ON THE FREQUENCY SCALINGS OF RF GUNS *

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Abstract

A frequency scaling law for RF guns is derived from the normalized Vlasov-Maxwell equations. It shows that higher frequency RF guns can generate higher brightness beams under the assumption that the accelerating gradient and all beam and structure parameters are scaled with the RF frequency. Numerical simulation results using MAGIC confirm the scaling law. The scaling of wakefield is discussed. A discussion of the range of applicability of the law is presented.

I. INTRODUCTION

The brightness achieved by conventional DC guns followed by RF bunchers may not meet the stringent requirements of future high-energy linear colliders and free electron lasers. This has motivated research on high brightness photocathode RF guns [1]. The frequencies of operational systems range from 500MHz to 3GHz, and a 17GHz gun [2] is to be tested shortly. In this paper we examine the frequency scaling of RF guns. Our analysis is based on the Vlasov-Maxwell equations. By defining coordinates normalized with the RF frequency, all the frequency dependencies are absorbed into two parameters: α^{rf}, which characterizes the accelerating gradient, and α^{sc}, which characterizes the space-charge force. The resultant dimensionless Vlasov-Maxwell equations have the same solution for any frequency as long as α^{rf}, α^{sc} and the functional form of the initial distribution in phase space are fixed. This gives a frequency scaling which shows that higher brightness bunches can be generated at higher frequencies, albeit with less total charge. Our conclusions are confirmed by numerical simulation. Practical limits on high frequency operation are discussed.

II. NORMALIZED VLASOV-MAXWELL EQUATIONS

Consider a RF gun operated at frequency ω. Since collisions are insignificant for the parameters of the RF gun, the electron distribution function, $f(\boldsymbol{x},\boldsymbol{p},t)$, evolves according to the Vlasov equation:

$$\left\{\frac{\partial}{\partial t}+\boldsymbol{v}\cdot\frac{\partial}{\partial \boldsymbol{x}}+e\left(\boldsymbol{E}+\frac{\boldsymbol{v}\times\boldsymbol{B}}{c}\right)\cdot\frac{\partial}{\partial \boldsymbol{p}}\right\}f(\boldsymbol{x},\boldsymbol{p},t)=0. \tag{1}$$

The fields $\boldsymbol{E}$ and $\boldsymbol{B}$ are determined self-consistently from Maxwell equations and an appropriate boundary condition. The boundary condition on a perfectly conducting wall, namely that the transverse electric field must vanish on the surface S, can be written as

$$\hat{\boldsymbol{n}}\times\boldsymbol{E} = 0, \tag{2}$$

where $\hat{\boldsymbol{n}}$ is normal to the surface. The total fields $\boldsymbol{E}$ and $\boldsymbol{B}$ can be split into two parts

$$\boldsymbol{E} = \boldsymbol{E}^{rf}+\boldsymbol{E}^{sc}, \tag{3}$$
$$\boldsymbol{B} = \boldsymbol{B}^{rf}+\boldsymbol{B}^{sc}, \tag{4}$$

where $\boldsymbol{E}^{rf}$ and $\boldsymbol{B}^{rf}$ are the fields in the absence of the beam and $\boldsymbol{E}^{sc}$ and $\boldsymbol{B}^{sc}$ are the fields due to the electron beam. Since the RF fields $\boldsymbol{E}^{rf}$ and $\boldsymbol{B}^{rf}$ already satisfy Maxwell equations and the boundary condition, $\boldsymbol{E}^{sc}$ and $\boldsymbol{B}^{sc}$ satisfy

$$\nabla\times\boldsymbol{E}^{sc} = -\frac{1}{c}\frac{\partial}{\partial t}\boldsymbol{B}^{sc} \tag{5}$$
$$\nabla\times\boldsymbol{B}^{sc} = \frac{1}{c}\frac{\partial}{\partial t}\boldsymbol{E}^{sc}+\frac{4\pi e}{c}\int \boldsymbol{v}f(\boldsymbol{x},\boldsymbol{p},t)d^3\boldsymbol{p} \tag{6}$$
$$\nabla\cdot\boldsymbol{E}^{sc} = 4\pi e\int f(\boldsymbol{x},\boldsymbol{p},t)d^3\boldsymbol{p} \tag{7}$$
$$\nabla\cdot\boldsymbol{B}^{sc} = 0, \tag{8}$$

together with the boundary condition $\hat{\boldsymbol{n}}\times\boldsymbol{E}^{sc}=0$.

The total number of electrons in the cavity, $N(t)$, can be found from the distribution function $f(\boldsymbol{x},\boldsymbol{p},t)$:

$$N(t) = \int f(\boldsymbol{x},\boldsymbol{p},t)d^3\boldsymbol{x}d^3\boldsymbol{p}. \tag{9}$$

It is convenient to define $\tau=\omega t$, $\boldsymbol{\xi}=\frac{\omega\boldsymbol{x}}{c}$, and $\boldsymbol{\beta}=\frac{\boldsymbol{v}}{c}$. The RF fields are now

$$\boldsymbol{E}^{rf} = E_0\boldsymbol{e}(\boldsymbol{\xi},\tau), \tag{10}$$
$$\boldsymbol{B}^{rf} = E_0\boldsymbol{b}(\boldsymbol{\xi},\tau), \tag{11}$$

where $\boldsymbol{e}$ is a dimensionless field profile and $\boldsymbol{b}$ is determined from Faraday's law

$$\boldsymbol{b}(\boldsymbol{\xi},\tau) = -\int^{\tau}\nabla_{\xi}\times\boldsymbol{e}(\boldsymbol{\xi},\tau')d\tau'. \tag{12}$$

The normalized electron distribution

$$\tilde{f}(\boldsymbol{\xi},\boldsymbol{\beta}\gamma,\tau) = \frac{1}{N(t)}(\frac{mc^2}{\omega})^3 f(\boldsymbol{x},\boldsymbol{p},t), \tag{13}$$

is defined so that

$$\int\tilde{f}(\boldsymbol{\xi},\boldsymbol{\beta}\gamma,\tau)d^3\boldsymbol{\xi}d^3(\boldsymbol{\beta}\gamma) = 1. \tag{14}$$

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With the definitions

$$\alpha^{rf} \equiv \frac{eE_0}{mc\omega}, \quad (15)$$

$$\alpha^{sc} \equiv \frac{4\pi\omega r_e}{c}N(\tau), \quad (16)$$

$$\boldsymbol{e}^{sc} \equiv \frac{\boldsymbol{E}^{sc}}{E_0}, \quad (17)$$

$$\boldsymbol{b}^{sc} \equiv \frac{\boldsymbol{B}^{sc}}{E_0}, \quad (18)$$

where $r_e = e^2/mc^2$ is the classical electron radius, we obtain the normalized Vlasov equation

$$\{\frac{\partial}{\partial\tau} + \boldsymbol{\beta}\cdot\nabla_\xi + \alpha^{rf}\left[(\boldsymbol{e}+\boldsymbol{e}^{sc})\boldsymbol{\beta}\times(\boldsymbol{b}+\boldsymbol{b}^{sc})\right]\cdot\nabla_{\beta\gamma}\} \tilde{f}(\boldsymbol{\xi},\boldsymbol{\beta}\gamma,\tau) = 0. \quad (19)$$

In Eq. (19), the normalized space-charge fields $\boldsymbol{e}^{sc}$ and $\boldsymbol{b}^{sc}$ satisfy the normalized Maxwell equations

$$\nabla_\xi \times \boldsymbol{e}^{sc} = -\frac{\partial}{\partial\tau}\boldsymbol{b}^{sc}, \quad (20)$$

$$\nabla_\xi \times \boldsymbol{b}^{sc} = \frac{\partial}{\partial\tau}\boldsymbol{e}^{sc} + \frac{\alpha^{sc}}{\alpha^{rf}}\int \boldsymbol{\beta}\tilde{f}(\boldsymbol{\xi},\boldsymbol{\beta}\gamma,\tau)d^3(\boldsymbol{\beta}\gamma), \quad (21)$$

$$\nabla_\xi \cdot \boldsymbol{e}^{sc} = \frac{\alpha^{sc}}{\alpha^{rf}}\int \tilde{f}(\boldsymbol{\xi},\boldsymbol{\beta}\gamma,\tau)d^3(\boldsymbol{\beta}\gamma), \quad (22)$$

$$\nabla_\xi \cdot \boldsymbol{b}^{sc} = 0, \quad (23)$$

with the boundary condition $\hat{\boldsymbol{n}} \times \boldsymbol{e}^{sc} = 0$. Note that Eq. (19)–(23) are dimensionless and all frequency dependencies have been absorbed in the normalized field strengths α^{rf} and α^{sc}.

The performance of an RF gun can be easily scaled in frequency. As long as α^{rf}, α^{sc}, and the initial particle distribution in the normalized phase space are fixed, the particle distribution in the normalized phase space is independent of the frequency. In other words, if the accelerating gradient is proportional to the frequency (so that α^{rf} is fixed, see Eq.(15)), the total bunch charge is inversely proportional to the frequency (so that α^{sc} is fixed, see Eq.(16)), and (for a photocathode gun) the laser temporal and spatial profiles scale with wavelength (which fixes the initial particle distribution in normalized phase space), then the current, beam divergence, mean energy, and energy spread will be invariant, the beam radius, bunch length, and emittance will be inversely proportional to the frequency, and therefore the electron beam brightness and laser power density will be proportional to the frequency square. These scalings are summarized in Table 1.

III. SCALINGS OF WAKEFIELDS

Some wakefield effects have been included in our formulation when we impose the perfectly conducting boundary condition. The effects of finite conductivity will be dealt with later in this paper. Our frequency scalings hold not only for RF guns, but also for general accelerating structures. However, they seem to differ from traditional wakefield scalings.

Traditional wakefild theory [5] assumes an axisymmetric disk-loaded structure and uses a cylindrical coordinate (r,ϕ,z) to treat the problem. Assume a is the radius of the disk opening,

parameter	scaling
cavity dimensions	ω^{-1}
accelerating gradient	ω^1
peak current	ω^0
bunch charge	ω^{-1}
bunch energy	ω^0
bunch energy spread	ω^0
bunch emittance	ω^{-1}
bunch radius	ω^{-1}
bunch length	ω^{-1}
bunch divergence	ω^0
bunch brightness	ω^2
laser peak power	ω^0

Table I
Frequency scaling of a photocathode RF guns.

r_q is the transverse displacement of the particle, E_{0n} is the field strength of the n- the traveling wave mode at the disk opening, and ω_n, m, and w_n are the frequency, azimuthal wavenumbeer, and the field energy per unit length of that mode, respectively. Then the longitudial and transverse wakefields can be written as [7]

$$w_{\parallel n}(\rho,\phi,\tilde{\tau}_n) = -2qk_n\rho_q^m\rho^m\cos m\phi\cos\tilde{\tau}_n, \quad (24)$$

$$\boldsymbol{w}_{\perp n}(\rho,\phi,\tilde{\tau}_n) = \hat{\boldsymbol{r}}2m\frac{k_n c}{\omega_n a}\rho_q^{m-1}\rho^m\cos m\phi\cos\tilde{\tau}_n - \hat{\boldsymbol{\phi}}2m\frac{k_n c}{\omega_n a}\rho_q^{m-1}\rho^m\sin m\phi\sin\tilde{\tau}_n, \quad (25)$$

where $k_n = E_{on}^2/(4w_n)$ is the loss parameter, and $\rho = r/a$, $\rho_q = r_q/a$, and $\tilde{\tau}_n = \omega_n(t - z/c)$ are the normalized variables. Wilson [5] assumes that the transverse displacement r_q is fixed regardless of the operating frequency ω_n. This regards the transverse displacement as limited by practical constraints such as the machining tolerance and the alignment. However, if the transverse displacement is scaled as ω_n and thus ρ and ρ_q are fixed, the scalings will be

$$w_{\parallel n} \sim \omega_n^2 \quad \text{for any } m, \quad (26)$$

$$w_{\perp n} \sim \omega_n^2 \quad \text{for any } m. \quad (27)$$

These are scalings of wakefields induced by a single particle. For a bunch of particles, the total wakefield is obtained by integrating the wakefields induced by all particle. Assuming the functional form of the particle distribution in the normalized coordinate $(\rho_q,\phi,\tilde{\tau}_n)$ is fixed regardless of ω_n yet the magnitude is scaled as ω_n^{-1} (so that the total charge scales as ω_n^{-1}), then the total wavefield will scale as

$$\boldsymbol{w}_n^{tot} \sim \omega_n^1 \quad \text{for any } m, \quad (28)$$

with ρ, $\tilde{\tau}_n$, and ϕ being fixed. Therefore, the total wakefield scales in the same way as the RF field and the space-charge field.

RF frequency (GHz)	2.856	17.136
Peak field on cathode (MV/m)	41.7	250
RF phase for laser pulse	$12°$	$12°$
Exit beam energy (MeV)	2.1	2.1
Energy spread (%)	0.22	0.22
Laser spot radius (mm)	2.9	0.49
Laser pulse length (ps)	9.6	1.6
Charge in bunch (nC)	0.6	0.1
Beam emittance (πmm-mrad)	6.6	1.1
Beam bunch length (mm)	0.96	0.16
Beam radius (mm)	4.0	0.67
Beam divergence (mrad)	19	19
Peak current (A)	180	180
Beam brightness (A/π^2m^2)	4.1×10^{12}	1.5×10^{14}

Table II

MAGIC simulation results for BNL's 2.856GHz photocathode RF gun and a scaled gun at 17.136GHz.

It confirms that our frequency scalings are correct even when the wakefields are included, as long as the beam size and total charge are scaled down with frequency.

Nonetheless, the field profile of the n- the traveling wave mode will change slightly when we scale the operating frequency, since the skin-depth does not scale linearly with freqeuncy. On way to study the skin-depth effect on RF gun beam dynamics is to use numerical simulation. As will be shown in next section, the skin-depth effect in RF guns is negligible in the regime of practical interest, since the scaling law still holds when the skin-depth effect is taken into account.

IV. NUMERICAL RESULTS AND DISCUSSION

Numerical simulations with the particle-in-cell code MAGIC confirm the frequency scaling. The performance of the BNL photocathode RF gun [3] operated at 2.856GHz was compared with that of a scaled BNL design at 17.136GHz ($\times 6$ higher frequency). The results are summarized in Table 2.

The practical assumptions made herein are (1) that very high gradients can be (more readily) achieved at higher frequencies, (2) that the bunch length and beam size can be scaled inversely with frequency, (3) that the laser peak power remains below the damage threshold to the cathode, and (4) that that changes to coupling geometry do not affect the field profile in the cavity. This implies that sufficiently powerful RF sources are available at the higher frequency, and that short laser pulses can be mode locked to RF with the same degree of relative error in temporal and spatial jitters. Since the skin depth does not scale linearly with frequency, coupling from the RF source to the cavity cannot be linearly scaled.

In summary, while high frequency systems may be difficult to realize technically, they may prove to be compact sources of high brightness bunches.

References

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