ANALYTIC COMPUTATION OF BEAM IMPEDANCES IN COMPLEX HETEROGENOUS ACCELERATOR GEOMETRIES

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Abstract

A general framework has been developed for computing longitudinal and transverse beam impedances in accelerator pipes consisting of several coaxial tubes with non simple transverse geometry, possibly made of composite materials and/or bearing special features like e.g. holes or slots, based on the combined use of Lorentz reciprocity theorem, Debye potentials, extended impedance boundary conditions, and generalized trasmission line (waveguide) circuit concepts. The results are applied to the proposed LHC design.

I. INTRODUCTION

Rounded corners, multi-layered or composite walls, pumping holes, etc., make accelerator cross-sectional pipe geometries *not simple*. Beam coupling impedances must then computed by numerical methods, analytic solutions being available only for simple (transverse) geometries where, e.g., the Laplacian is separable, and simple (e.g., perfect conductor) boundary conditions. Analytic, even approximate, solutions on the other hand are relatively appealing, as they provide an immediate insight into the role played by the design parameters.

In this paper we briefly summarize a general approach for the analytic computation of beam coupling impedances in complex structures, together with some representative results pertinent to the proposed LHC liner.

II. PERTURBED COUPLING IMPEDANCES

Stationary perturbative formulae for the beam (complex, frequency dependent) coupling impedances per unit length [1] of pipes with non-simple cross sections and/or boundary conditions can be obtained from the electromagnetic reciprocity (Lorentz) theorem, and relate the beam coupling impedance $Z^0_\parallel$, $Z^0_\perp$ of a *simple, unperturbed* pipe assumed known, to that of another pipe differing from the former by some *perturbation* in the boundary geometry and/or constitutive properties [2], [3][1]. They read:

$$Z_\parallel - Z^0_\parallel = \frac{\epsilon_0}{\beta_0 c Q^2}\left\{ Y_0 \oint_{\partial S} Z_{wall}\, E^{*irr.}_{0n} \cdot \right.$$
$$\left. \cdot \left[\beta_0 E^{irr.}_{0n} + \beta_0^{-1} E^{sol.}_{0n} \right] d\ell - \oint_{\partial S} E^*_{0z} E^{irr.}_{0n}\, d\ell \right\}, \quad (1)$$

for the longitudinal impedance, and:

$$\bar{\bar{Z}}_\perp - \bar{\bar{Z}}{}^0_\perp = \frac{\epsilon_0}{\beta_0 c Q^2 k} \lim_{\vec r_0, \vec r_1 \to 0} \left\{ Y_0 \oint_{\partial S} Z_{wall} \nabla_{\vec r_0} E^{irr.*}_{0n}(\vec r, \vec r_0) \otimes \right.$$
$$\nabla_{\vec r_1} \left[\beta_0 E^{irr.}_{0n} + \beta_0^{-1} E^{sol.}_{0n} \right] (\vec r, \vec r_1) d\ell +$$
$$\left. - \oint_{\partial S} \left[\nabla_{\vec r_0} E^*_{0z}(\vec r, \vec r_0) \otimes \nabla_{\vec r_1} E^{irr.}_{0n}(\vec r, \vec r_1) \right] d\ell \right\} \quad (2)$$

for the transverse one[2]. In (1) and (2) $\vec r$ is the transverse position, $\nabla_{\vec r}$ is the transverse gradient, $\beta_0 = v_0/c$=beam velocity/ligth velocity (in vacuum), ∂S is the pipe cross-section boundary, $E_0^{sol.}$, $E_0^{irr.}$ are the (known, k-domain) solenoidal and irrotational parts (Helmholtz theorem) of the electric field, in the *simple, unperturbed* pipe, Q is the beam charge[3], Y_0 is the free-space wave-admittance, Z_{wall} the (complex, frequency dependent) surface impedance describing the *local* properties of the pipe wall, and $k = \omega/\beta_0 c$.

The first integral on the r.h.s of (1) and (2) accounts for the effect of constitutive perturbations of the boundary, and thus is nonzero if and only if Z_{wall} is not identically zero on ∂S. The second integral on the r.h.s. of eq.s (1) and (2), on the other hand, accounts for the effect of geometrical perturbations of the boundary, and is non-zero if and only if the *unperturbed* axial field component E_{0z} is not identically zero on ∂S. Accordingly the second integral in (1) and in (2) effectively spans only the *geometrically perturbed* boundary subset $\partial S - \partial S_0$.

III. IMPEDANCE BOUNDARY CONDITIONS

Equations (1) and (2) are based on a simple Leontóvich (impedance) boundary condition (BC), at the pipe wall [4]:

$$\vec n \times (\vec n \times \vec E - Z_{wall} \vec H)|_{wall} = 0 \quad (3)$$

$\vec n$ being the local normal unit vector. In the spirit of Leontóvich BC, the penetration of EM fields from vacuum into *multilayered* lossy media can be viewed as lossy transverse electromagnetic (TEM) wave propagation in the direction (locally) normal to the interfaces.[4]

The equivalence between (TEM) waves in stratified media and voltage waves through cascaded transmission lines (TL) can thus be used to compute the wall impedance Z_{wall} at the inner surface of the beam screen, by repeated application of the impedance transport formula across a homegenous TL section with length ℓ characteristic impedance Z_c and propagation constant k:

$$Z_{in} = Z_c \frac{Z_\ell + j Z_c \tanh(jk\ell)}{Z_c + j Z_\ell \tanh(jk\ell)} \quad (4)$$

where Z_ℓ is the impedance connected to the output port, and Z_{in} is the impedance seen at the input port.

[1] Equations (1) and (2) are accurate for suitably *small* perturbations; they are exact whenever the coupling impedances depend linearly on Z_{wall}.

[2] Note that $\bar{\bar{Z}}_\perp$ is a tensor, in general. See [2], and references quoted therein.

[3] The impedances are obviously independent of Q, since the fields in (1) and (2) are proportional to Q.

[4] Leontóvich BC can be applied provided: *i)* the magnitude of the relative index of refraction of the (first) medium where the field penetrates is large, and *ii)* the penetration depth is small compared to the (minimum) thickness of the medium and the curvature radius of its boundary [5].

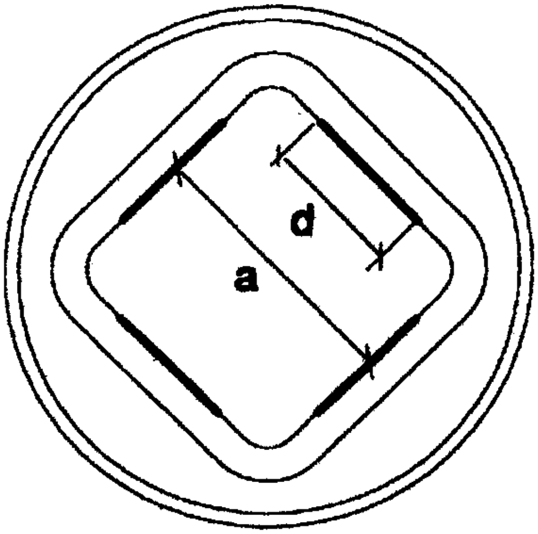

Figure 1. Tentative LHC design.

Even the boundary condition at a perfectly conducting screen bearing a regular array of holes can be modeled in terms of a wall impedance [6]:

$$Z_{wall} = -j\frac{Z_0 k_0}{s_t s_z}(\alpha_e + \alpha_m) \tag{5}$$

in agreement with [7], [8], where $\alpha_{e,m}$ are the (inside) electrical and magnetical polarizabilities of the holes (Bethe approximation implied, hole diameter $\ll$ wavelength) [5], and s_t, s_z are the inter-hole spacings in the transverse and longitudinal pipe directions, respectively.

IV. SOME RESULTS PERTINENT TO LHC

The tentative design of LHC is shown in Fig. 1 [6].

The main contribution to the beam impedances accordingly comes from the perforated stainless steel rounded corners of the beam screen. Using (1) and (2) and the exact solution for the fields produced by a relativistic particle traveling parallel to the axis of a perfectly conducting square-section pipe [10], one gets[7]:

$$Z_{||,\perp} - Z_{||,\perp\ 0} = C_{||,\perp}\ G^{(1)}_{||,\perp}(\xi) + D_{||,\perp}\ G^{(2)}_{||,\perp}(\xi), \tag{6}$$

where:

$$C_{||} = \frac{\epsilon_0^{-1} Y_0 Z_{wall}}{4\pi^2 ca} = ka^2 C_{\perp} \tag{7}$$

$$D_{||} = -jk\frac{(1-\beta_0^2)\epsilon_0^{-1}}{4\pi^2\beta_0 c} = ka^2 D_{\perp}, \tag{8}$$

The functions $G^{(1)}_{||,\perp}$ and $G^{(2)}_{||,\perp}$ are displayed in Fig.s 2.[8]. The wall impedance on the rounded corners of the LHC liner (7) can be computed from the equivalent TL circuit shown in Fig. 3, by repeated application of eq. (4),

Its real part is accordingly displayed in Fig. 4.

[5] For a multi-coaxial pipe, the polarizabilities should be computed *in the presence* of the outer shells [6].

[6] A square beam-screen has been chosen in view of its better performance in terms of Laslett tune shifts [9].

[7] For the present case the transverse impedance is proportional to the unit dyadic $\bar{\bar{I}}$, and can thus be described by a scalar.

[8] The second term in (6) related to boundary shape perturbation is imaginary, thus giving no contribution to power losses. As a space charge effect, it vanishes in the limit $\beta_0 \longrightarrow 1$.

The longitudinal impedance can be used to compute the energy lost by the beam per unit pipe length (parasitic loss, $\Delta\mathcal{E}/L$ [1]). For a Gaussian bunch with r.m.s. length σ_z, using eq. (1), one has [6]:

$$\frac{\Delta\mathcal{E}}{L} = \frac{a^2}{Q^2 c Z_0}\, W\left(\frac{\sigma_z}{a}\right) G^{(1)}_{||}\left(\frac{d}{a}\right) \tag{9}$$

where for LHC $a^2 Q^{-2} c^{-1} Z_0^{-1} = 41.91\ Joule\ m^{-1}$, the function $G^{(1)}_{||}$ has been already defined, and:

$$W\left(\frac{\sigma_z}{a}\right) = Y_0 \int_{-\infty}^{+\infty} e^{-\frac{\sigma_z^2}{a^2}\frac{y^2}{\beta_0^2}}\, Re\left[Z_{wall}\left(\frac{cy}{a}\right)\right] dy \tag{10}$$

is displayed in Fig. 5.

V. CONCLUSIONS

We introduced a general and systematic framework for computing beam coupling impedances and related quantities in possibly composite, multilayered, complex-shaped accelerator pipes, yielding accurate results in analytic form. We believe that the above could be a valuable tool for predicting the performance and optimizing the design of planned and/or existing accelerators.

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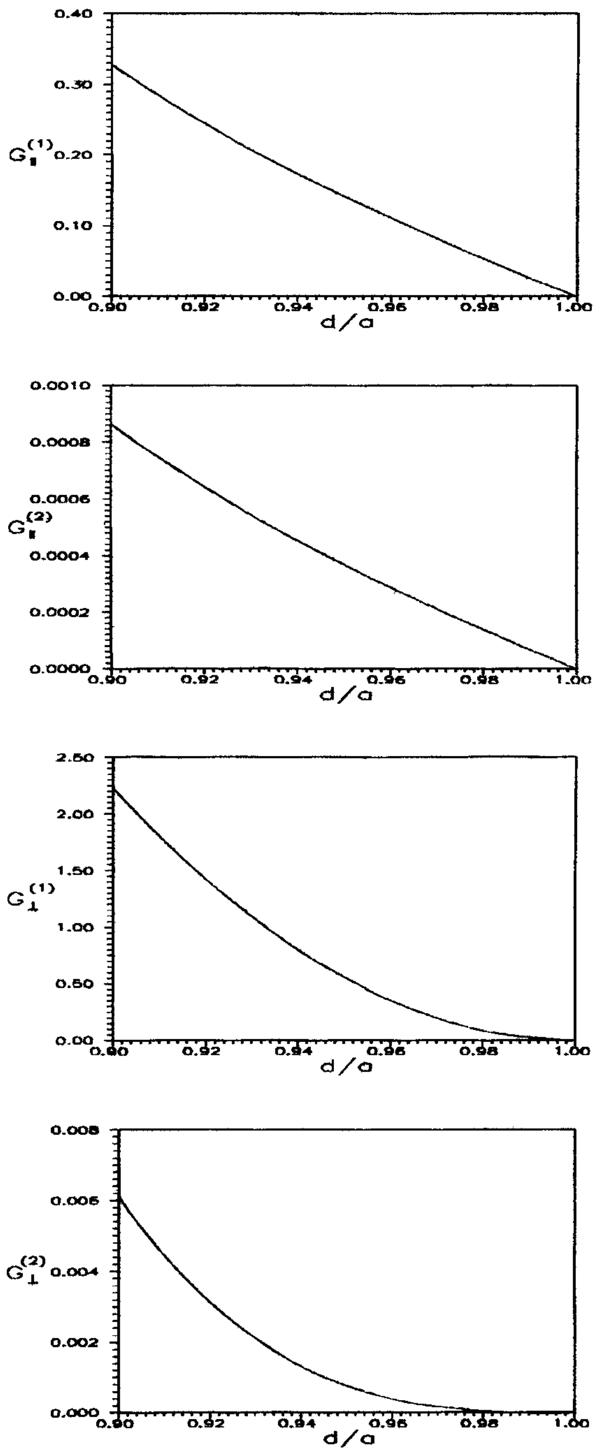

Figure 2. The functions $G_{||,\perp}^{(1)}$ and $G_{||,\perp}^{(2)}$.

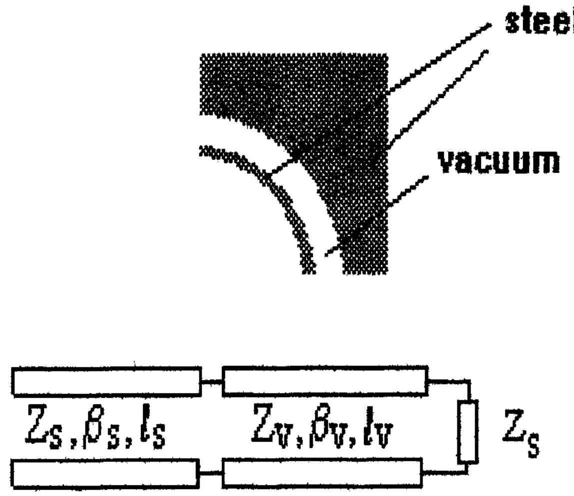

Figure 3. Multilayered wall and TL equivalent circuit.

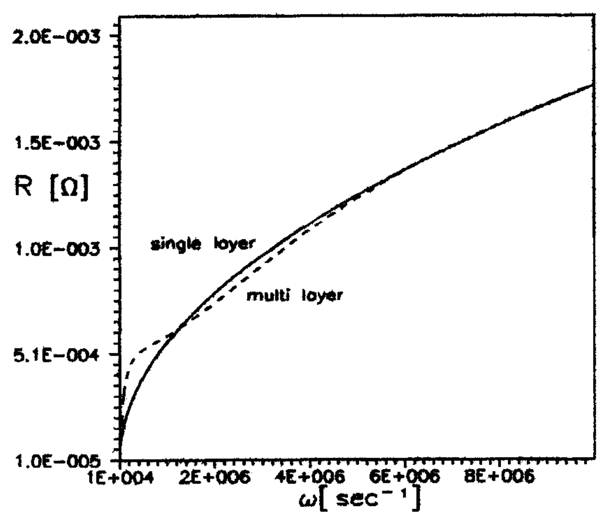

Figure 4. LHC wall impedance (rounded corners), realpart.

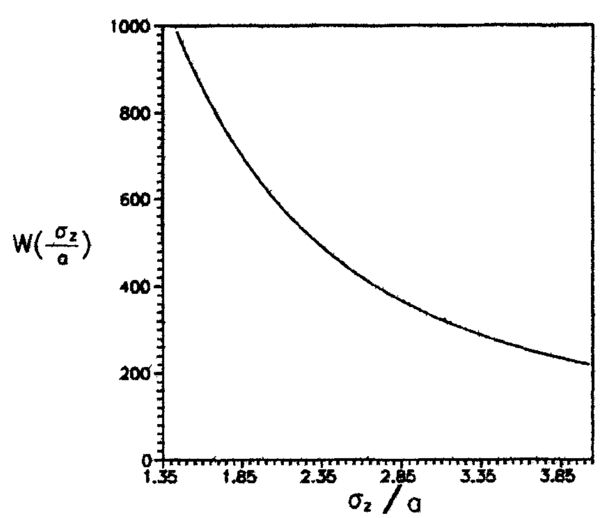

Figure 5. The function $W(\sigma_z/a)$.