# Computer Simulation of the Tevatron Crystal Extraction Experiment 

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#### Abstract

The Fermilab crystal-extraction experiment E853 at Tevatron was simulated by Monte Carlo code CATCH [1] tested earlier in the CERN-SPS experiment [2-4]. Predictions for the extraction efficiency, angular scans and extracted beam profiles are presented. Several ideas are proposed and tested by the simulation, how to get in E853 the key information of the extraction experiment: the "septum width" of a crystal and dependence of extraction efficiency on it, the impact parameters of protons at crystal, and the contribution of the first and multi passes to the extraction. The ways to optimize E853 are analyzed.


## I. Introduction

The crystal-extraction experiments at CERN SPS [2-4] and Fermilab Tevatron [5] have in view possible application of channeling for proton extraction from a multi-TeV machine [6]. The technique employs a bent crystal placed in the beam halo, which traps and bends the particles parallel to the crystallographic plane within Lindhard angle $\theta_{c}$. The halo particles hit a crystal very close to its edge, with impact parameter $b$ in the range $\sim \AA$ to $\sim \mu \mathrm{m}$. This calls for a good perfection of the crystal edge. Alternatively, one should investigate how crystal extracts particles in the multipass mode, which involves several scatterings in the crystal of the circulating particles.

As the extraction includes many passes, there is no easy way to extrapolate the results with energy. This makes the detailed comparison of the measurements with computer simulation essential. Such an analysis made [7] for the CERN-SPS experiment has shown good agreement of the theory with measurements [2,3]. The major result of [7] was a prediction of the edge imperfection of the crystals used at SPS. The new SPS experiment, employing a crystal with an amorphous edge-layer to testify this idea, has proved much the same efficiency indeed [4]. Another prediction, made for the "U-shaped" crystal - much the same efficiency but narrow ( $70 \mu \mathrm{rad} f w h m$ ) angular scan[7] has also been confirmed [3]. With the simulation code [1] tested at SPS, here we model the extraction of $900-\mathrm{GeV}$ protons from Tevatron, matching E853 [5,8].
The real crystal has an irregularity of the surface, which defines a range of inefficient $b$ at the edge ("septum width" $t$ ). The following information is essential for understanding the crystal extraction process:
(a) efficiency $F$, and contributions to it from the first $/$ multi passes; (b) distribution over $b$ at the crystal; (c) septum width $t$; (d) dependence $F(t)$.

We propose the ways to get this information in E853.
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## II. Qualitative discussion

The essential feature of E853 is the fact that the crystal atomic planes are perpendicular to the crystal face in touch with the beam. In E853 one should align a crystal in 2 planes: the channeling plane (vertical, $y$ ) with the accuracy of $\theta_{c}$, and the horizontal plane $(x)$ to keep the crystal face parallel to the incident protons (Fig. 1). At first glance, the need to tune two angles is an inconvenience. Here we show that this extra degree of freedom is an excellent possibility to study extraction in many details!


Figure 1. Horizontally tilted crystal: (a) aligned, (b) tilt $x^{\prime}>0$, (c) $x^{\prime}<0$.

Consider a crystal disaligned from the beam horizontally at $x^{\prime}$. Depending on the sign of $x^{\prime}$, either the upstream end approaches the beam (we define $x^{\prime}<0$ ), or the downstream one $\left(x^{\prime}>0\right)$; fig. 1. Because of $x^{\prime}$, a septum width as thick as $t=\left|x^{\prime}\right| L$ occurs at the crystal edge; $L$ is the crystal length ( 4 cm in E853). Protons incident in the range $0<b<t$ do not traverse the full length of crystal. The result depends dramatically on $x^{\prime}$ sign.
In case of $x^{\prime}>0$, protons traverse the downstream edge. It is disaligned by $\sim 0.64 \mathrm{mrad}$ (the bending angle) w.r.t. the beam. Therefore, protons traverse it like an amorphous substance. This case imitates a crystal with an amorphous near-surface layer as wide as $t \approx x^{\prime} L$. Measuring $F\left(x^{\prime}\right)$ for $x^{\prime}>0$, one measures $F(t)$. Theory [9] predicts very weak $F(t)$ dependence at high energies. The confirmation would be encouraging for the multiTeV crystal extraction. Notice that the step of $t$ scan could be very fine: with $\delta x^{\prime}=2.5 \mu \mathrm{rad}$ and $L=4 \mathrm{~cm}$ one has $\delta t=0.1 \mu \mathrm{~m}$.
In case of $x^{\prime}<0$, protons traverse the upstream edge which is aligned w.r.t. the beam. Therefore, many particles are trapped in channeling. However, those incident in the range $0<b<x^{\prime} L$ traverse a reduced ( $<4 \mathrm{~cm}$ ) length, thus getting a reduced ( $<0.64$ $\mathrm{mrad})$ deflection and therefore are lost. The inequality of two cases, $x^{\prime}>0$ and $x^{\prime}<0$, causes a strong asymmetry of $F\left(x^{\prime}\right)$ dependence. The difference $\Delta F=F\left(x^{\prime}\right)-F\left(-x^{\prime}\right)$ is proportional to the number of protons incident with $0<b<x^{\prime} L$. Varying $x^{\prime}$
and observing $\Delta F$, one investigates the distribution over $b$ at crystal, with accuracy of $\delta b=0.1 \mu \mathrm{~m}$.

This is complicated by another interesting phenomenon. The protons incident on an aligned imperfect crystal with $b_{\max }<t$, have to traverse the full length of the crystal, and to experience a substantial nuclear scattering. Suppose, this crystal is disaligned so that $b_{\max } / x^{\prime} L \approx 0.1$. Then at first incidence the protons traverse only the crystal edge, with the "length" $\leq 0.1$ that of crystal. The respective scattering and losses over 0.1 L are much smaller. In this case the protons retain better chances for extraction with later passes than in the former case (perfect alignment). The secondary $b$ of the scattered protons are still sufficiently large ( $\approx 30 \mu \mathrm{~m} \gg x^{\prime} L$ here ), so the "gap" $x^{\prime} L$ is not dangerous.

We come to conclusion that a peak efficiency with imperfect crystal is achieved at some tilt $x^{\prime} \neq 0$. In the real experiment one scans $x^{\prime}$ while searching the peak, and comes to this case automatically. We used the case $b_{\max } / x^{\prime} L=0.1$ as an illustration; the optimal $x^{\prime}$ will be found automatically in the scan. Further on, we refer to this case as to the "pre-scatter" case, when protons first gently pre-scatter in the crystal edge to come later with low divergence but high $b$. Understandably, with imperfect crystal the prescatter case may appear also for a small negative tilt, $x^{\prime}<0$. Then, $F\left(x^{\prime}\right)$ may have two peaks, with a dip at $x^{\prime}=0$. The width of the dip at $x^{\prime} \approx 0$ is also an indicator for $b_{\text {max }}$.

## III. Simulation procedure

The crystal was located 61 m upstream of C 0 point of Tevatron lattice, with the edge at the horizontal distance of $X=1.75 \mathrm{~mm}$ from the beam axis. At the crystal location, the machine parameters were $\beta_{x}=105.7 \mathrm{~m}, \alpha_{x}=0.109$ (horizontally), and $\beta_{y}=21.5 \mathrm{~m}, \alpha_{y}=0.148$ (vertically); tunes $Q_{x}=20.5853$ and $Q_{y}=20.5744$. The beam invariant rms emittance was 2.5 $\mathrm{mm} \cdot \mathrm{mrad}$, which corresponds to vertical rms divergence 11.5 $\mu \mathrm{rad}$ and width 0.24 mm at the crystal location.

Crystal was a $\mathrm{Si}(110)$ slab 40 by 3 by $3 \mathrm{~mm}^{3}, 0.64$ mrad bent, with a perfect lattice, and curved with a constant curvature to deflect protons in vertical direction. As an option, we model an amorphous layer at its edge and/or irregularities of the surface. The horizontal parameters $x, x^{\prime}$ of incident particles are defined by the mechanism of diffusion. The two processes, diffusion and crystal extraction, are unfold in E853. Beam parameters in the channeling plane (vertical) are not disturbed by the diffusion. The exact value of $b_{\text {max }}$ matters only w.r.t. $t$. Since $t$ is unknown for the real crystal, we can postulate $b_{\max }=1 \mu \mathrm{~m}$, and then model crystals with different $t$.

## IV. Results

Fig. 2 shows the $F\left(y^{\prime}\right)$ angular scan for $x^{\prime}=0$. The peak $F$ of an ideal crystal is $\simeq 44 \%$. The same fig. shows scans for the crystals with $t=1 \mu \mathrm{~m}$ (i.e. $t=b_{\max }$ ) and $t=50 \mu \mathrm{~m}$, where at $y^{\prime}=x^{\prime}=0 \quad F \approx 36 \%$ and $32 \%$ respectively. However, for the imperfect crystals the real peak was found at $x^{\prime} \neq 0$ (Fig. 3). With optimized $x^{\prime}$, one has peak $F \approx 42 \%$ and $35 \%$ for $t=1 \mu \mathrm{~m}$ and $t=50 \mu \mathrm{~m}$ respectively. Notably, the efficiencies and scans are quite weakly dependent on the crystal perfection. The width of $y^{\prime}$ scan was $50-55 \mu \mathrm{rad} f w h m$ in these cases.


Figure 2. $\quad F\left(y^{\prime}\right)$ scan for $x^{\prime}=0$. Ideal crystal: (o) first-pass and ( $\bullet$ ) overall efficiencies. Imperfect crystal: overall efficiency with $t=1 \mu \mathrm{~m}(\star)$ and $t=50 \mu \mathrm{~m}(*)$.

The angular scan $F\left(x^{\prime}\right)$ is in Fig. 3. The depth of the dip at $x^{\prime} \approx 0$ (i.e. at perfect alignment) is $\simeq 14 \%$ and $\simeq 7 \%$ w.r.t. the peak for $t=1 \mu \mathrm{~m}$ and $t=50 \mu \mathrm{~m}$ respectively. The width $\Delta x^{\prime}$ of the peculiarity (either peak or dip) near $x^{\prime} \approx 0$ is roughly $b_{\text {max }} / L$ which is $25 \mu \mathrm{rad}$ in our simulation. $F$ is $1 / 2$ of the


Figure 3. $F\left(x^{\prime}\right)$ scan near the peak; see fig. 2 caption.
maximum at $x^{\prime} \simeq 14 \mathrm{mrad}$ and -0.3 mrad for an ideal crystal (fwhm of the horizontal scan is $\simeq 14 \mathrm{mrad}$ ), at $x^{\prime} \simeq 15 \mathrm{mrad}$ and -1.2 mrad for the crystal with $t=1 \mu \mathrm{~m}$ ( $f w h m \simeq 16 \mathrm{mrad}$ ), and at $x^{\prime} \simeq 18 \mathrm{mrad}$ and -5 mrad for the crystal with $t=50 \mu \mathrm{~m}$ ( $f w h m \simeq 23 \mathrm{mrad}$ ).

The asymmetry of the scan, $F\left(x^{\prime}\right) \neq F\left(-x^{\prime}\right)$, is due to the loss of the protons trapped in channeling near the crystal edge. With an ideal crystal, the asymmetry exists for any $x^{\prime}$. With a septum width $t$, the asymmetry can be seen for an angling $\pm x^{\prime}$ larger than $t / L$ only. In our simulation with $t=50 \mu \mathrm{~m}$, the scan is symmetric indeed within $\pm 1.3 \mathrm{mrad}$ but asymmetric outside this range of $x^{\prime}$; note that $50 \mu \mathrm{~m} / 40 \mathrm{~mm}=1.25 \mathrm{mrad}$. We expect therefore this $x^{\prime}$-threshold for an asymmetry to be a good measure of the septum width $t$. If one plots the magnitude of asymmetry, $F\left(x^{\prime}\right)-F\left(-x^{\prime}\right)$, as a function of $x^{\prime} L$, he obtains a rough estimate of the beam distribution over $b$ at crystal. The minimal step $\delta b=\delta x^{\prime} L=0.1 \mu \mathrm{~m}$ is much finer than the precision of coordinate detectors $\approx 0.1 \mathrm{~mm}$ !

Notice an abrupt decrease in $F$ of the ideal crystal over the range of $x^{\prime} L$ from 0 to $-b_{\max }$ : from $44 \%$ at $x^{\prime}=0$ to $28 \%$ at $x^{\prime}=-b_{\text {max }} / L$. This drop is an excellent opportunity to measure


Figure 4. $F(L)$ for ideal (o) and $t=1 \mu \mathrm{~m}(\bullet)$ crystals.
the primary $b_{\text {max }}$ with a precision of $\delta b=0.1 \mu \mathrm{~m}$. With an ideal crystal one can measure a distribution over the primary $b$ (in the range of $\sim 1 \mu \mathrm{~m}$ ). With imperfect crystal in the same way one measures the distribution over secondary $b$ (in a broad range from $\sim t$ to $\sim 1 \mathrm{~mm})$. Finally, the dependence $F\left(x^{\prime}\right)$ for $x^{\prime}>0$ gives actually the dependence of $F$ on the septum width $t \simeq$ $x^{\prime} L$.

The distribution of the extracted particles over $x$ at the crystal face is essential for understanding both the crystal interplay with other accelerator elements and the requirements for the crystal face perfection. We have found that one half of the extracted protons have penetrated into the crystal depth by $>0.3 \mathrm{~mm}$; another half had $b<0.3 \mathrm{~mm}$. The $y^{\prime}$ divergence of the extracted beam was defined by the channeling properties of $\operatorname{Si}(110)$ crystal; its full width $2 \theta_{c}$ was $\approx 12.8 \mu \mathrm{rad}\left(\theta_{c} \simeq 6.4\right.$ $\mu \mathrm{rad})$, and $f w h m \approx 9 \mu \mathrm{rad}$. The $x^{\prime}$ divergence was $\simeq 5 \mu \mathrm{rad} f w h m$ with the ideal crystal and $\sim 12 \mu \mathrm{rad} f w h m$ with $t=1 \mu \mathrm{~m}$. It was increased due to scattering in inefficient passes. After the doublet of quadrupoles and the Lambertson-type magnet, two detectors (hodoscopes with 0.1 mm bins) were placed at 80.5 m (D1) and 120.5 m (D2) downstream of the crystal to measure the bent-beam profiles. The horizontal profiles had width $\simeq 0.3$ and 0.4 mm fwhm for the ideal and $t=1 \mu \mathrm{~m}$ crystals at D1, and $\simeq 0.5$ and $0.7-0.9 \mathrm{~mm}$ fwhm at D2.

## V. Optimization

The extraction efficiency $F$ is defined by the processes of channeling, scattering, and nuclear interaction in crystal. All the processes depend essentially on the crystal length $L$. Fig. 4 shows $F(L) . F$ is maximal, near $70 \%$, in $L$ range $0.4-1.0 \mathrm{~cm}$, irrespective of the crystal perfection.

## VI. Conclusions

The key information of the multi-pass crystal extraction can be obtained from the analysis of the horizontal angular scan of efficiency. In the considered way one can study the impact parameters of halo particles and the structure of the crystal edge with an accuracy as fine as $0.1 \mu \mathrm{~m}$.

The extraction efficiency is expected as high as $\simeq 40 \%$ irrespective of the crystal septum width, and can be increased up to $\sim 70 \%$ with the use of a shorter $(\leq 1 \mathrm{~cm})$ crystal. The dif-
ference in efficiency between the ideal and imperfect crystals is very low, because of predominance of the multi-passes in extraction at high energies, and partly due to the found effect of a gentle "prescattering" in the edge of a crystal tilted horizontally. This provides an elementary solution to the problem of a finite septum width and infinitesimal impact parameters.

One general trend in the results of simulations, from SPS [7] to Tevatron to LHC [6], is worthwhile to mention: the difference in efficiency of the ideal crystal and crystal with imperfect surface vanishes with energy $E$, because the scattering angle reduces faster $(\sim 1 / E)$ than $\theta_{c}$ does $(\sim 1 / \sqrt{E})$.

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