ASYMMETRIC EMITTANCE BEAM GENERATION USING ROUND BEAM RF GUNS AND NON-LINEAR OPTICS

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ABSTRACT

Future generation electron-positron linear colliders need asymmetric flat beams in order to properly control the beam-beam interaction at the final focus. Room temperature collider designs ask for normalized rms transverse emittances which look not attainable by RF Photo-Injectors, whilst the requirements of the TESLA superconducting collider ($\varepsilon_{nx} = 20$ mm mrad, $\varepsilon_{ny} = 1$ mm mrad) are at the edge of the present RF gun status of the art, except for the asymmetric emittance requirements. In this paper we explore the efficiency of conventional round-beam RF guns in conjunction with non-linear optical devices, namely sextupole triplets, as converters of round beams into flat beams, in order to match the TESLA specifications. Basically the beam emittance, at the exit of the gun, must be dominated by non-linear effects (spherical aberrations). A quadrupole doublet and a sextupole triplet, placed downstream the beam transport line, provide respectively a geometric astigmatism and an asymmetric emittance correction in the vertical and in the horizontal phase-spaces. The possibilities to produce asymmetric beams using this technique are presented here. In Section 1 we introduce the subject, premises and general performances of the device. In Section 2 we give results of some analytical scaling laws. In Section 3 we show some preliminary simulations of ray-tracing throughout the structure.

I. INTRODUCTION

The present study arises from the following considerations:

1 - a consolidated knowledge of state of the art RF-guns to generate high quality round beams ($\varepsilon_{nr} = 2$-4 mm mrad).

2 - a great interest in finding "slim" structures to convert round beams into asymmetric beams, which may offer an alternative to the damping rings.

3 - there exists no linear transformation that allows to increase the emittance aspect ratio $\varepsilon_{nx}/\varepsilon_{ny}$: belonging to the class of simplectic matrixes[1], any of them conserves the squared sum $\varepsilon_{nx}^2 + \varepsilon_{ny}^2$ and the product $\varepsilon_{nx} \cdot \varepsilon_{ny}$.

Starting from these assumptions, we found that non-linear optical components could produce asymmetric emittances if the original beam emittance (equal in both planes) is dominated by spherical aberrations. In Fig. 1 we present a layout of the non linear optical system (the *flat beam converter*) which accomplishes this task.

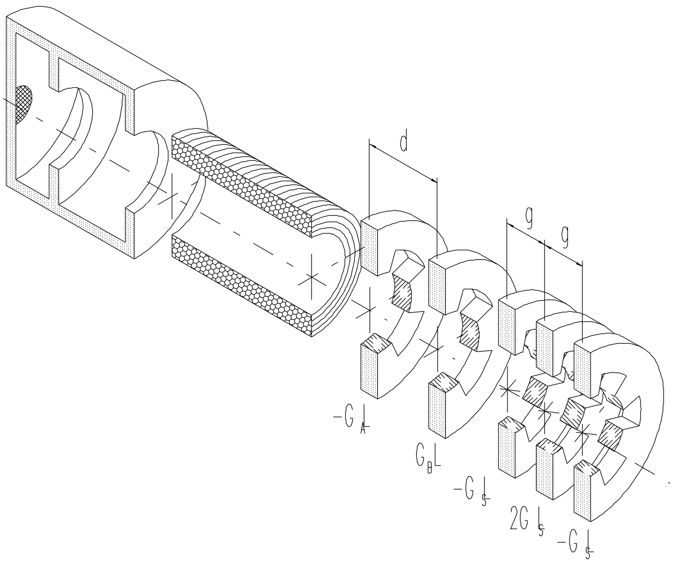


Fig. 1: Layout of the flat beam converter.

The input phase space can be described by:

$$p_{r0}(r_0) = c_1 \cdot r_0 + c_3 \cdot r_0^3$$
$$p_{x0} = c_1 x_0 + c_3 \left(x_0^2 + y_0^2\right) x_0 \qquad 0 < r_0 < R_0, c_3 > 0 \qquad 1.1$$
$$p_{y0} = c_1 y_0 + c_3 \left(x_0^2 + y_0^2\right) y_0$$

It has been shown[2] that disk-like bunches (geometric aspect ratio R/L>>1), generated by high gradient RF guns (100 MV/m), present the desired phase-space profile: in laminar flow regime, the linear correlation gives negligible emittance $\left(\varepsilon_{nr} = \left(\langle r^2 \rangle \cdot \langle p_r^2 \rangle - \langle r \cdot p_r \rangle^2\right)^{1/2}\right)$, while the third order correlation becomes dominant. Moreover, depending on the radial charge density profile at the cathode, from pure uniform to pure gaussian, spherical aberrations assume positive, null or negative values. For pure uniform distributions the emittance of the generated beam shows a positive spherical aberration, $\varepsilon_{nr0} = c_3 R_0^4 / \sqrt{72}$, ($\varepsilon_{nr0} = 2\varepsilon_{nx_i0}$) .

The beam generated by the RF gun is then driven through a solenoid, a quadrupole doublet and a sextupole triplet. The quadrupole lens is set to provide the beam with a proper astigmatism ratio, $y_{\max} / x_{\max} = t$, and cancel (double waist) both horizontal and vertical linear correlation. The sextupole lens[3] applies a non linear radial focusing kick $\Delta p_r = -\alpha \cdot r^3$, which depends on the integrated gradient of the sextupoles $G_s L$, the drift distance between the sextupoles

g, and the longitudinal momentum of the particles p_z, through the coefficient

$$\alpha = -4 \cdot \frac{g}{p_z}\left(\frac{q}{m_0 c} \cdot G_s L\right)^2 \cdot r^3$$

The sextupole triplet has been properly matched to correct at best the third order correlation in the horizontal phase-space. Due to the astigmatism, in the vertical phase-space, emittance correction cannot be optimized at the same time. This is the origin of the asymmetry.

In this scheme the solenoid has a secondary role, it reduces the transverse divergence of the electrons at the entrance of the quadrupole lens: actually, the higher is the particle radial momentum, p_{r-in}, the stronger has to be the quadrupole focusing powers (to have a double waist. Transverse momentum corrections consistently influence the particles motion, changing their longitudinal momentum, $p_z = \sqrt{\gamma^2 - 1 - p_{tr}^2}$. At first order, $\Delta p_{z-out} \approx p_{r-in}^2 / p_{z-in}$; the solenoid is then inserted to lower p_{r-in} and damp undesired non-linear effects.

II. ANALYTICAL RESULTS.

In this section we study the dynamics of the particles, through the quadrupole doublet and the sextupole lens. We want to write some scaling laws to describe the output beam characteristics, emittance aspect ratio and beam brilliance. To describe the efficiency of the device: we follow the track of a particle beam, starting with an emittance dominated by spherical aberrations, in a laminar flow regime. The results show that the efficiency of the device depends slightly on the input characteristic of the initial beam (charge, energy, divergence, strength of the spherical aberration...), the dominant parameter being only the beam astigmatism ratio t, introduced by the quadrupole doublet.

Let $Q_{y,x}$ be the matrixes of the quadrupole doublet,

$$Q_{y,x} = \begin{pmatrix} 1 & 0 \\ \mp 1/f_A & 1 \end{pmatrix}\begin{pmatrix} 1 & \frac{d}{p_z} \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ \pm 1/f_B & 1 \end{pmatrix}; \qquad (2.1)$$

where $1/f_{A,B} = q/mc \cdot G_{A,B} L / p_z$ (the focusing powers of the quadrupoles) and d (the drift distance), represent the free parameters ($G_{A,B} L$ are the integrated gradients). Since we assume a *laminar flow* (the divergence depends only on the radial position in the bunch) we may apply a *point to point* computation instead of matrix transport. The double waist condition, $Q_x\begin{pmatrix} x \\ C \cdot x \end{pmatrix} = \begin{pmatrix} m \cdot x \\ 0 \end{pmatrix}$, $Q_y\begin{pmatrix} y \\ C \cdot y \end{pmatrix} = \begin{pmatrix} n \cdot y \\ 0 \end{pmatrix}$, allows to reduce the free parameters to one, and to express, $Q_{y,x}$, $1/f_{A,B}$, and the resulting magnitude factors m and n, as functions of d:

$$\frac{1}{f_A(d)} = \sqrt{1 + \frac{p_z}{d}C} \quad m(d) = \left(1 + \frac{d}{f_A(d) p_z}\right) + \frac{dC}{p_z},$$
$$\frac{1}{f_B(d)} = \frac{p_z / g}{\sqrt{1 + \frac{p_z}{d}}} \quad n(d) = \left(1 - \frac{d}{f_B(d) p_z}\right) + \frac{d}{p_z} \qquad (2.2a\text{-}d)$$

In this matrix computation we have disregarded the existence of the third order term in the momentum expression, thus finding the best quadrupole settings to cancel the linear correlation. Now we insert expressions for A and B into matrices $Q_{y,x}$ and apply a point to point transport, using the third order expression (1.1) for the input phase-space (Fig.2). Having done so, the beam distribution at the exit of the quadrupoles is described by the equations (Fig.3):

$$p_{x1} = \frac{c_3}{m^4}\left(x_1^2 + y_1^2\right) \cdot x_1 + \frac{c_3}{m^4} \cdot \left(\frac{m^2}{n^2} - 1\right) y_1^2 \cdot x_1$$
$$p_{y1} = \frac{c_3}{n^4}\left(x_1^2 + y_1^2\right) \cdot y_1 + \frac{S}{n^4} \cdot \left(\frac{n^2}{m^2} - 1\right) x_1^2 \cdot y_1 \qquad (2.3a,b)$$

The particles are uniformly distributed over an ellipse of semi-axes $x_{1\max} = m \cdot r_0$ and $y_{1\max} = n \cdot r_0$. Eqs (2.3) do not return two pure spherical aberrations. We can look at them as the combination of a pure cubic term, respectively of coefficient c_3 / m^4 and c_3 / n^4, and a residual part, given by

$$\frac{c_3}{m^4} \cdot \left(\frac{m^2}{n^2} - 1\right) y_1^2 \cdot x_1 \text{ and } \frac{c_3}{n^4} \cdot \left(\frac{n^2}{m^2} - 1\right) x_1^2 \cdot y_1,$$

that is not removable with sextupoles. It is clear now that the efficiency of the flat beam converter has to deal with these terms. Referring to the astigmatism ratio, $t=n/m$, we see that:

- the closer is t to unity, the weaker are residuals: no asymmetry has been generated, but we can completely cancel the emittance;

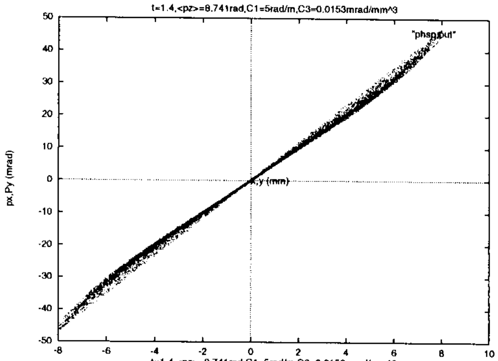
- the farther is t to unity, the larger are the residuals: there is big asymmetry but we have bad chance to correct any of the two phase-spaces for their consistent residuals.

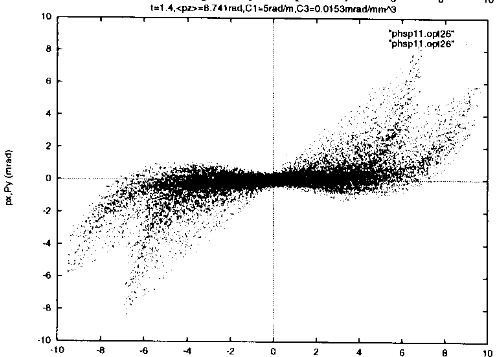
Here below we report the scaling low for the emittance variations, $\varepsilon x = \varepsilon_{nx} / \varepsilon_{nx0}$ and $\varepsilon y = \varepsilon_{ny} / \varepsilon_{ny0}$, the aspect ratio $\varepsilon_{ny} / \varepsilon_{nx}$, and the emittance cross product $(\varepsilon_{ny}\varepsilon_{nx}) / (\varepsilon_{ny0}\varepsilon_{nx0})$ (in units of the initial values), obtained correcting the spherical aberration in the horizontal motion, by means of a sextupole triplet of strength $\alpha = c_3 / m^4$:

$$R = \frac{\varepsilon_{ny}}{\varepsilon_{nx}} = \frac{\sqrt{5}}{5} \cdot \left(\sqrt{8 + 12 \cdot t^2 + 9 \cdot t^4}\right) \qquad (2.6)$$

$$\varepsilon x = \frac{\sqrt{10}}{4}\left|1 - t^2\right|, \ \varepsilon x = \frac{\sqrt{2}}{4}\left|1 - t^2\right|\sqrt{8 + 12 \cdot t^2 + 9 \cdot t^4} \qquad (2.7a,b)$$

$$P = \varepsilon x \cdot \varepsilon y = \frac{\sqrt{5}}{8}\left(1 - t^2\right)^2 \cdot \sqrt{8 + 12 \cdot t^2 + 9 \cdot t^4} \qquad (2.8)$$





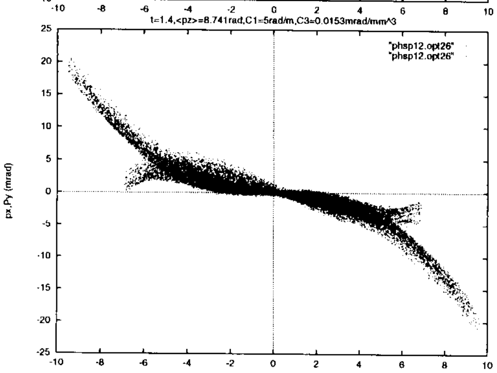


Fig. 2: Phase-space of a spherically aberrated round beam
Fig.3: Beam phase-space after the quadrupoles (since t=1.4>1, the larger dimension is y, the smaller is x).
Fig.4: Phase-space after the sextupoles (spherical aberration has been corrected).

As brilliance scales like $\left(\varepsilon_{ny}\varepsilon_{nx}\right)^{-1}$, the product P is an index of the beam degrade. We remind that t is the astigmatism ratio m/n between the vertical and the horizontal orientation. Hence, choosing to damp the horizontal emittance, for t <1 the lower emittance is assigned to the larger dimension, while for t >1 the lower emittance is assigned to the smaller one.

III.COMPARISON BETWEEN SIMULATIONS AND ANALYTICAL SCALING LAWS

We briefly present in the following the results of simulations for a flat beam converter. The beam has been generated with the code ITACA, from a 1+1/2 cell injector, assuming a cathode field of 100 MV/m and a geometrical aspect ratio of 10. The beam dynamics through the flat beam converter has been simulated with a ray-tracing code, based on the thin lens approximation.

A comparison between simulations (dotted line) and analytical expressions (solid line) is shown in Figs. 5 and 6, where the emittance aspect ration *R(t)* and the emittance product *P(t)* are plotted as a function of the astigmatism ratio t. When the astigmatism is null (t =1) the solid curves do not agree with the results from the scaling laws, because the latter assume zero thermal emittances; in effect the thermal emittance generated by ITACA prevents the product *P(t)* from dropping to zero and forces the aspect ratio to be one. Fig. 5 and 6 also show that high values of aspect ratios cannot be reached without degrading the beam brightness: the optimal work-point for applications is close to t =1 , but strictly different than 1. For 1<t<2 we may reach aspect ratios much larger than 1 without degrading the beam too much: in particular for t =1.30 and R=3.2 we fit the condition of conservation of beam brightness, ε_{xyN} =1.

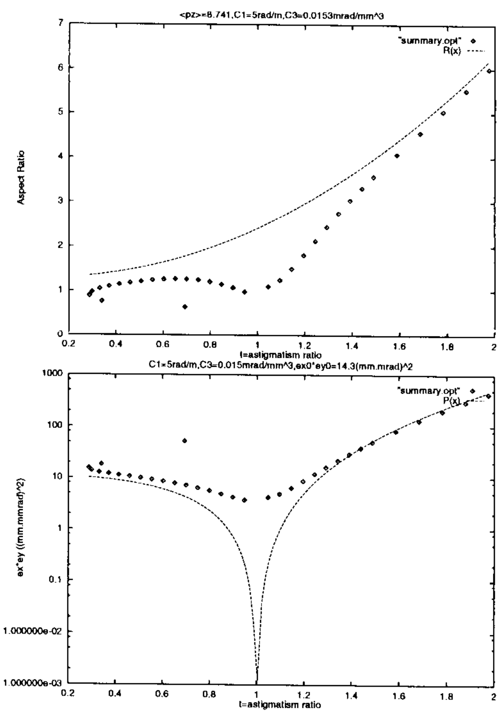


Fig. 5 and 6: aspect ratio and emittance product comparison (see text for further details).

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