

# ON ENHANCEMENT OF LIMITED ACCELERATING CHARGE

Yu. Tur, National Science Center - Kharkov Institute of Physics & Technology,  
310108 Kharkov, UKRAINE

## Abstract

Results are presented of theoretical analysis of electron flow interactions with the accelerating field of superposed multiple parts of the first and second harmonics. It is shown that at certain phases and amplitudes relationships the limited charge value can be increased in bunches of considerable phase lengths owing to the preservation of identical accelerating conditions and decreasing of the losses to radiation by the beam.

One of the fundamentals problem of the physics of accelerators is the one of increasing the accelerated charge limit at restrained levels of rf-power input and rf emittance growth suppression. Underlying this proposal are the effects of the non-linear relationship of bunch-radiated field amplitude vs. its phase length and, also, the feasibility of providing the identical acceleration conditions for all particles in a lengthy bunch and emittance preservation in the "table-type" rf-field, realized by way of inclusion in the acceleration mode of multiple frequency harmonic superposition. It must be noted that the acceleration technique, providing for excitation in each of the accelerating space of rf-fields at multiply frequencies with the resulting envelope shape being close to rectangular, is well known and employed, for instance, in proton synchrotron for widening the region of phase stability and lowering the space charge effect (see, for instance [1]). Below is given the evaluation of this technique to the problems of increasing the accelerating charge limit.

Considered is a cavity of arbitrary transverse cross section with a length of the interaction region  $d$  and two resonances at frequencies  $\omega$  and  $2\omega$ . Let's consider excitation of this cavity in the approximation assigned current from an external oscillator by the current with density

$$\vec{j} = \vec{j}_{11} \cos(\omega\tau) + \vec{j}_{12} \sin(\omega\tau) + \vec{j}_{21} \cos(2\omega\tau) + \vec{j}_{22} \sin(2\omega\tau) \quad (1)$$

as well as by a beam with pulsed current  $I$ , bunches of which of the length  $2\tau$  moving with the velocity  $v$ , are taken to be rectangular, then expanding into the Fourier series we have for the beam current density

$$j_{xn} = j_{yn} = 0$$

$$j_{zn} = j_0 f(x, y) \left[ \frac{\sin \omega\tau}{\omega\tau} \cos \omega \left( t - \frac{z}{v} \right) + \frac{\sin 2\omega\tau}{2\omega\tau} \cos 2\omega \left( t - \frac{z}{v} \right) \right] \quad (2)$$

Expanding the vector-potential of the excited field into the cavity eigen-modes

$$\vec{A} = \sum_i q_i(t) \vec{A}(\vec{r}),$$

the set of equations for cavity field will be written down as follows:

$$\ddot{q}_1 + 2\beta_1 \dot{q}_1 + \omega^2 q_1 = 4\pi \cdot c \frac{\int (\vec{j}_{12} + \vec{j}_{1n}) \vec{A}_1 dv}{\int \vec{A}_1^2 dv}.$$

$$\ddot{q}_2 + 2\beta_2 \dot{q}_2 + 4\omega^2 q_2 = 4\pi \cdot c \frac{\int (\vec{j}_{22} + \vec{j}_{2n}) \vec{A}_2 dv}{\int \vec{A}_2^2 dv} \quad (3)$$

Considering for the sake simplicity that  $E$  is independent of  $z$  and doing the integration, we will obtain:

$$q_1 = \frac{D_1 I_0 \frac{\sin \omega\tau}{\omega\tau} - I_1}{2\beta_1 \omega_1} \sin \omega_1 t; \quad q_2 = \frac{D_2 I_0 \frac{\sin 2\omega\tau}{2\omega\tau} + I_2}{2\beta_2 \omega_2} \sin \omega_2 t \quad (4)$$

where

$$I_1 = -4\pi c \frac{\int \vec{j}_{11} \vec{A}_1 dv}{\int \vec{A}_1^2 dv}; \quad I_2 = 4\pi c \frac{\int \vec{j}_{12} \vec{A}_2 dv}{\int \vec{A}_2^2 dv} \quad (4a)$$

$$D_1 = 4\pi c \frac{2 \frac{v}{\omega} \sin \frac{\omega d}{2v} \int f(x, y) \vec{A}_{z1} dx dy}{\int \vec{A}_1^2 dv} \quad (4b)$$

$$D_2 = 4\pi c \frac{\frac{v}{\omega} \sin \frac{\omega d}{v} \int f(x, y) \vec{A}_{z2} dx dy}{\int \vec{A}_2^2 dv}$$

At which, the harmonic phases are chosen such that  $I_1 > 0$  and  $I_2 > 0$ , while

$$\int \vec{j}_{11} \vec{A}_1 dv = \int \vec{j}_{22} \vec{A}_2 dv = 0$$

Hence the expression for the resulting cavity field in the steady-state regime:

$$\bar{E} = \frac{I_1 - D_1 j_0 \frac{\sin \omega \tau}{\omega \tau}}{2\beta_1 c} \bar{A}_1(\bar{r}) \cos \omega t - \frac{I_2 + D_2 j_0 \frac{\sin 2\omega_0 \tau}{2\omega_0 \tau}}{2\beta_2 c} \bar{A}_2(\bar{r}) \cos(2\omega t) \quad (5)$$

We'll find the increment of particle energy:

$$\Delta W = e \int_{-d/2}^{d/2} \bar{E}(t = t_0 + z/v) dz \quad (6)$$

here  $t$  is the particle time-of-flight through the center of cavity.

Substituting (5) and integrating (6), we will obtain:

$$\Delta W = \Delta W_{\bar{A}_1} \cos \omega t_0 - \Delta W_{\bar{A}_2} \cos 2\omega t_0 - c_1 j_0 \frac{\sin \omega \tau}{\omega \tau} \cos \omega t_0 - c_2 j_0 \frac{\sin 2\omega \tau}{2\omega \tau} \cos 2\omega t_0 \quad (7)$$

where

$$\Delta W_{\bar{A}_1} = \frac{evI_1}{\beta_1 \omega \cdot c} A_{1z}(x_0, y_0) \sin\left(\omega \frac{d}{2v}\right) \quad (7a)$$

$$\Delta W_{\bar{A}_2} = \frac{evI_2}{2\beta_2 \omega \cdot c} A_{2z}(x_0, y_0) \sin\left(\omega \frac{d}{v}\right)$$

$$c_1 = \frac{evD_1}{\beta_1 \omega \cdot c} A_{1z}(x_0, y_0) \sin\left(\omega \frac{d}{2v}\right) \quad (7b)$$

$$c_2 = \frac{evD_2}{2\beta_2 \omega \cdot c} A_{2z}(x_0, y_0) \sin\left(\omega \frac{d}{v}\right)$$

$x_0, y_0$  are the beam transverse coordinates.

Let's compare the obtained result with the single-harmonic interaction case with small phase space of the bunch ( $\omega\tau \rightarrow 0$ )

$$\frac{\Delta W_{\bar{A}_1}^{\max}}{\Delta W_1^{\max}} = \frac{\Delta W_{\bar{A}_1} - \Delta W_{\bar{A}_2} - c_1 j_0 \frac{\sin \omega \tau}{\omega \tau} - c_2 j_0 \frac{\sin 2\omega \tau}{2\omega \tau}}{\Delta W_{\bar{A}_1} - c_1 j_0} \quad (8)$$

and at ( $\omega\tau > 1$ )

$$\frac{\Delta W_{\bar{A}_1}^{\max}}{\Delta W_1^{\max}} = \frac{\Delta W_{\bar{A}_1} - \Delta W_{\bar{A}_2}}{\Delta W_{\bar{A}_1} - c_1 j_0} \quad (9)$$

As an example, let's consider beam acceleration in the cavity excited by the first and second harmonic at power input ratio 10:1 with angle of flight at the fundamental frequency  $\frac{\omega \cdot d}{2v} = \pi/2$ . In this case, the identical conditions for accelerating all particles become feasible to fulfill for bunches with the phase space up to 30°, and, as it is easy to deduce from the above relationships, the losses to radiation by the beam can be decreased as compared to the single-frequency case, by  $\approx \frac{\omega \tau}{\sin \omega \tau}$ , i.e. the limit of accelerated charge is thus raised.

## REFERENCES

[1] L.Z. Barabash, P.I. Lebedev et al. Enhancement of particle capture in proton synchrotron by adding the second harmonic to the accelerating field-Proc. All-Union Seminar on Charge Particle Accelerators, Moscow, 1968, vol.2, 123-127.