}$$

$$[R_m(L2)] = [R_a] [R_{L2}] [R_b] \tag{8}$$

Measurement of two shorts yields:

$$\Gamma_1 = S_{11A} + \frac{S_{12A}S_{21A}}{\frac{1}{\Gamma_R} - S_{22B}} \tag{9}$$

$$\Gamma_2 = S_{22B} + \frac{S_{12B}S_{21B}}{\frac{1}{\Gamma_R} - S_{11B}} \tag{10}$$

Thus, S parameters of error box A and B and the propagation parameter (γ) of a transmission line with length of (L2-L1) can be obtained by solving equations (7),(8),(9) and (10). In the process of solving equations (7),(8),(9) and (10), however, two complex quadratic equations have to be solved. Each time, only one of the two conjugate roots of the quadratic equation is the right choice. If the correct roots are chosen, the following three conditions should be met [2]: (1) $|R_{a21}/R_{a22}| < |R_{a11}/R_{a12}|$, (2) $\left| e^{2\gamma(L2-L1)} \right| < 1$, and (3) the difference between the argument of deduced $e^{2\gamma(L2-L1)}$ and the estimated phase difference $\gamma(L2 - L1)$ from the real length of L2 and L1 should not exceed 90 degrees. Therefore, theoretically any one of these three conditions can be used as a criterion to determine the correct root. In our computer code implementing the calibration and de-embedding procedure, all of these three conditions are checked simultaneously at each data point. The conditions (1) and (3) are used as criteria to chose roots and to check each other. This approach is effective to ensure the reliability of the calibration.

The condition (2), however, can not be used practically as a reliable criterion from our observation which is in agreement with the statement in [2]

To check the generalized TRL algorithm and our computer code, three sections of semi-rigid coaxial lines with SMA connectors were carefully made. Two of them and an SMA short are used as calibration standards. The third section is used as DUT. The results are very close to theoretical values.

III. Measurement and Results

III.1. Repeatability

The repeatability is mainly affected by changing/stretching of wires, and opening/closing of the mirror plane (copper plate) during calibration and measurement. The repeatability was checked at the resonant frequency of the first dipole mode (1489 MHz). The mirror plane was opened and re-connected. The results show that the relative errors of magnitudes of S parameters between these procedures were less than 1%, and the phase errors were less than 0.5 degree. Also wires with a difference of 1 mm in length were used which affected the position of the matching resistors and the penetration of the movable coaxial cables in the matching section. The relative errors of magnitudes of S parameters between the two measurements is also less than 1%, and the phase errors are less than 0.5 degree. These results are in agreement with the repeatability of the final results (Z/Q) of this mode.

III.2. Results

The measurement results for the first three dipole modes are shown in Tables 1 and 2. Theoretically, Z/Q of a cavity is determined only by the geometry of the cavity. However, due to the high Q value of the cavity the magnitude of |S21| was very small (-50~-60 db) which may affect accuracy of the measurement. To check such a possible problem, the Q of the cavity was lowered by inserting a metal wire into the cavity or magnetically coupling resistors through a hole which is 4 cm from the mirror plane. The results show that there is no systematic change of Z/Q when Q and the magnitude of |S21| vary.

As a comparison, the results from measurements using bead pull technique and the calculated results from computer code URMEL-T are also listed.

Table 1. Results of Z/Q (wire spacing: 0.5 inch)

Mode	f_0	Q	Z/Q	Z/Q	Z/Q
	MHz		Wire	Bead pull	Urmel-T
TM110	1482	9188			2.97
	1487	1319		2.65	
	1489	2539	3.15		
	1487	1890	3.20		
	1486	1524	3.15		
	1488	1111	3.15		
	1487	956	2.90		
	1492	639	2.70		
TM120	2689	11685			7.28
	2703	2800		7.29	
	2723	3019	5.60		
	2736	2461	5.80		
	2725	1826	5.05		
	2736	502	5.55		
TM130	3898	13525			8.79
	3923	3230		13.10	
	3979	3248	15.65		
	3977	2015	15.50		

Table 2. Results of Z/Q (wire spacing: 0.3 inch)

Mode	f_0	Q	Z/Q	Z/Q	Z/Q
	MHz		Wire	Bead pull	Urmel-T
TM110	1482	9184			1.07
	1487	1867		1.17	
	1488	2127	1.15		
	1485	1753	1.15		
	1480	879	1.15		
	1480	331	1.20		
TM120	2690	11677			2.63
	2706	2612		4.31	
	2712	2641	3.00		
	2711	2540	3.00		
	2712	2364	3.00		
	2724	2234	3.15		
	2714	1471	2.75		
	2724	612	3.20		

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